Discrete Symmetries

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5 Golden Rules (2004)

- Spinors of SO(10) (for families)
- Incomplete multiplets (for Higgs)
- Repetition of families (from extra dimensions)
- $N = 1$ supersymmetry
- Importance of discrete symmetries

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These rules have bottom-up and top-down motivation

- grand unification (evolution of couplings)
- quark and lepton (neutrino) masses
- proton stability (R-Parity)

Rule 1 and 5

- Spinors if SO(10) might be important even in absence of GUT gauge group
- one can incorporate top-Yukawa coupling and neutrino see-saw mechanism
- discrete symmetries with many applications

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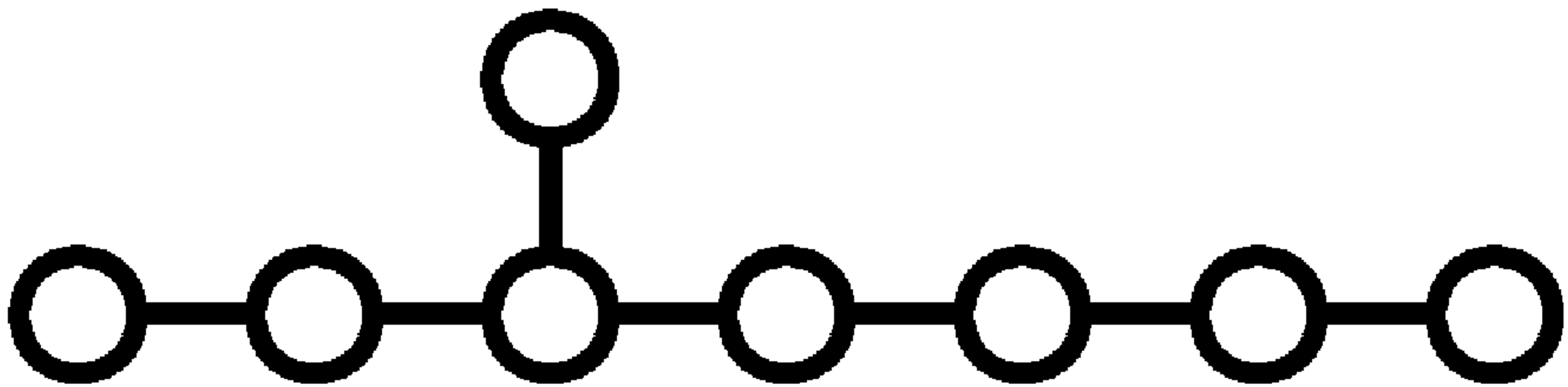
From the mathematical structure we would prefer exceptional groups

- There is a maximal group: E_8,
- but E_8 and E_7 do not allow chiral fermions in $d = 4$.
- How does this fit with our usual picture of unification based on SU(5) or SO(10)?

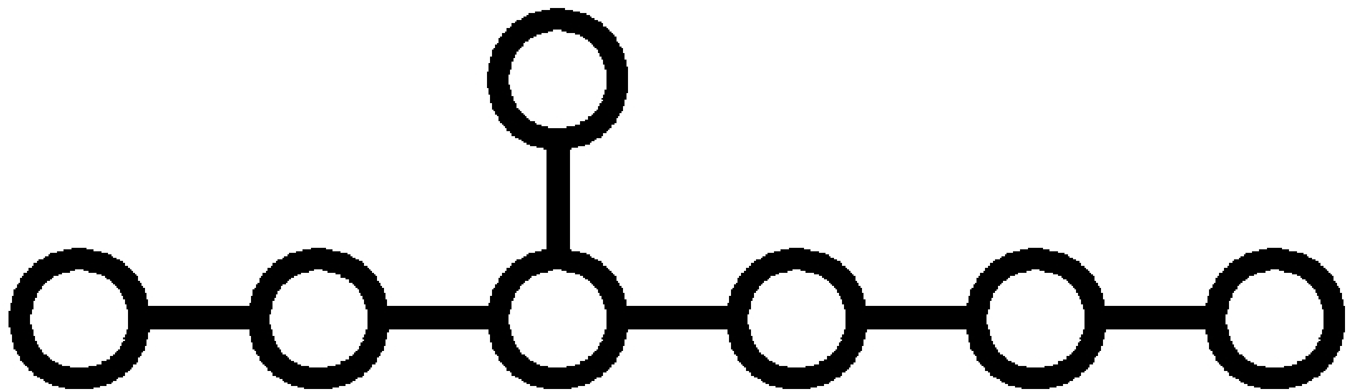
Maximal Group



E_8 is the maximal group.

There are, however, no chiral representations in $d = 4$.

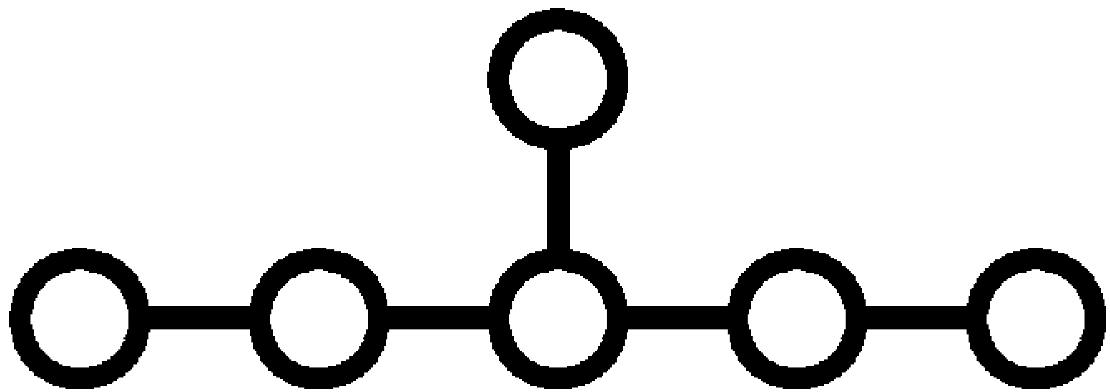
E_7



Next smaller is E_7.

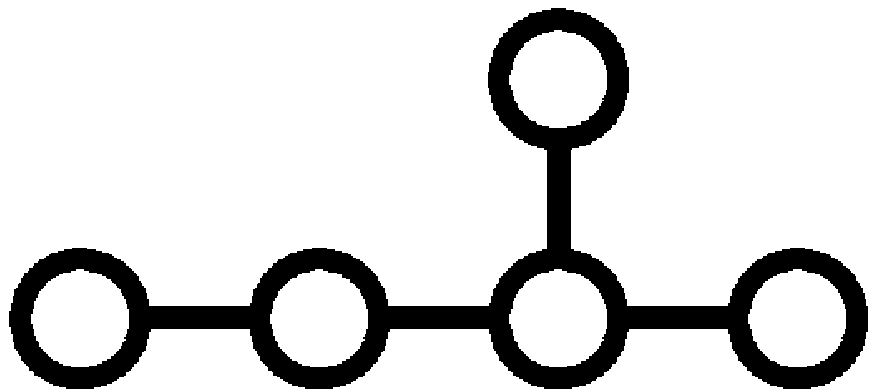
No chiral representations in $d = 4$ either

E_6



E_6 allows for chiral representations even in $d = 4$.

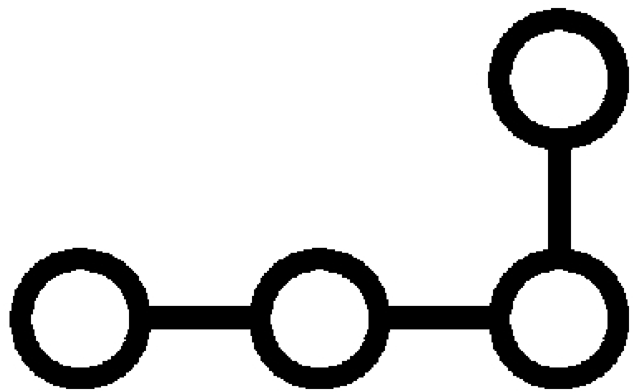
$E_5 = D_5$



E_5 is usually not called exceptional.

