Axions, Alignment and Clockworks

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Axions

Axions have been first discussed in the framework of solutions to the strong CP problem. An axion is a

- pseudo-scalar particle a
- pseudo-Goldstone boson of a global symmetry $U(1)_A$
- $U(1)$ is anomalous
- coupling to gauge fields $aF\tilde{F}$ via the anomaly

Relevant parameters include

- the axion decay constant f_a: $\frac{a}{f_a}F\tilde{F}$
- its mass m_a
- coupling to photons and gluons

Use of Axions

Axions can play a role for

- the strong CP problem in QCD (Peccei, Quinn, 1977)
- the mechanism of inflation (Freese, Frieman, Olinto, 1990)
- the source of quintessence (Frieman, Hill, Stebbins, Waga, 1995)
- the relaxion (Graham, Kaplan, Rajendran, 2015)

Axions are abundant in string theory constructions, where they originate from various anti-symmetric tensor fields

- there is an opportunity for multi-axion systems
- that seems to be helpful for the consistency of some of the axionic models (alignment of axions)

Scales and hierarchies

Usually the axions come with specific scales

- the QCD axion requires a specific window for f_a between 10^9 and 10^{12} GeV (large compared to $M_{\rm weak}$)
- trans-Planckian field values in axionic inflation
- quintessential axion requires extremely flat potential and small mass of 10^{-33} eV
- extreme fine tunings for the relaxion

One of the challenges is to understand these scales within a consistent UV-complete theory

- in string theory the scale for f_a set by $M_{\rm string}$
- the relevance of alignment enters here

Alignment

Alignment has been first suggested in natural inflation.
We shall discuss here (Kim, Nilles, Peloso, 2005)

- axionic inflation
- tensor modes in CMB fluctuations
- trans-Planckian excursions of the (axionic) inflaton field
- the alignment of axions

Other application of multi-axion systems

- axionic domain walls for QCD axion (Choi, Kim, 1985)
- alignment of quintessential axions (Kaloper, Sorbo, 2006)
- the relaxion mechanism (Choi, Im, 2015)

The Quest for Flatness

The mechanism of inflation requires a “flat” potential.
We demand

- symmetry reason for flatness of the potential
- slightly broken symmetry to move the inflaton

The obvious candidate is axionic inflation

- axion has only derivative couplings to all orders in perturbation theory
- broken by non-perturbative effects (instantons)

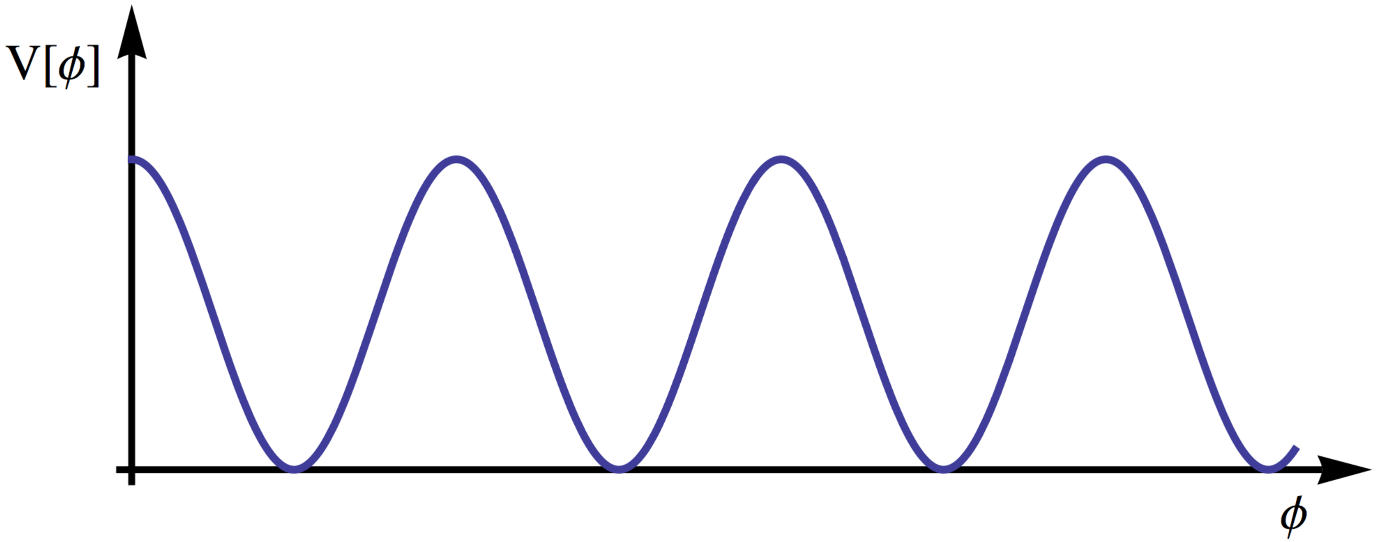
Motivated by the QCD axion

(Freese, Frieman, Olinto, 1990)

The Axion Potential

The axion exhibits a shift symmetry $\phi \to \phi + c$

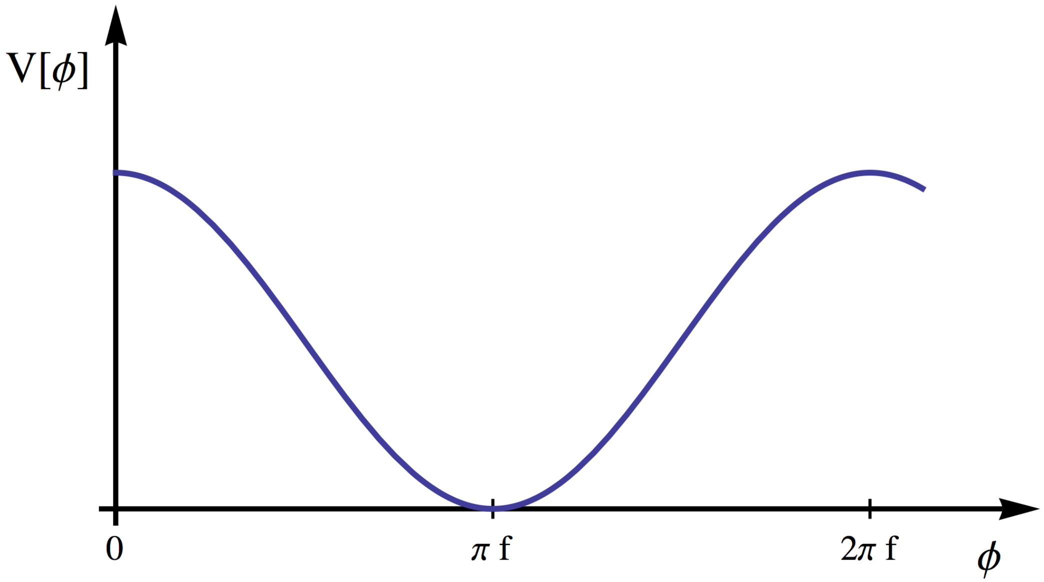
Nonperturbative effects break this symmetry to a remnant discrete shift symmetry



$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{2\pi\phi}{f}\right)\right]$$