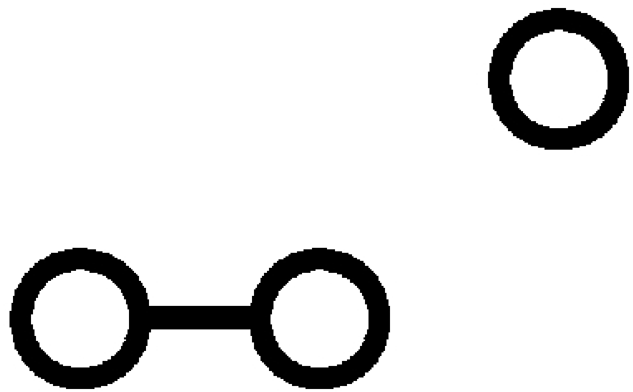
It coincides with $D_5 = SO(10)$.

$E_4 = A_4$



E_4 coincides with $A_4 = SU(5)$

E_3



E_3 coincides with $A_2 \times A_1$ which is $SU(3) \times SU(2)$.

Exceptional groups in string theory

String theory favours E_8

- $E_8 \times E_8$ heterotic string
- E_8 enhancement as a nonperturbative effect (M- or F-theory)

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Strings live in higher dimensions:

- chiral spectrum possible even with E_8
- E_8 broken in process of compactification
- provides source for more discrete symmetries
- from $E_8/SO(10)$ and $SO(6)$ of the higher dimensional Lorentz group

The use of additional symmetries

Symmetries are very useful for

- absence of FCNC (solve flavour problem)
- Yukawa textures à la Frogatt-Nielsen
- solutions to the μ problem
- creation of hierarchies
- proton stability

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Continuous global symmetries might be destroyed by gravitational effects. We have to rely on

- gauge symmetries and
- discrete symmetries

(Banks, Seiberg, 2010)

Heterotic Braneworld

The heterotic braneworld is based on

- orbifold compactification of the heterotic string
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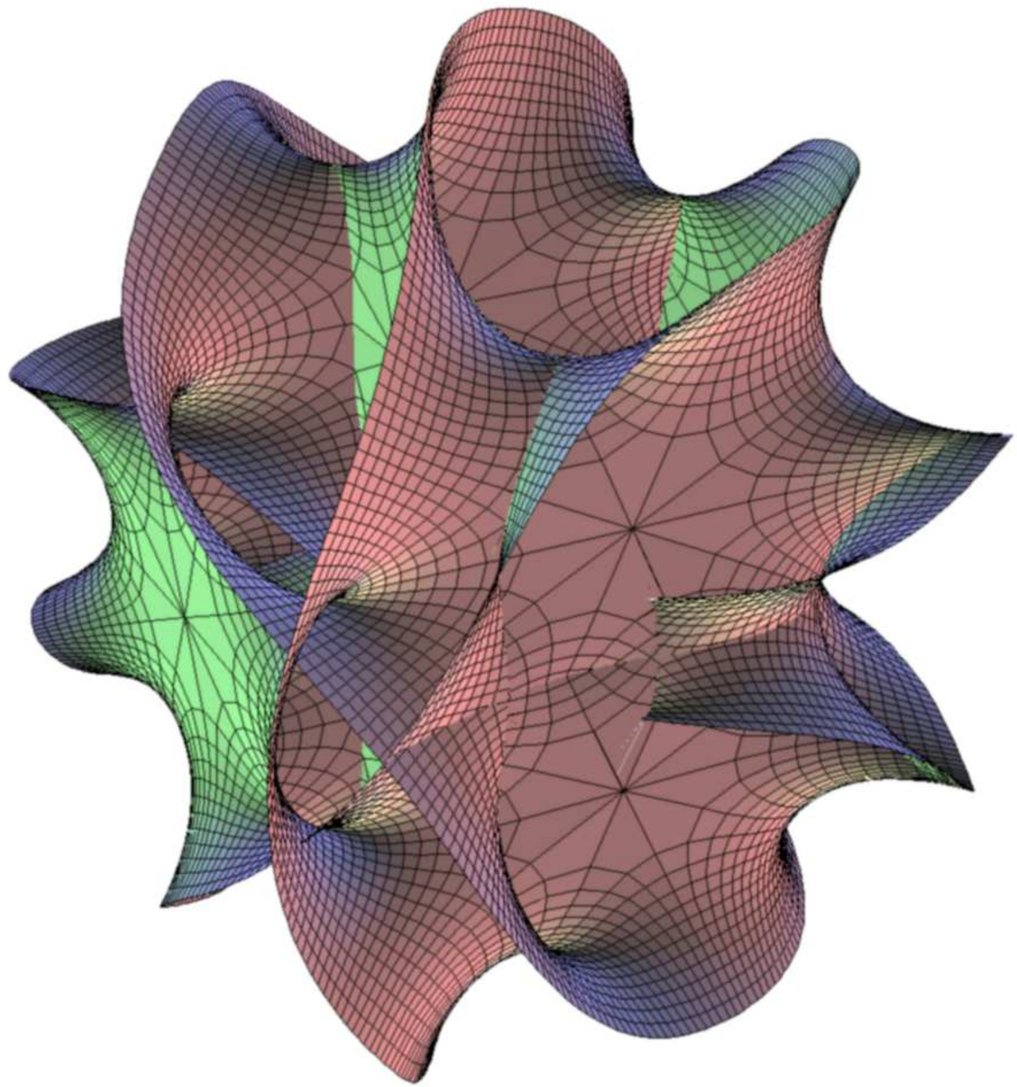
Fields can propagate

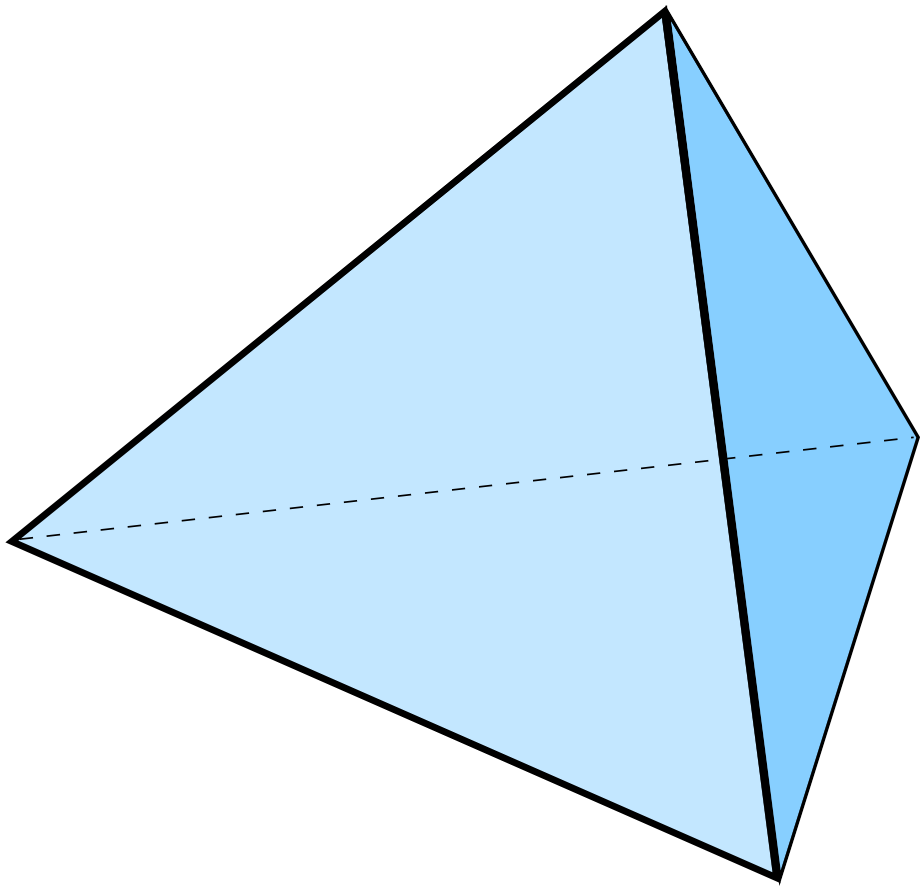
- in the Bulk ($d = 10$ untwisted sector)
- on 3-Branes ($d = 4$ twisted sector fixed points)
- on 5-Branes ($d = 6$ twisted sector fixed tori)

This localization is an important property of the set-up and should be taken seriously (it is not just an approximation to obtain calculability)