The Axion Potential

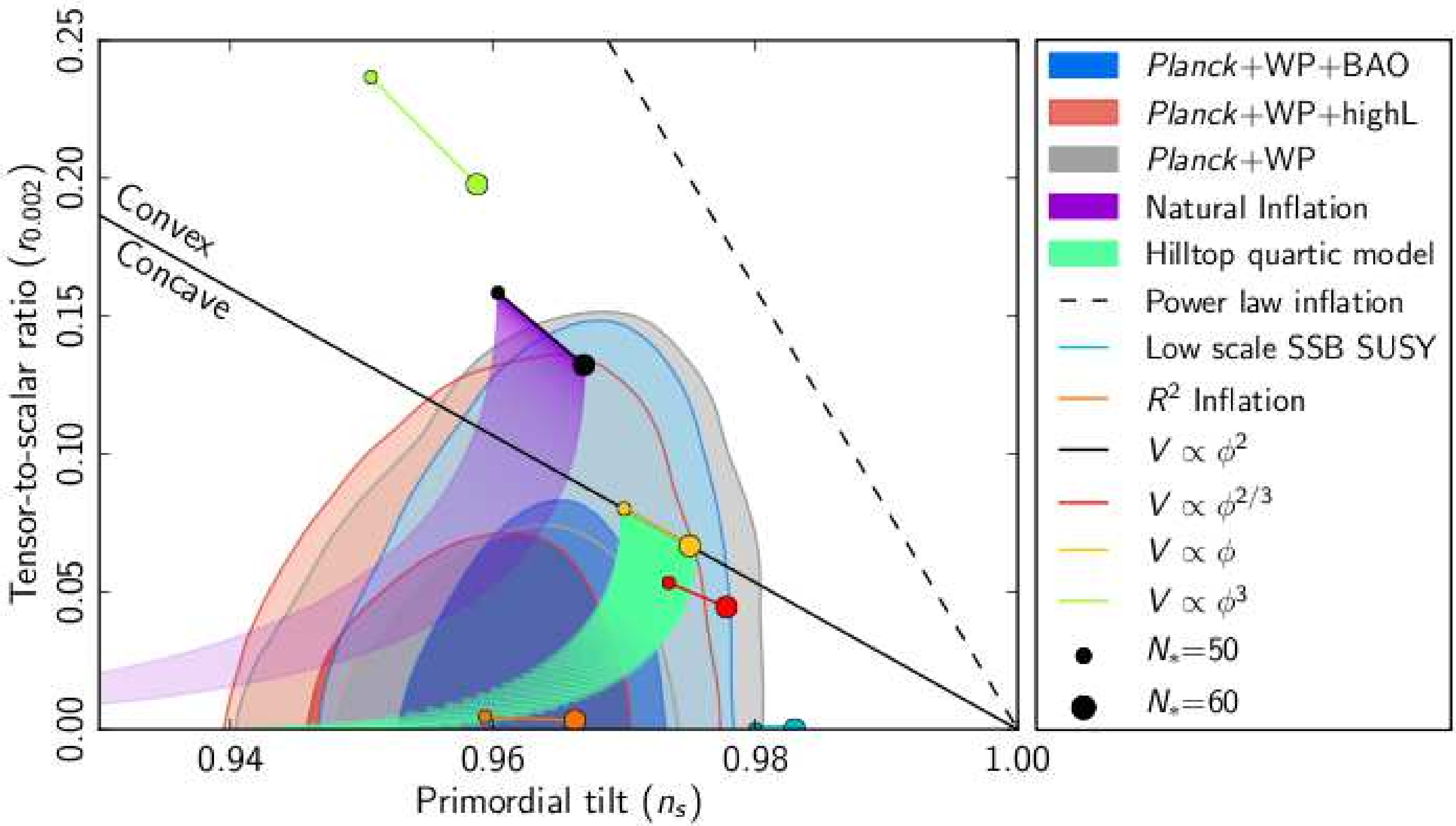
Discrete shift symmetry identifies $\phi = \phi + 2\pi n f$



$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{2\pi\phi}{f}\right)\right]$$

ϕ confined to one fundamental domain

Planck results (Spring 2013)



BICEP2 (Spring 2014)

Tentatively large tensor modes of order $r \sim 0.1$ had been announced by the BICEP collaboration

- large tensor modes brings us to scales of physics close to the Planck scale and the so-called "Lyth bound"
- potential $V(\phi)$ of order of GUT scale few $\times 10^{16}$ GeV
- trans-Planckian excursions of the inflaton field
- For a quadratic potential $V(\phi) \sim m^2\phi^2$ it implies $\Delta\phi \sim 15 M_{\mathrm{P}}$ to obtain 60 e-folds of inflation

Axionic inflation, on the other hand, seems to require the decay constant to be limited: $f \leq M_{\mathrm{P}}$.

So this might be problematic. (Banks, Dine, Fox, Gorbatov, 2003)

Aligned axions

A way out is the consideration of two (or more) fields.

(Kim, Nilles, Peloso, 2004)

- top-down approach favours a multi-axion picture
- we require $f \leq M_{\rm P}$ for the individual axions

The alignment extends the fundamental domain of the aligned axion to super-Planckian values, as the axionic inflaton spirals down in the potential of the second axion

Alternative mechanisms, like e.g. ”Axion Monodromy" give a similar qualitative picture (McAllister, Silverstein, Westphal, 2008)

The KNP set-up

We consider two axions

$$\mathcal{L}(\theta,\rho) = (\partial\theta)^2 + (\partial\rho)^2 - V(\rho,\theta)$$

with potential

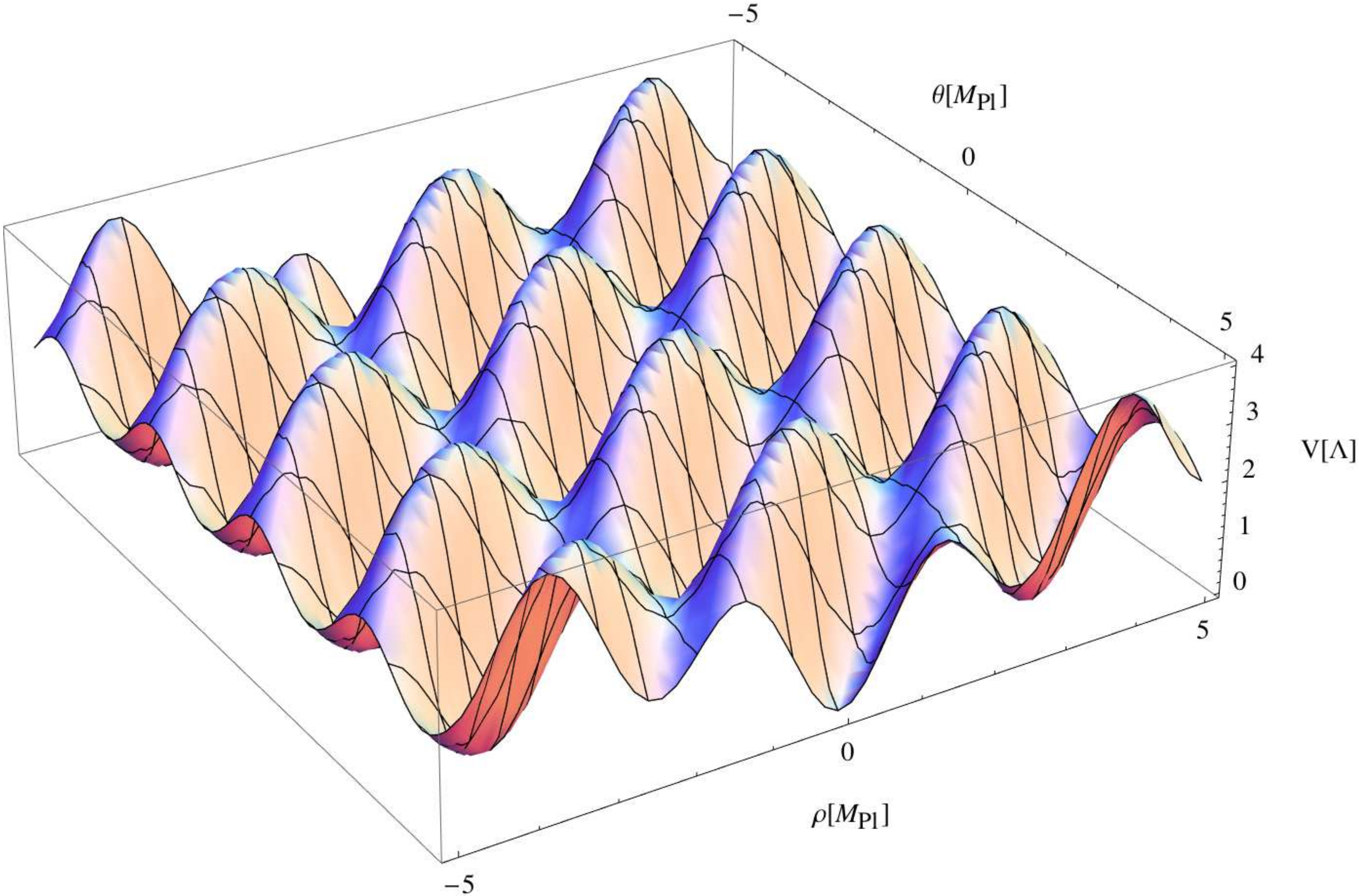
$$V(\theta,\rho) = \Lambda^4\left(2 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1}\right) - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2}\right)\right)$$