Calabi Yau Manifold



Orbifold



Local Grand Unification

String theory gives us a variant of GUTs

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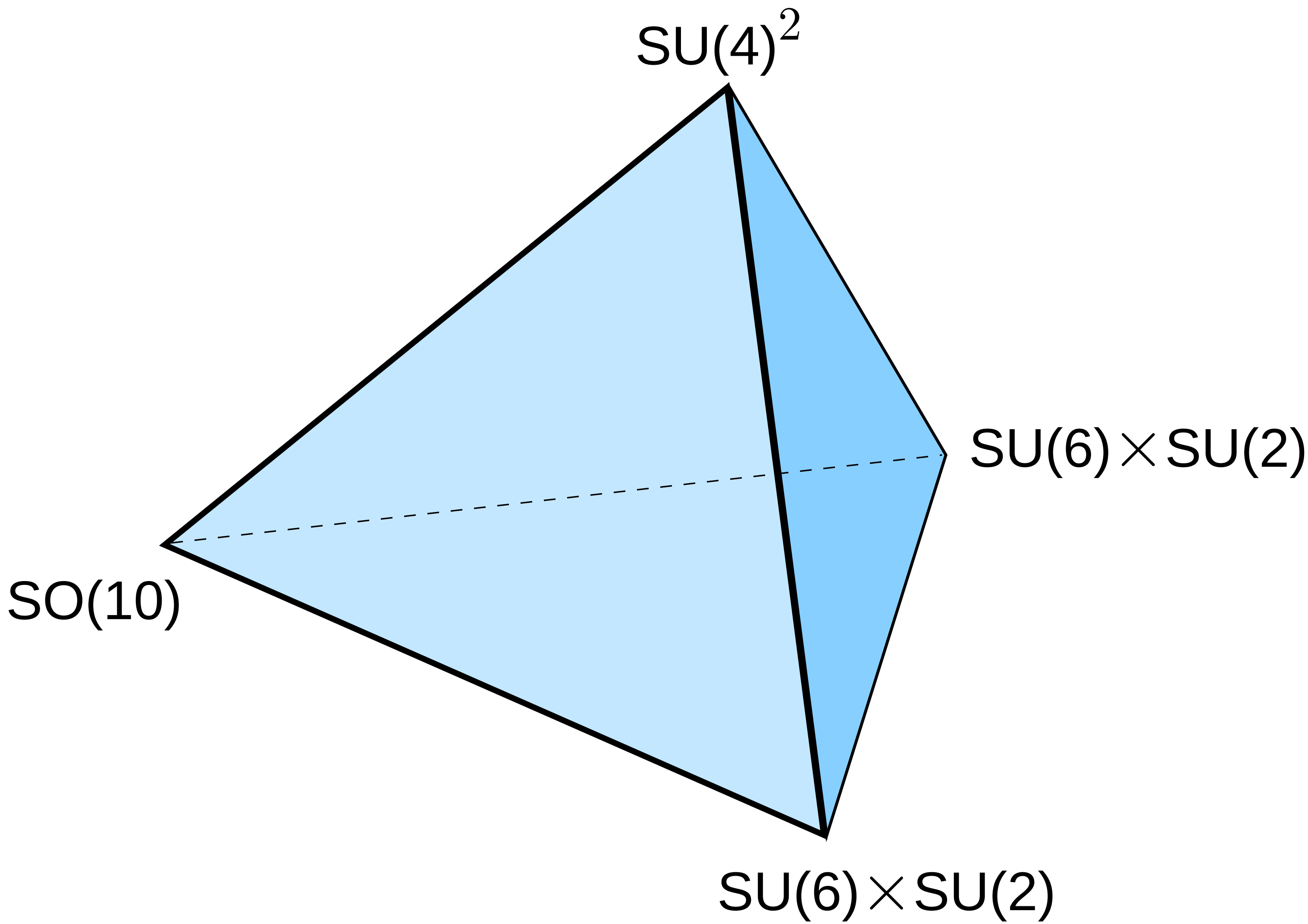
Key properties of the theory depend on the geography of the fields in extra dimensions.

This geometrical set-up is called local grand unification.

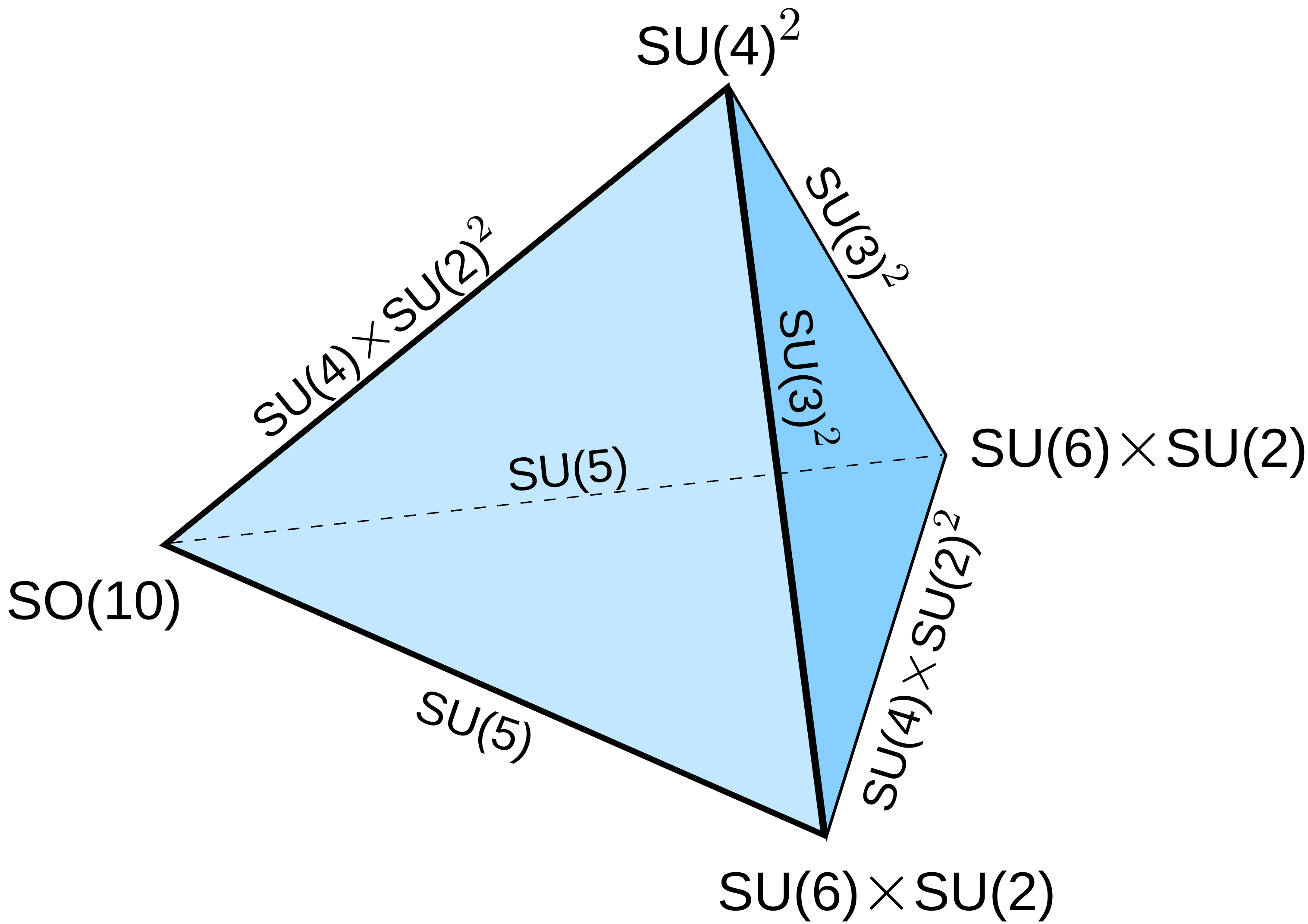
The localization of matter as well as the local structure of the gauge group determines the properties of the theory.