This potential has a flat direction if $\frac{f_1}{g_1} = \frac{f_2}{g_2}$

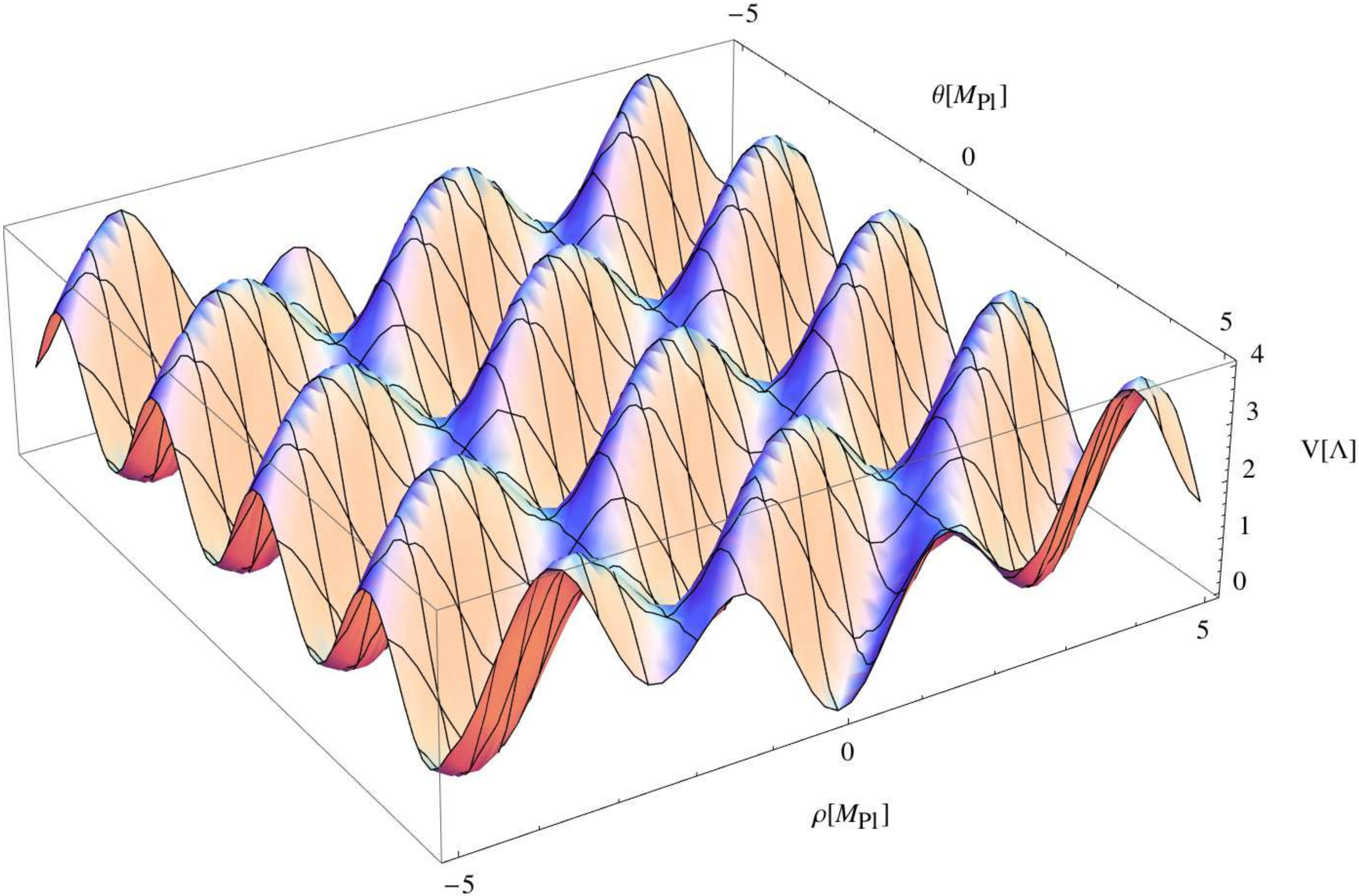
Alignment parameter defined through $\alpha = g_2 - \frac{f_2}{f_1} g_1$

For $\alpha = 0$ we have a massless field ξ.

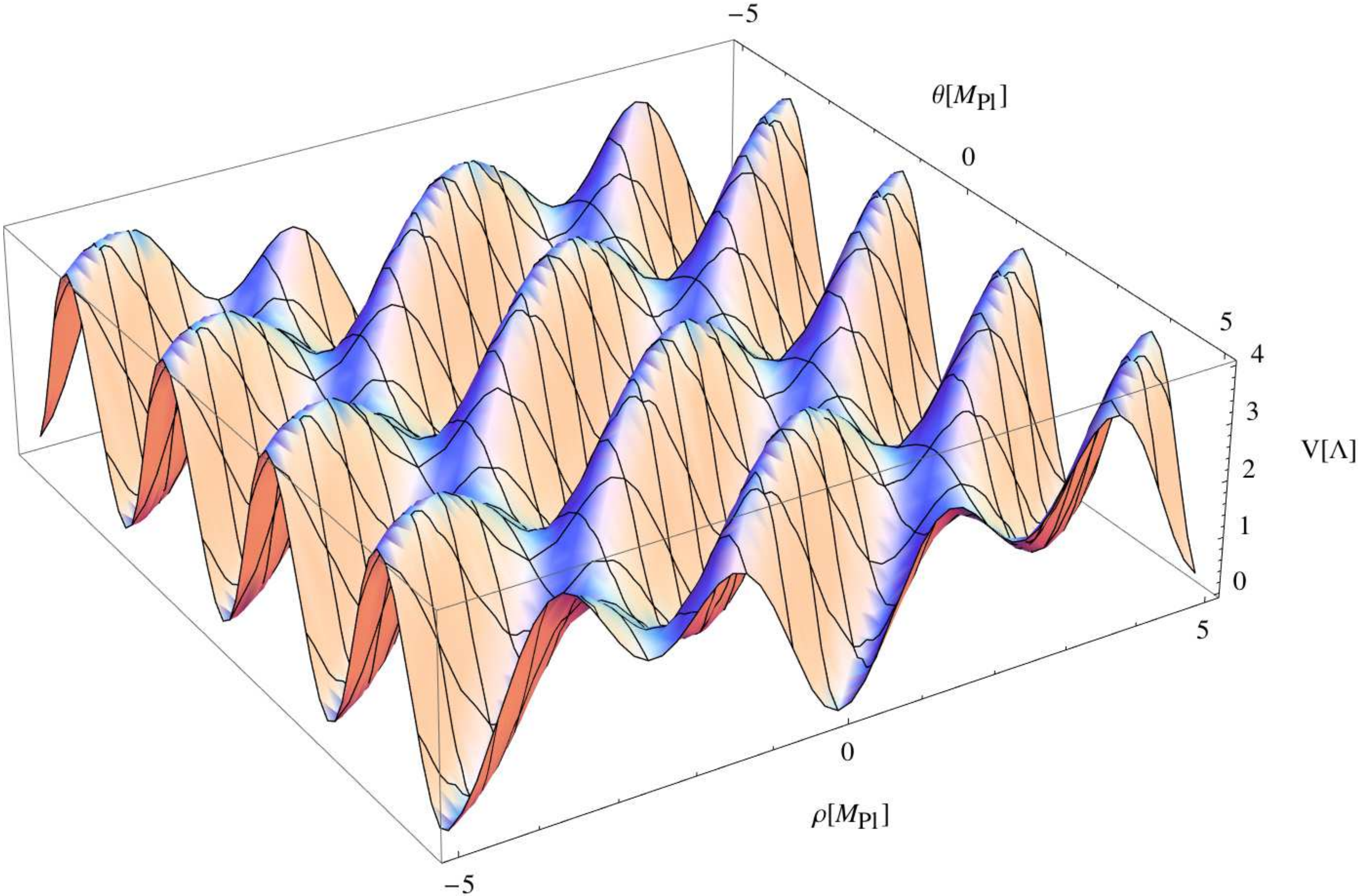
Potential for $\alpha = 1.0$



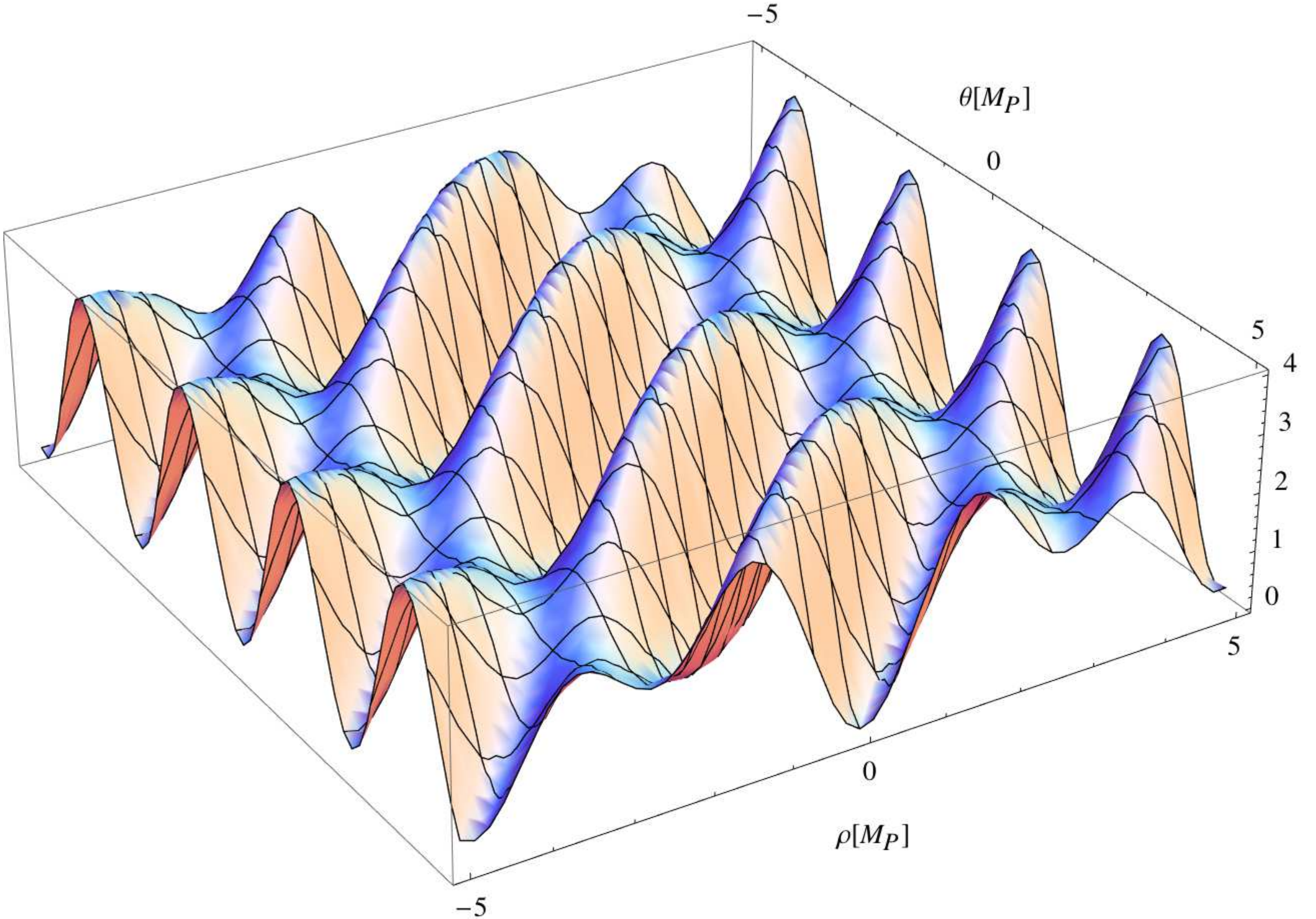
Potential for $\alpha = 0.8$



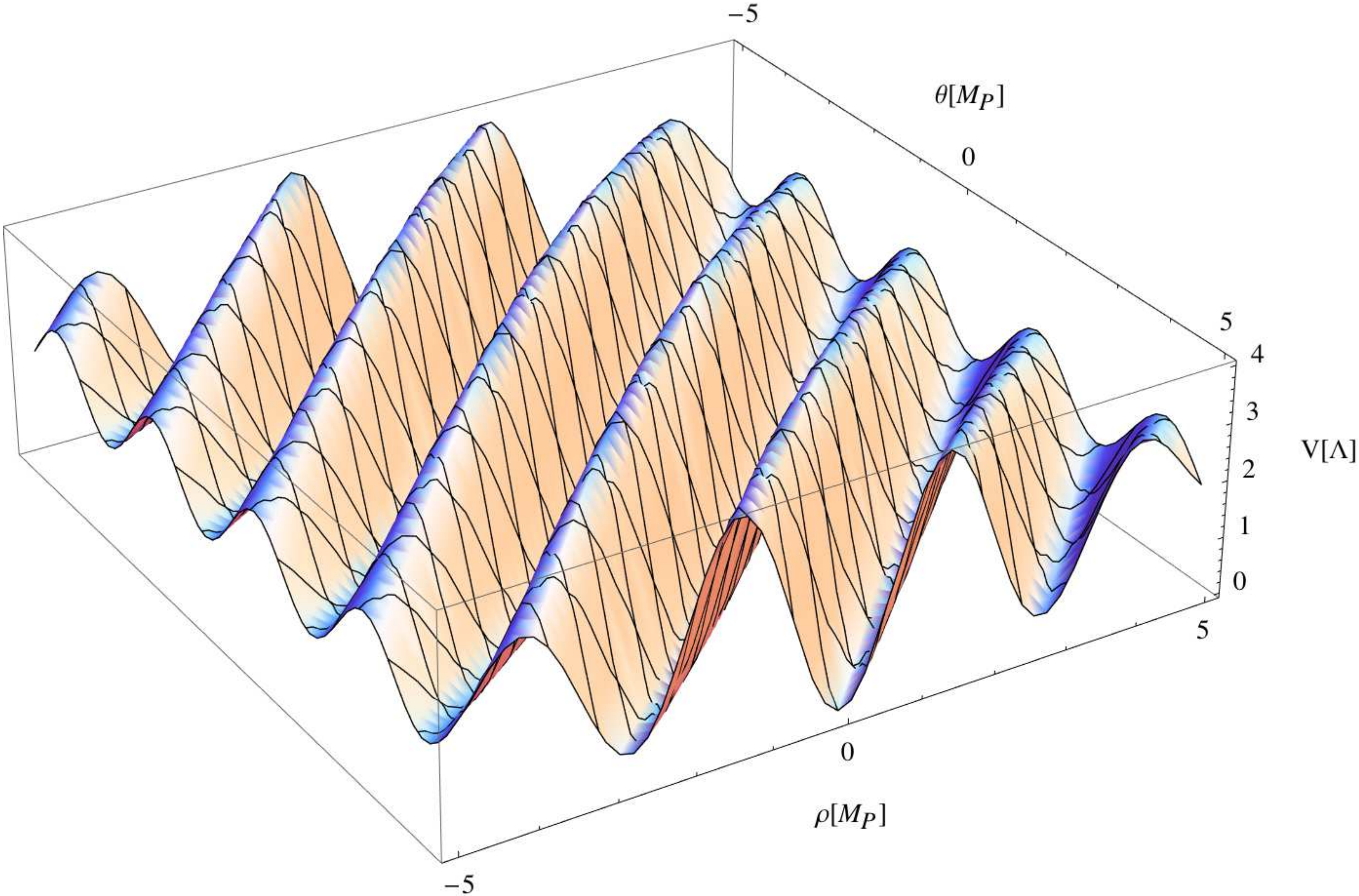
Potential for $\alpha = 0.5$



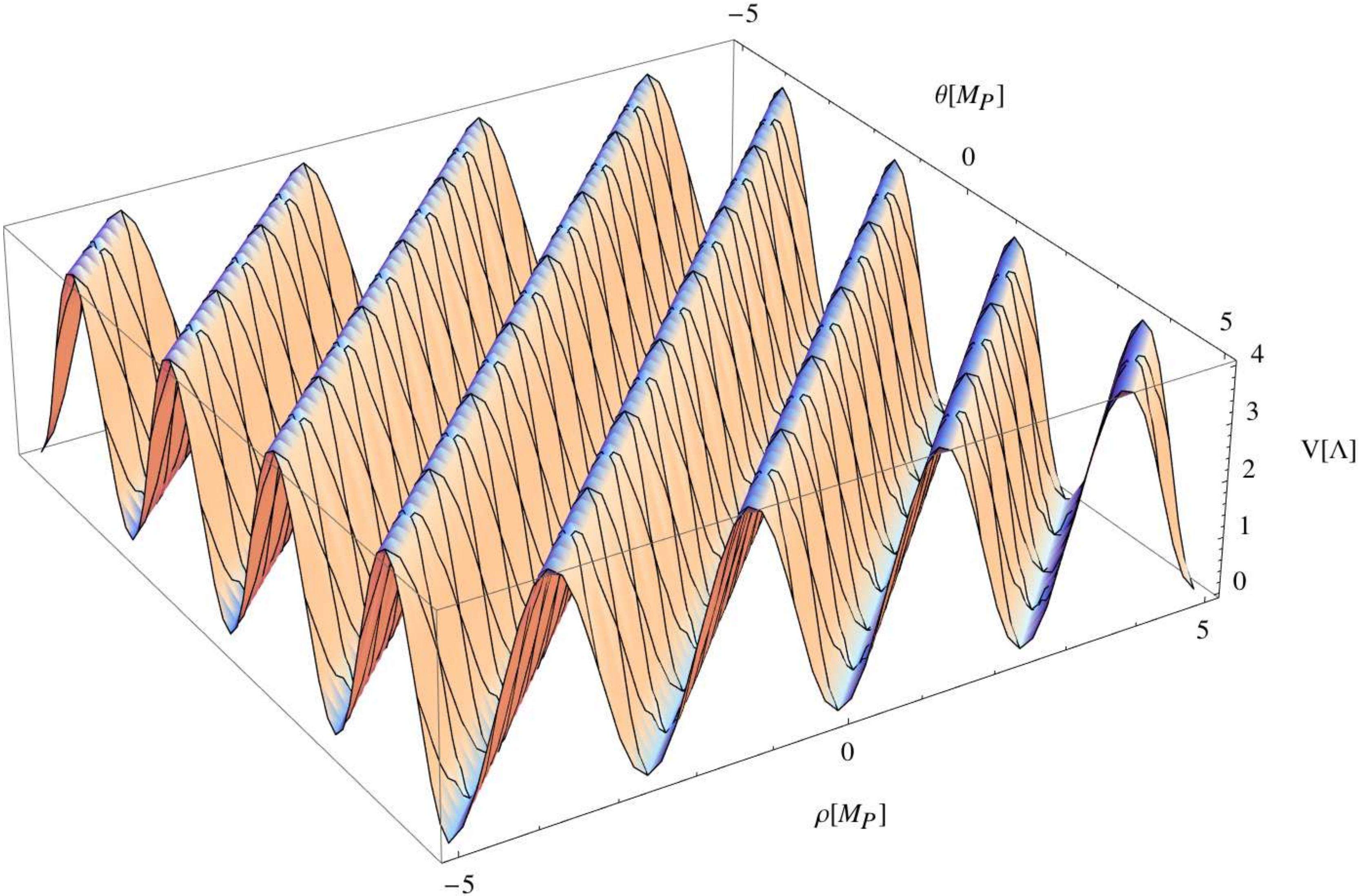
Potential for $\alpha = 0.3$



Potential for $\alpha = 0.1$



Potential for $\alpha = 0$



The lightest axion

Mass eigenstates are denoted by (ξ, ψ). The mass eigenvalues are