Localized gauge symmetries



(Förste, HPN, Vaudrevange, Wingerter, 2004)

Standard Model Gauge Group



Symmetries

In the heterotic braneworld we find

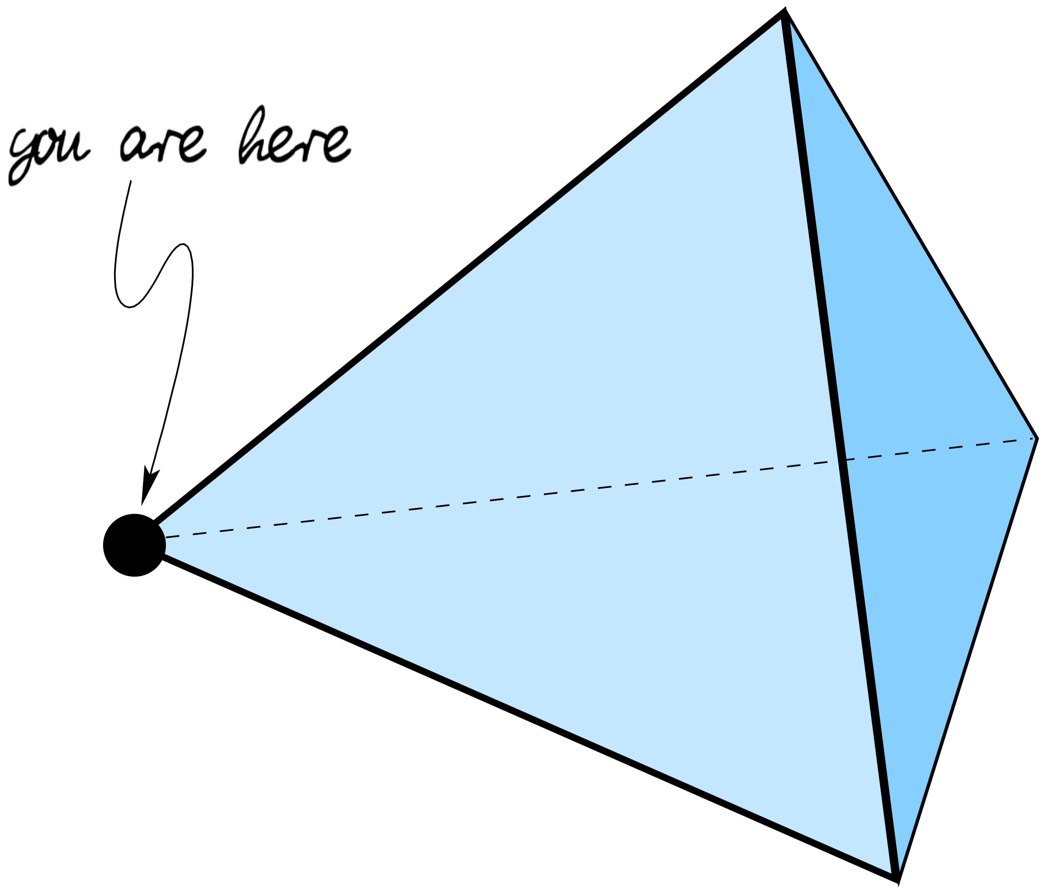
- gauge symmetries (no continuous global symmetries)
- discrete symmetries from geometry and stringy selection rules (Kobayashi, HPN, Plöger, Raby, Ratz, 2006)

The orbifold point is a special point in the moduli space of the compact extra dimensions with enhanced symmetries.

These symmetries might be slightly broken. This will introduce small parameters that lead to a creation of hierarchies.

We might live close to the orbifold point.

Location matters



Symmetries in heterotic braneworld

Applications of discrete symmetries:

- (nonabelian) family symmetries (and FCNC)

  (Ko, Kobayashi, Park, Raby, 2007)

- Yukawa textures (via Frogatt-Nielsen mechanism)
- a solution to the μ-problem

  (Lebedev, HPN, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, 2007)

- creation of hierarchies

  (Kappl, HPN, Ramos-Sanchez, Ratz, Schmidt-Hoberg, Vaudrevange, 2008)

- proton stability via “Proton Hexality” or Z_4^R

  (Förste et al. 2010; Lee et al. 2011)

- approximate global $U(1)$ for a QCD accion

  (Choi, Kim, Kim, 2006; Choi, HPN, Ramos-Sanchez, Vaudrevange, 2008)

The μ problem

In general we have to worry about

- doublet-triplet splitting
- mass term for additional doublets
- the appearance of "naturally" light doublets

In the heterotic braneworld we find models

- with only 2 doublets
- which are neutral under all selection rules
- if $M(s_i)$ allowed in superpotential
- then $M(s_i)H_uH_d$ is allowed as well

The μ problem II

We have verified that (up to order 8 in the superpotential)

- $F_i = 0$ implies automatically
- $M(s_i) = 0$ for all allowed terms $M(s_i)$ in the superpotential W

Therefore

- $W = 0$ in the supersymmetric (Minkowski) vacuum
- as well as $\mu = \partial^2 W / \partial H_u \partial H_d = 0$, while all the vectorlike exotics decouple
- with broken supersymmetry $\mu \sim m_{3/2} \sim < W >$

This solves the μ-problem

The creation of the hierarchy

Is there an explanation for a vanishing μ:

- string miracle or an underlying symmetry?

The μ-term is in fact forbidden by an R-smmetry.

For a continuous R-symmetry we would have

- a supersymmetric ground state with $W = 0$ and $\mathrm{U}(1)_R$ spontaneously broken
- a problematic R-Goldstone-Boson

However, the above R-symmetry appears as an accidental continous symmetry resulting from an exact discrete symmetry of (high) order N

Hierarchy

Such accidental symmetries lead to

- creation of a small constant in the superpotential
- explanation of a small μ term

(Kappl, HPN, Ramos-Sanchez, Ratz, Schmidt-Hoberg, Vaudrevange, 2008)

Even with a moderate hierarchy like $\phi/M_P \sim 10^{-2}$ one can generate small values for μ and $< W >$

$$m_{3/2} \sim W_{\text{eff}} = c + A\,\mathrm{e}^{-a\,S}$$

The second term in W_{eff} could be protected by an anomalous R-symmetry like e.g. Z_4^R

(Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg, Vaudrevange; 2010)

F-theory

F-theory with enhanced exceptional gauge symmetry is the way to incorporate rule 1 in Type II theories. It allows

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Phenomenological constructions are based on the concept of local models, e.g. at the local E_8 point. (Heckman, Vafa, 2010)

- a single gauge group like E_8
- containing other symmetries like R-parity as well
- there might not be a global completion!

Local E_8 point does not possess all the ingredients for realistic model building.

(Marsano, Schafer-Namecki, Saulina, 2011; Lüdeling, HPN, Stephan, 2011)

Clarification

Do not confuse

"Local Grand Unification" with "Local Model Building".

- Local Grand Unification appears in consistent (global) string models where the gauge symmetries are enhanced at special points in extra-dimensional space.
- Local Model Building is an attempt to construct models without the incorporation of gravity (these models are potentially inconsistent).

Do not trust the predictions of "Local Models" unless they are confirmed by a global completion!

Rule 6: Global Models

Sometimes it is said that globally consistent models are only relevant for questions like moduli stabilization......

- this needs not be correct (as experience shows)
- the really reliable (discrete) symmetries can only be understood within a global approach (e.g. R-parity)

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Phenomenological analyses of local models typically

- rely on continuous global U(1)s
- that might be broken in the full theory
- what are the remaining symmetries?

We need to answer this question before any predictions can be made!

Rule 7: Berechenbarkeit

Nowadays we need calculability that goes beyond the effective supergravity field theory approach, e.g. exact conformal field theory

- flat orbifolds, free fermionic constructions (Faraggi et al.)
- tensoring CFTs (Gepner models) (Schellekens et al.)