$$\lambda_{1/2} = F \pm \sqrt{F^2 + \frac{2g_1 g_2 f_1 f_2 - f_2^2 g_1^2 - f_1^2 g_2^2}{f_1^2 f_2^2 f_1^2 g_2^2}}$$

with $$F = \frac{g_1^2 g_2^2 (f_1^2 + f_2^2) + f_1^2 f_2^2 (g_1^2 + g_2^2)}{2 f_1^2 f_2^2 g_1^2 g_2^2}$$

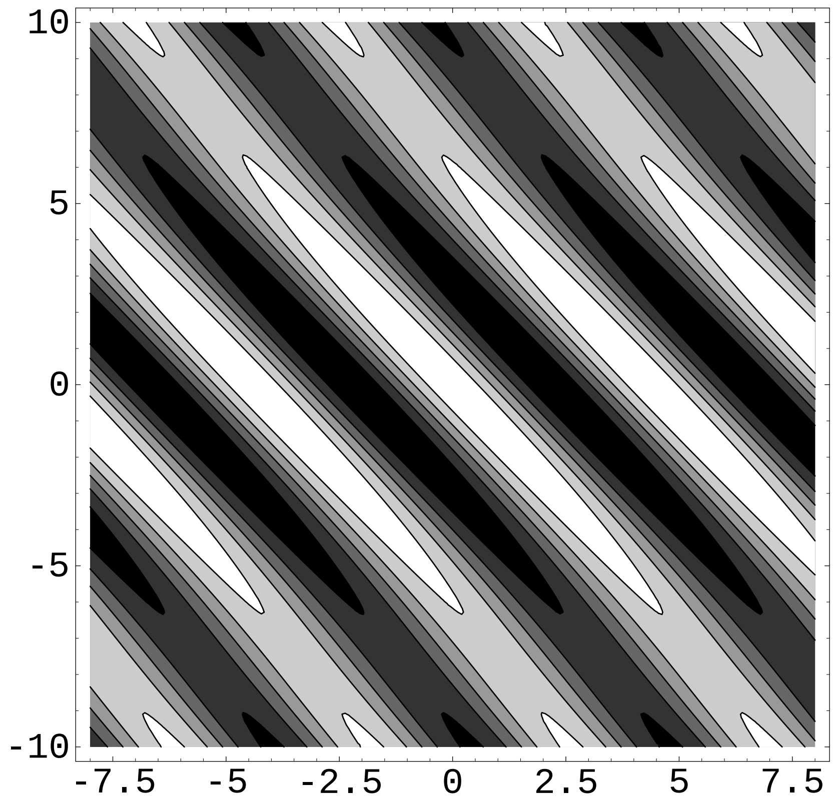
Lightest axion ξ has potential

$$V(\xi) = \Lambda^4 \left[2 - \cos\left(m_1(f_i, g_1, \alpha)\xi\right) - \cos\left(m_2(f_i, g_1, \alpha)\xi\right)\right]$$

leading effectively to a one-axion system

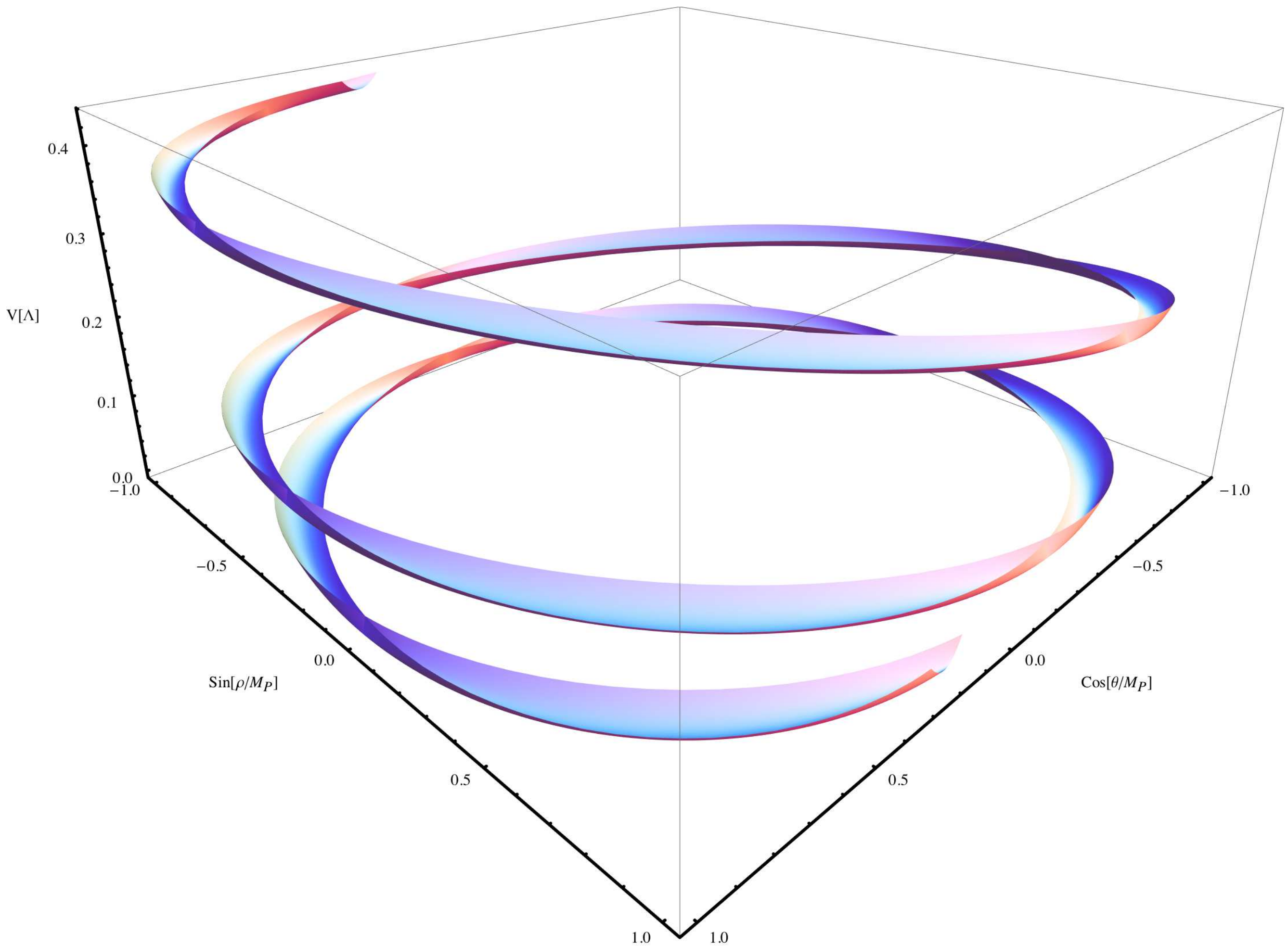
$$V(\xi) = \Lambda^4 \left[1 - \cos\left(\frac{\xi}{\tilde{f}}\right)\right] \quad \text{with} \quad \tilde{f} = \frac{f_2 g_1 \sqrt{(f_1^2 + f_2^2)(f_1^2 + g_1^2)}}{f_1^2 \alpha}$$

Axion landscape of KNP model



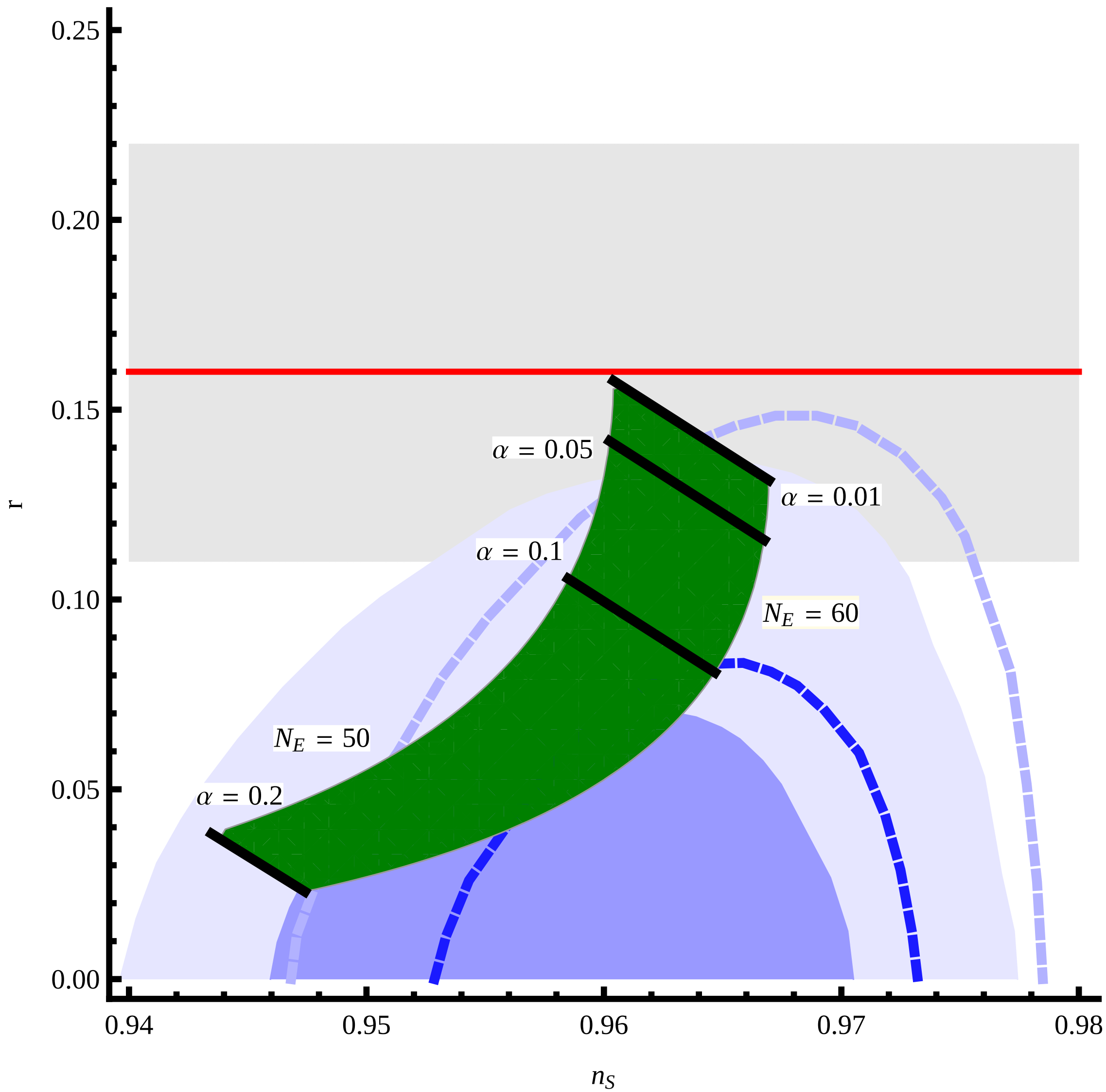
The field ξ rolls within the valley of ψ. The motion of ξ corresponds to a motion of θ and ρ over many cycles. The system is still controlled by discrete symmetries.