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- flat orbifolds, free fermionic constructions (Faraggi et al.)
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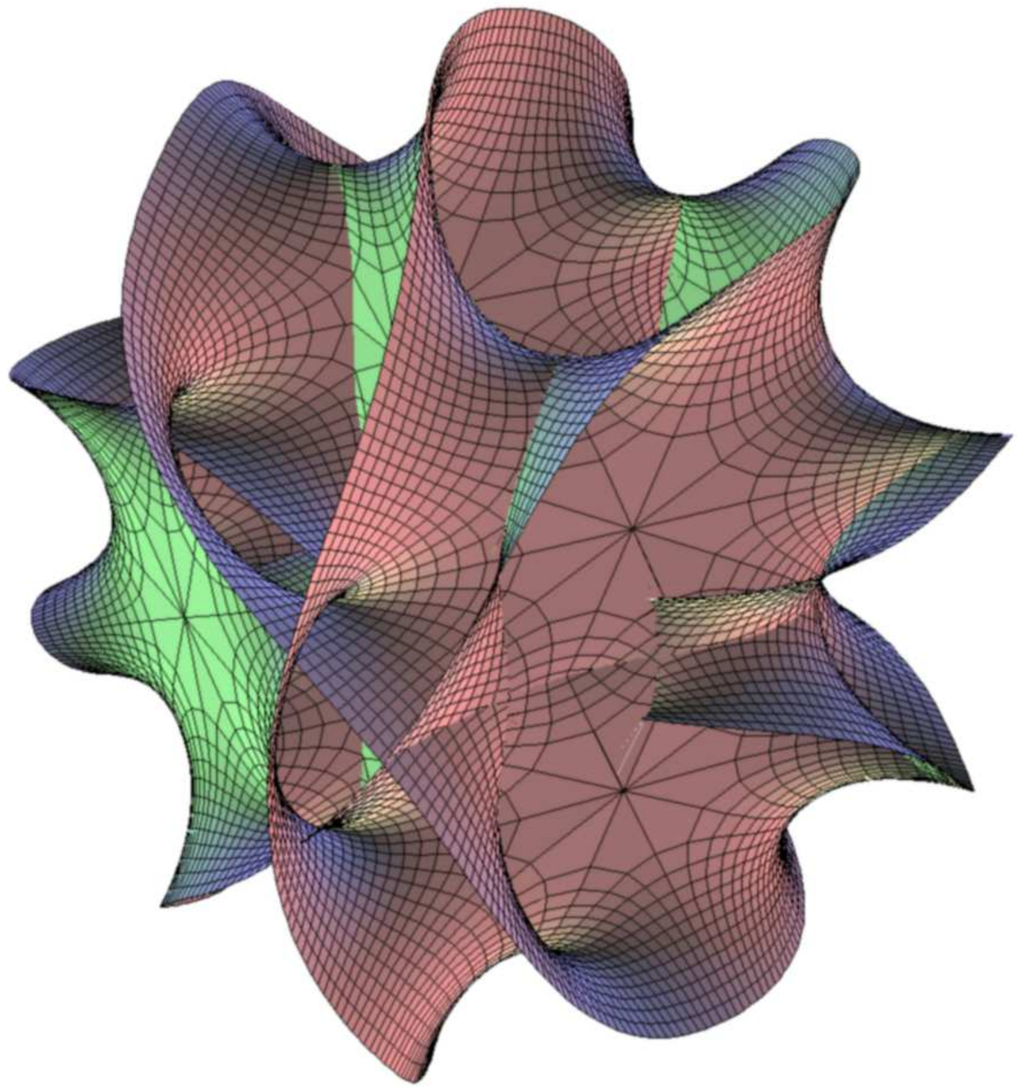
We have to analyze points of enhanced symmetries and enhanced particle spectra

- slightly broken symmetries (Frogatt-Nielsen)
- small parameters to create hierarchies

Hopefully nature is close to points of enhanced calculability.

Calabi Yau Manifold



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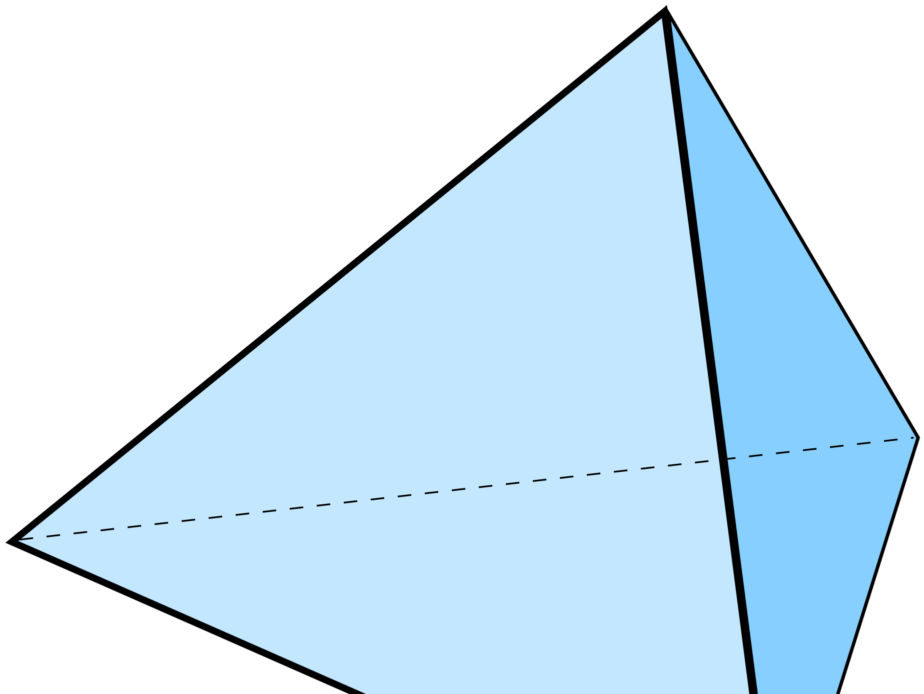

The fate of smooth compactification

Models on smooth manifolds describe generic points in moduli space

- limited calculability in practice (not full CFT)
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As a result, phenomenological analyses of these models often rely on continuous gobal symmetries

- an approximation is needed for “calculability”
- heterotic Calabi-Yau compactification should be related e.g. to a point with exact CFT

For F-theory it seems to be a real challenge to find a flat (CFT) approximation.

Improve calculability

Have to connect smooth compactification to e.g. flat orbifolds (Groot Nibbelink et al.; Blaszczyk et al.; 2009-2011)

- resolution of singularities within toric geometry
- is a good approximation in large volume limit

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But there are still some points that have to be clarified

- relation of number of massless states in orbifold and blow-up
- “missing” Yukawa couplings in large volume limit

Local anomalies might play an important role in the attempt to transfer calculability from orbifolds to smooth manifolds.

(Blaszczyk, Cabo Bizet, HPN, Ruehle, 2011)

The Anomaly Polynomial

The Green-Schwarz anomaly polynomial is a useful tool to study the relation between various schemes. The 12-form

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The Green-Schwarz anomaly polynomial is a useful tool to study the relation between various schemes. The 12-form

$$I_{12}(F_i, R) = I_4 \times I_8$$

contains crucial information on the properties of the model:

- can be computed independently in the different set-ups
- controls the coupling of “axions” to matter fields
- reveals broken and unbroken (discrete) symmetries.

Relate models of reduced calculability to those where explicit calculations can be done.

(Blaszczyk, Cabo Bizet, HPN, Ruehle, 2011)

Golden Rules (2011)

- Spinors of SO(10) (for families)
- Incomplete multiplets (for Higgs)
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- $N = 1$ supersymmetry
- Importance of discrete symmetries
- globally consistent models
- Berechenbarkeit
- study GS-anomaly polynomial