Monodromic Axion Motion



One axion spirals down in the valley of a second one.

The “effective” one-axion system



UV-Completion

Large tensor modes and $\Lambda \sim 10^{16}$GeV lead to theories at the "edge of control" and require a reliable UV-completion

- small radii
- large coupling constants
- light moduli might spoil the picture

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Large tensor modes and $\Lambda \sim 10^{16}$GeV lead to theories at the "edge of control" and require a reliable UV-completion

- small radii
- large coupling constants
- light moduli might spoil the picture

So it is important to find reliable symmetries

- axions are abundant in string theory
- perturbative stability of "shift symmetry"
- broken by nonperturbative effects
- potential protection through supersymmetry

Stability

We have a very flat direction and within the effective QFT we are at the "edge of control"

- is inflation perturbed by other effects?
- is there an upper limit on $f_{\rm eff}$?

Remember that in case of a single axion we had the limit

- $f_{\rm eff} \leq M_{\rm string}$ (Banks, Dine, Fox, Gorbatov, 2003)
- derived from dualities in string theory (e.g. T-duality)

In the multi-axion case these arguments are not directly applicable, but the question of trans-Planckian values should be tested in a given model

Weak Gravity Conjecture (WGC)

It is based on prejudice about black hole properties and is formulated to constrain $U(1)$ gauge interactions,

(Arkani-Hamed, Motl, Nicolis, Vafa, 2006)

- give limits on mass to charge ratio $|q/m| > 1$
- "convex hull" restrictions in multi-field case.

But our "knowledge" on black hole properties (no-hair conjecture and information paradox) has changed recently,

- fuzzballs, (Mathur, 2009-2015)
- brick- and fire-walls. (Almheiri, Marolf, Polchinski, Sully, 2012)

The motivation for the WGC might thus be less convincing.

WGC II

It is conjectured that the WGC (if true) might be applicable to axions, (Rudelius, 2015)

- based on a chain of string dualities,
- might give an upper limit on decay constants $f_{\rm eff}$.

This might lead to a no-go theorem for large axion decay constants, but

- there are loop-holes in the presence of subleading instantons, (Brown, Cottrell, Shiu, Soler, 2015)
- computationally we are at the “edge of control”.

Needs to be clarified in explicit constructions.

(Kappl, Nilles, Winkler, 2015)

$T-$**Duality**

String dualities give important constraints on the axion decay constants, especially $T-$duality $SL(2,Z)$:

$$T \rightarrow \frac{aT - ib}{icT + d}$$

generated by an inversion and a shift

$$T \rightarrow 1/T, \qquad T \rightarrow T + i.$$

$$G = K + \log |W|^2$$

must be invariant under T-duality.

$T-$Duality

K and W might transform nontrivially. Consider e.g.

$$K = -3 \log (T + \bar{T}).$$

This Kähler potential transforms under $SL(2, Z)$ as

$$K \to K + \log |icT + d|^6$$

and has to be compensated by a superpotential transforming as a modular form of weight -3:

$$W \to (icT + d)^{-3} W \ .$$

Explicit String Constructions

In string theory we do not just get cosine potentials, but obtain modular functions (e.g. Dedekind-functions) from

- world sheet instanton effects,
- gauge kinetic functions and gaugino condensates.

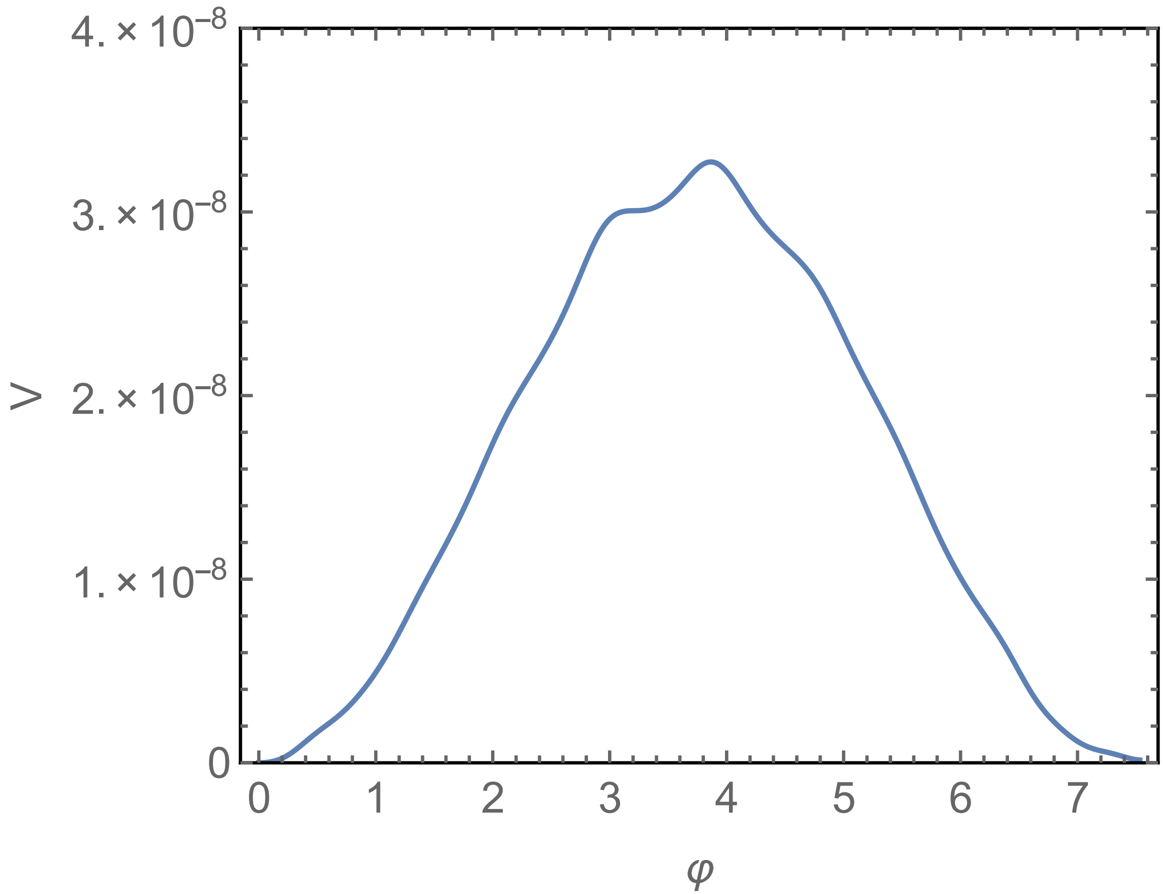
So we might consider instead

$$\eta(T) = e^{-\pi T/12} \times \prod_k \left(1 - e^{-2k\pi T}\right)$$

a modular function of weight $+1/2$.