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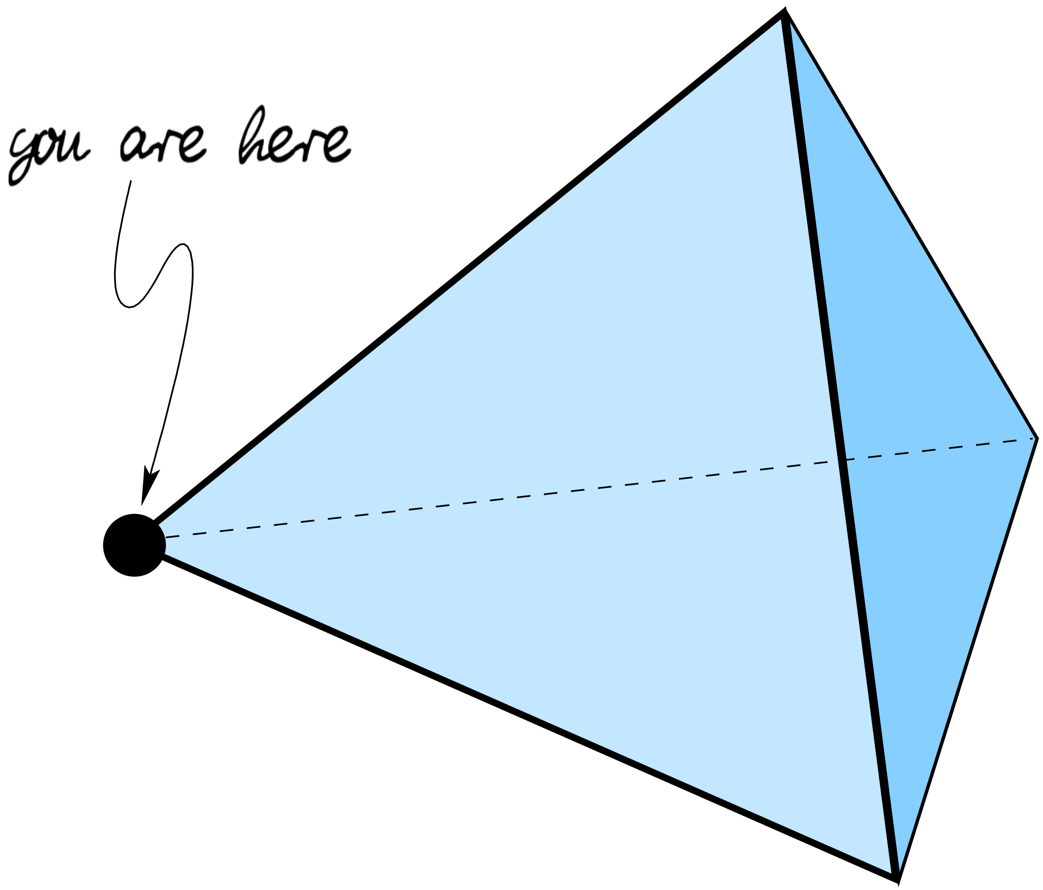
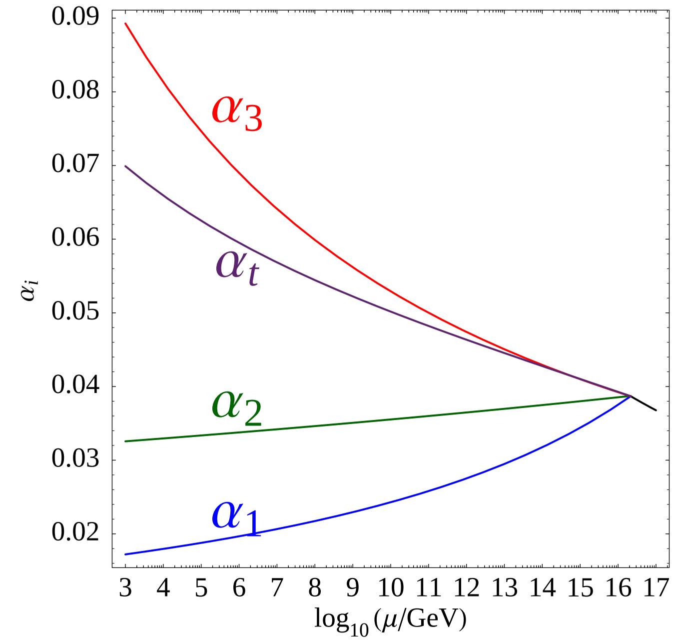
Let us hope that nature sits at a point of enhanced symmetry and calculability.

This is the place



Unification

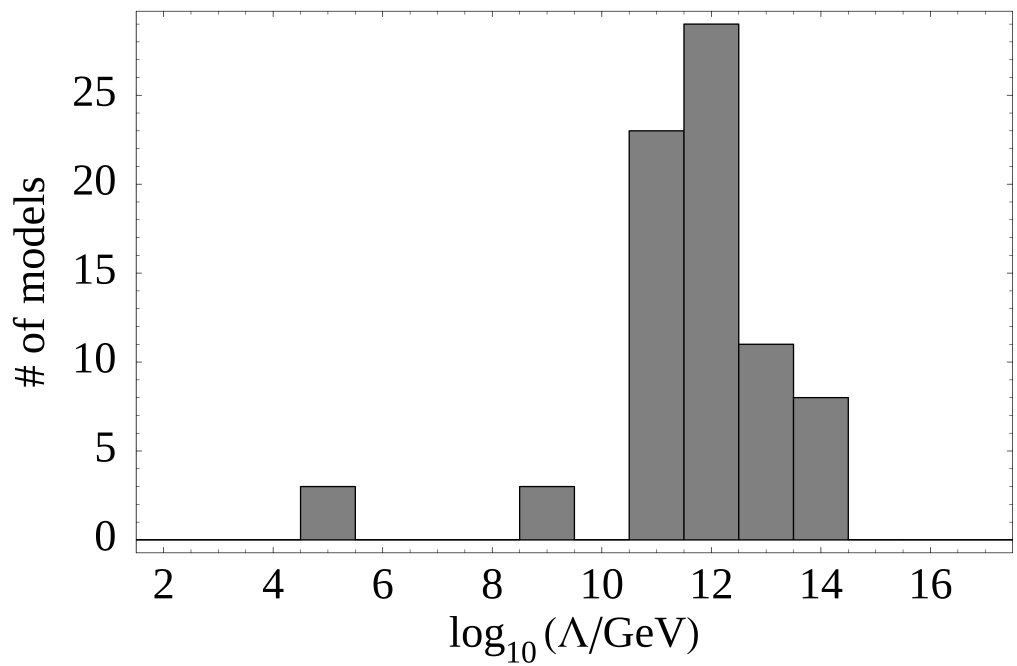
- Higgs doublets are in untwisted (U3) sector
- heavy top quark
- $\mu-$term protected by a discrete symmetry



- threshold corrections (“on third torus”) allow unification at correct scale around 10^{16} GeV
- natural incorporation of gauge-Yukawa unification

(Faraggi, 1991; Hosteins, Kappl, Ratz, Schmidt-Hoberg, 2009)

Hidden Sector Susy Breakdown



Gravitino mass $m_{3/2} = \Lambda^3 / M_{\rm Planck}^2$ is in the TeV range for the hidden sector gauge group $SU(4)$

(Lebedev, HPN, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, 2006)

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Proton stability

In the standard model Baryon number $U(1)_B$ is not a good symmetry

- Baryon and lepton number are anomalous
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Baryon number violation is needed for baryogenesis.

- Grand unification addresses these questions
- proton decay via dimension-6 operators
- GUT scale has to be sufficiently high

GUTs need SUSY

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But there is a problem

- dimension-4 and -5 operators
- more symmetries needed
- matter parity (or R-parity)
- baryon triality, proton hexality

(Ibanez, Ross, 1991; Dreiner, Luhn, Thormeier, 2005)

MSSM

The minimal particle content of the susy extension of the standard model includes chiral superfields

- Q, $\bar{U}$, $\bar{D}$ for quarks and partners
- L, $\bar{E}$ for leptons and partners
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with superpotential

$$W = QH_d\bar{D} + QH_u\bar{U} + LH_d\bar{E} + \mu H_u H_d.$$

Also allowed (but problematic) are dimension-4 operators

$$\bar{U}\bar{D}\bar{D} + QL\bar{D} + LL\bar{E}.$$

The question of proton stability

These dimension-4 operators could be forbidden by some symmetry

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Where does this symmetry come from?

- it could be a subgroup of $SO(10)$
- in consistent heterotic constructions it comes from $(E_8 \times E_8)/SO(10)$
- in local F-theory construction from $E_8/SO(10)$

Proton Hexality

But there are in addition dimension-5 operators that might mediate too fast proton decay $QQQL + \bar{U}\bar{U}\bar{D}\bar{E}$

	Q	$\bar{U}$	$\bar{D}$	L	$\bar{E}$	H_u	H_d	$\bar{\nu}$
$6\,Y$	1	-4	2	-3	6	3	-3	0
$\mathbb{Z}_2^{\text{matter}}$	1	1	1	1	1	0	0	1
B_3	0	-1	1	-1	2	1	-1	0
P_6	0	1	-1	-2	1	-1	1	3

Proton hexality is exactly what we need:

- dangerous dimension 4 and 5 operators forbidden
- neutrino Majorana masses allowed (LLH_uH_u)

(Dreiner, Luhn, Thormeier, 2005)

GUTs and Hexality

Combination of GUTs and proton hexality is perfect

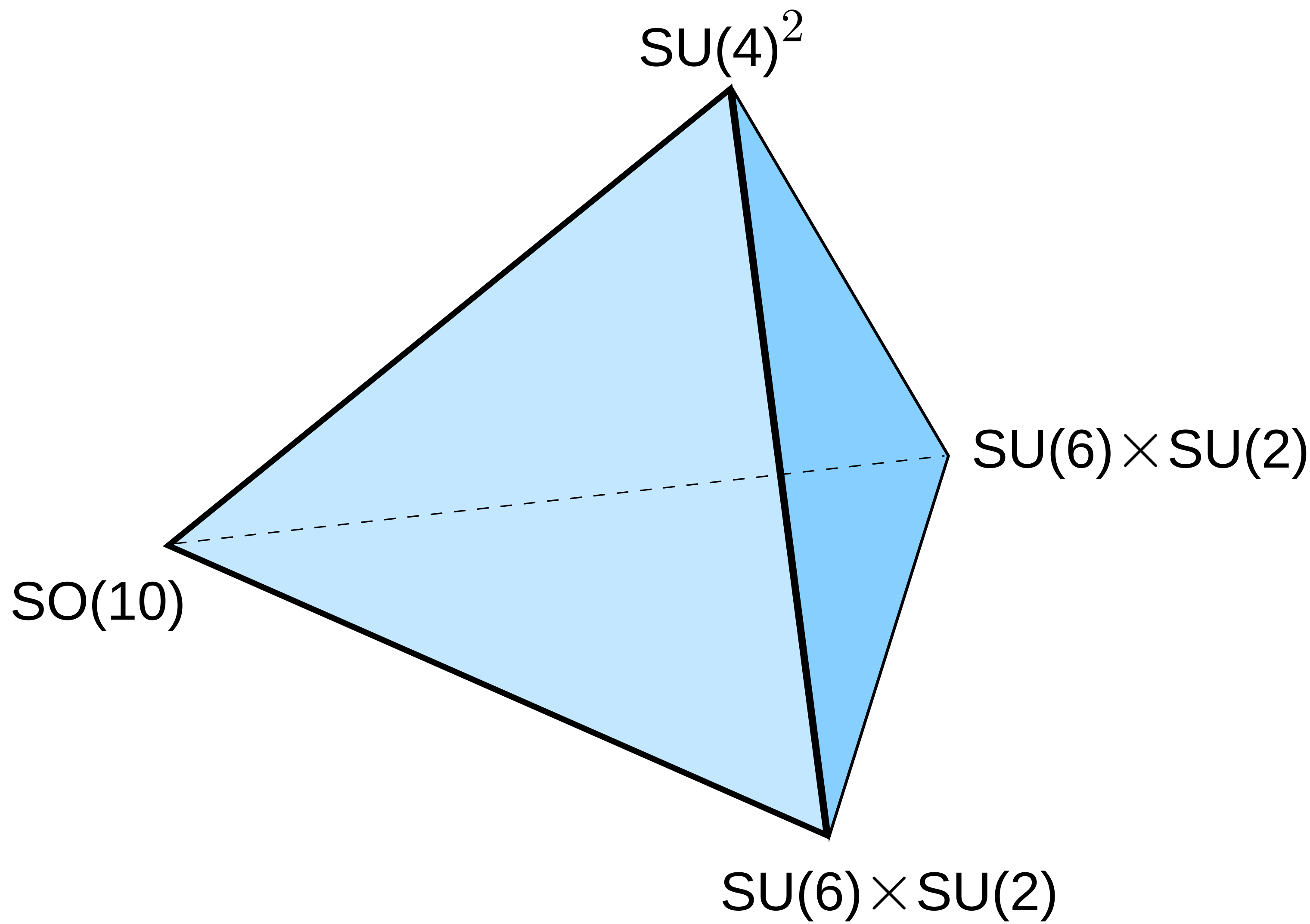
But GUTs and Hexality are incompatible (Luhn, Thormeier, 2007)

Example:
the 10-dimensional representation of SU(5) includes $\bar{U}$, Q and $\bar{E}$ and they cannot all have the same charge under hexality.

The problem is solved in

- Local Grand Unification
- need split multiplets for matter fields
- nonlocal structure of matter fields in compactified dimensions

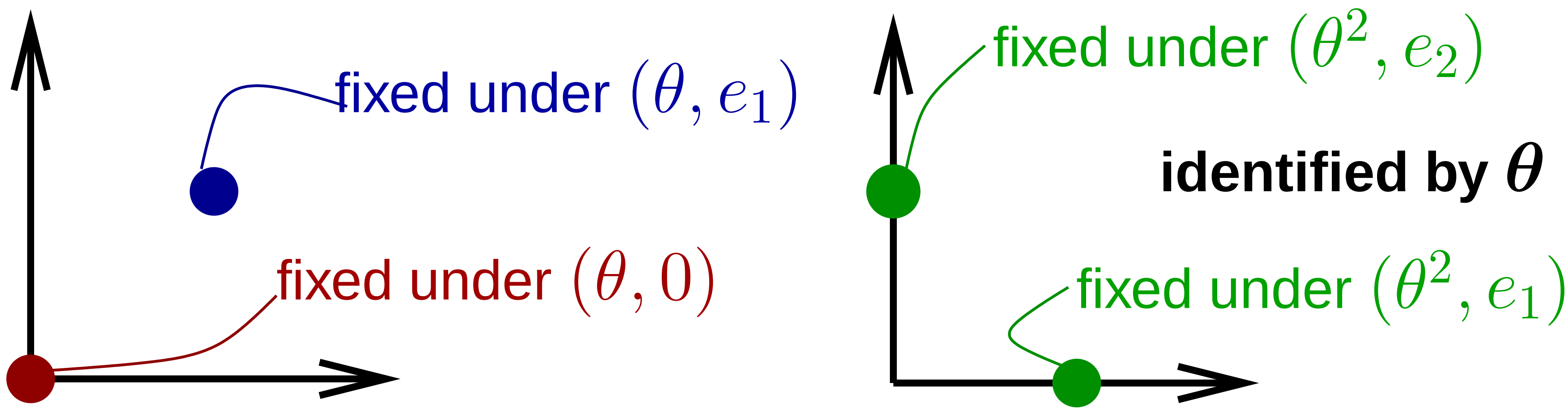
Localized gauge symmetries



(Förste, HPN, Vaudrevange, Wingerter, 2004)

A T_2/Z_4 toy example

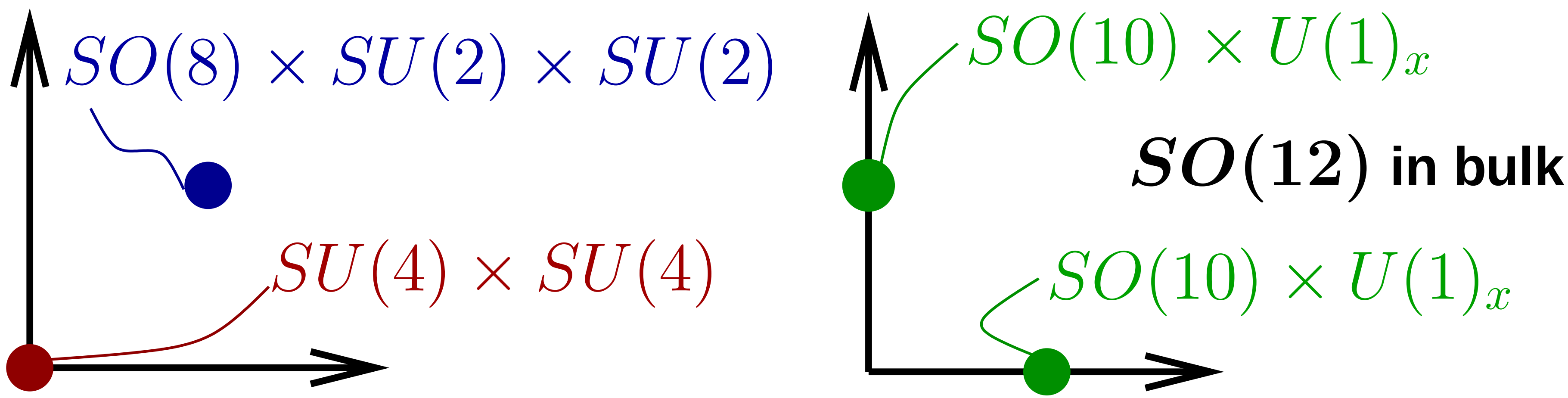
Consider the T_2/Z_4 orbifold, where we have two different types of fixed points



under rotation of $\theta = \pi/2$ and shift of the lattice vectors.

A T_2/Z_4 toy example

For a suitable embedding of twist and shift in the gauge group $SO(12)$ we have the following local gauge group structure



This allows split representations compatible with P_6 and does not require huge representations for the breakdown of $SO(12)$.

Lessons from the heterotic braneworld

The concept of local GUTs leads to a nontrivial structure of matter distribution in extra dimensions

- R-symmetries as subroup of SO(6) to solve the μ problem
- split multiplets for proton hexality
- Z_4^R and nonperturbative effects

  (Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg, Vaudrevange; 2010)
- discrete symmetries as subroups of $E_8 \times E_8 \times SO(6)$

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Note that we have consistent string models in a global construction. There is a plenitude of (discrete) gauge symmetries, both abelian and nonabelian.

(Kobayashi et al.,2006; Araki et al., 2008)

Conclusion

0 String theory might provide us with a consistent UV-completion of the MSSM including

- Local Grand Unification as a result of a consistent global construction,
- a plenitude of discrete symmetries,
- originating from some non-localities of matter distribution in extra dimensions.

Geography of extra dimensions plays a crucial role.

Local Grand Unification is the right way to proceed.

Discrete symmetries as subgroups of $E_8 \times E_8 \times SO(6)$ as a crucial prediction of string theory!

1984

1984

1984

1984

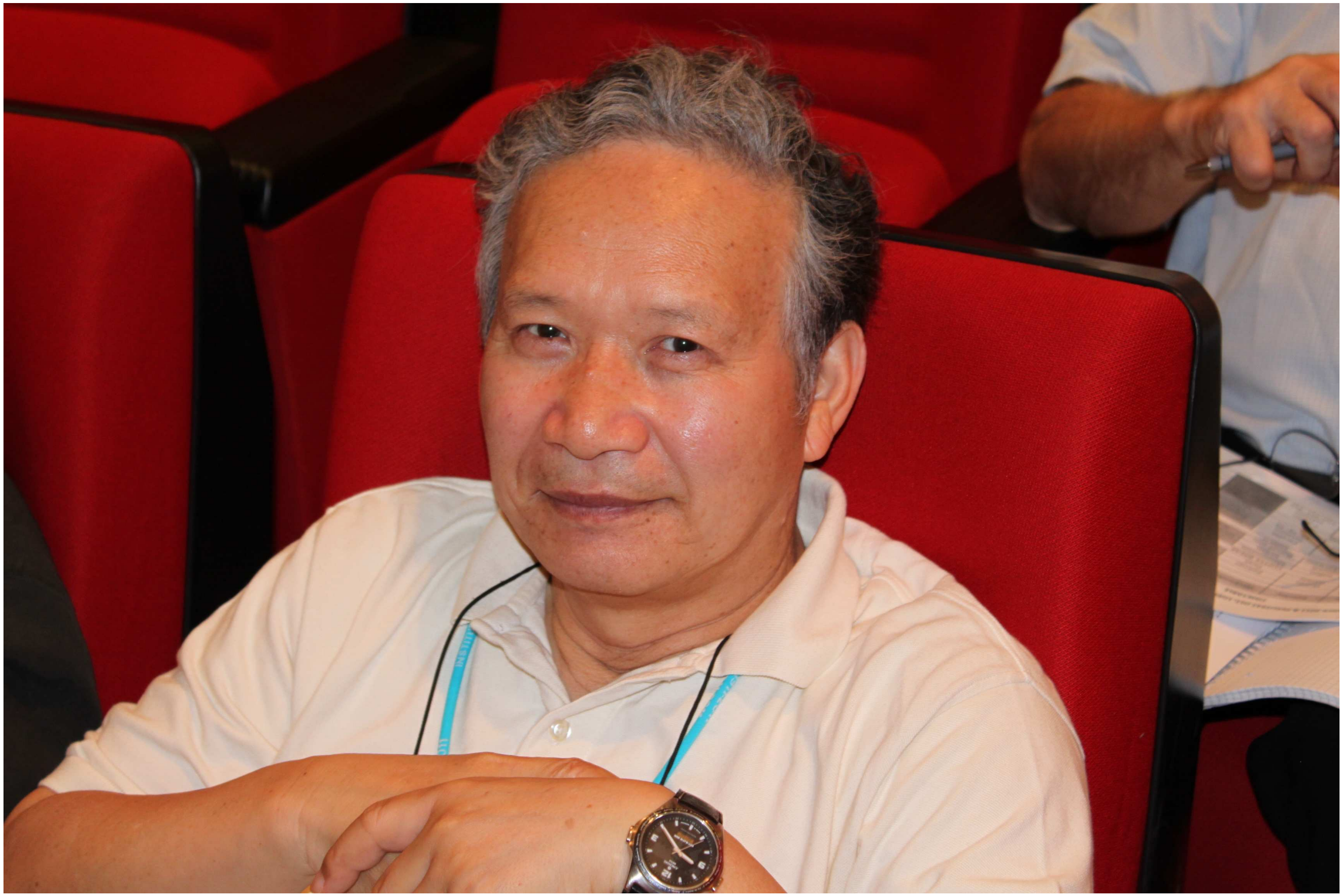
1987

1987

1987

90’s

2011

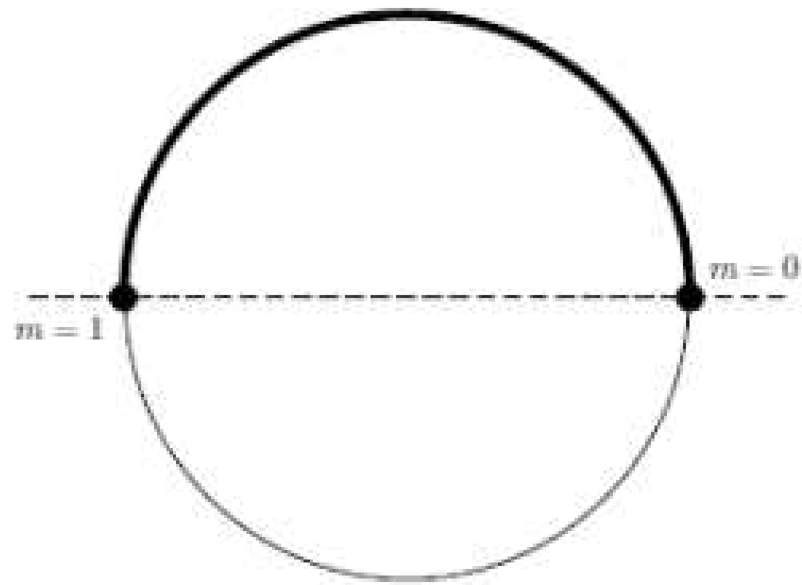


2011

2011

2011

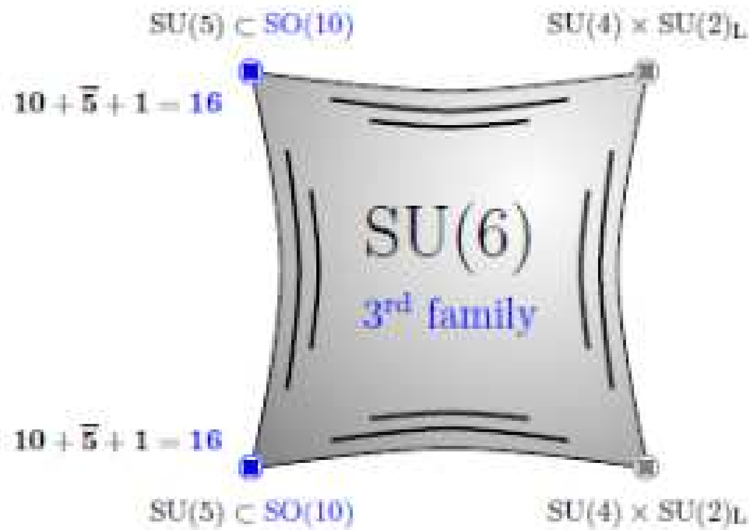
Origin of discrete symmetries



The semidirect product of $Z_2 \times Z_2$ and S_2

leads to the nonabelian group D_4

Local GUT picture



Family symmetries in local GUT models