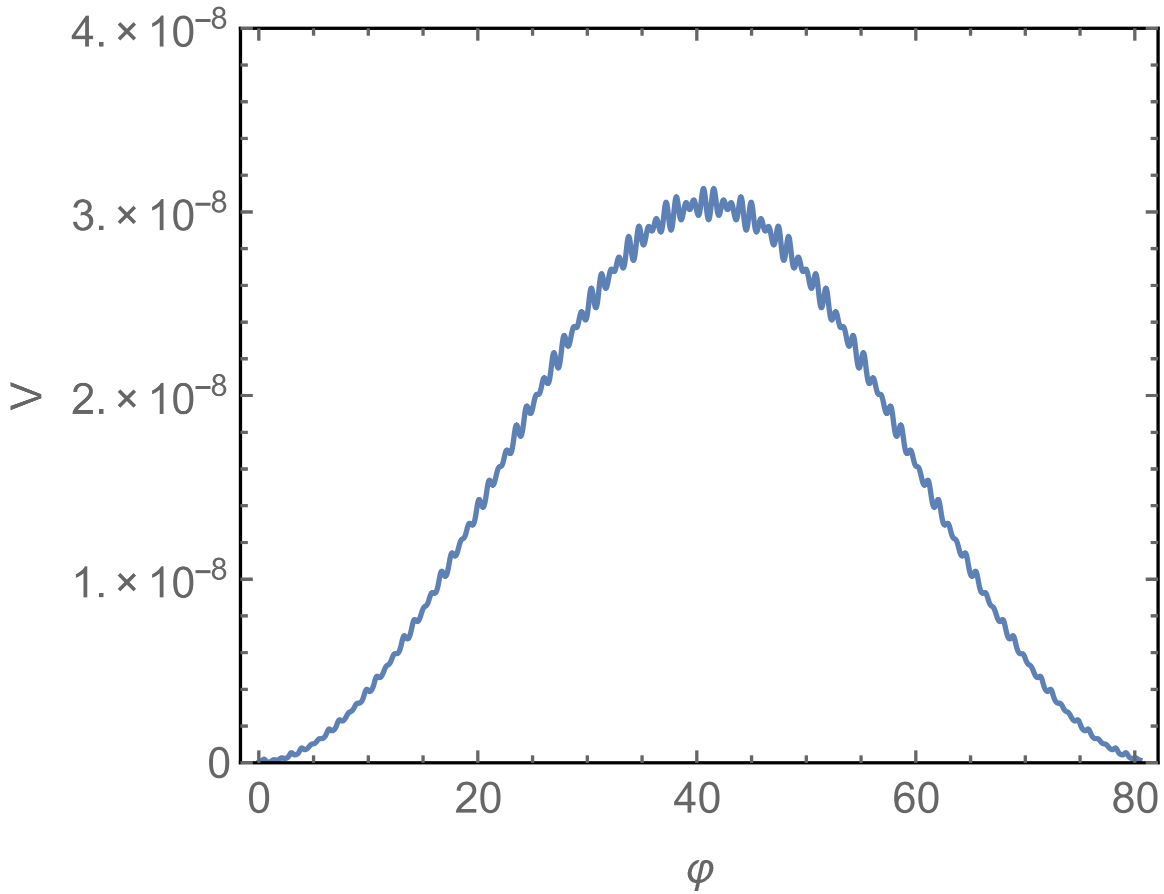
The higher harmonics give wiggles in the potential that perturb the flat direction and might stop inflation.

Wiggles in the aligned potential



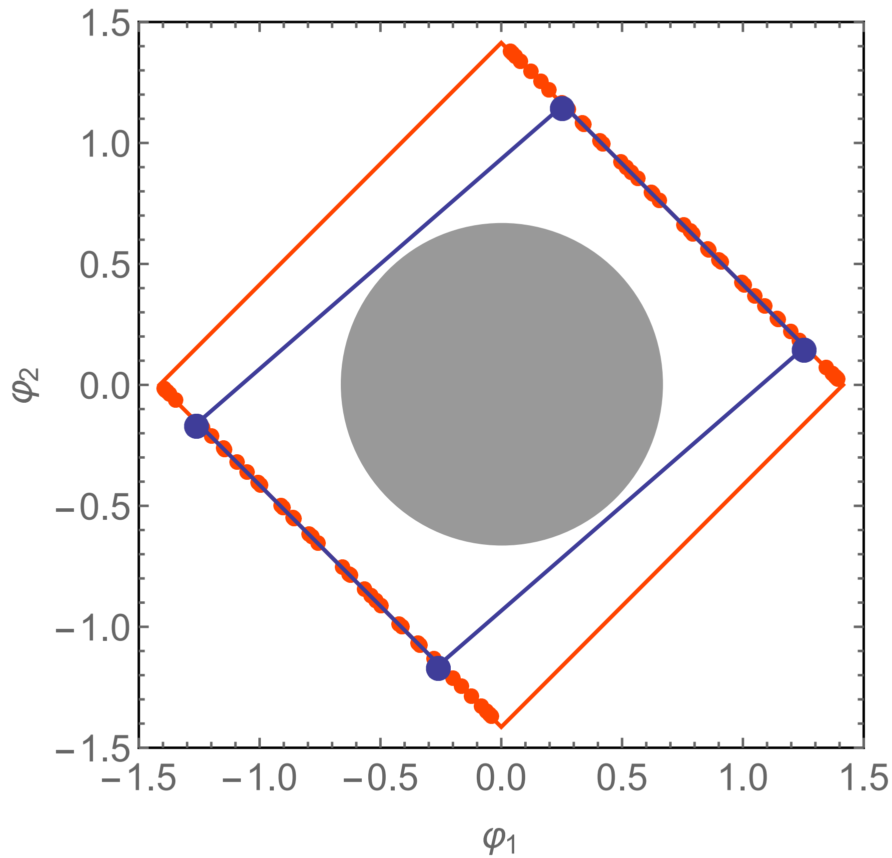
The wiggles in the case of weak alignment (small f)

Wiggles in the aligned potential



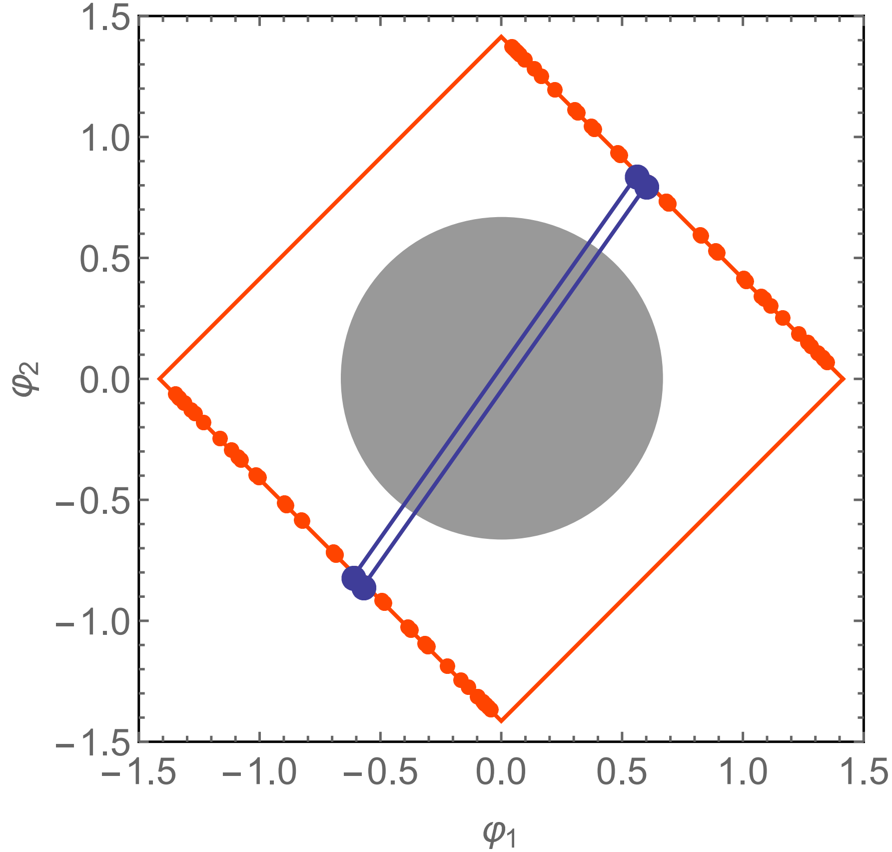
Strong alignment (potential superPlanckian f)

Weak alignment



The convex hull restrictions are trivially satisfied

Strong alignment



Subleading terms (red) satisfy the restrictions

Modulated natural Inflation

It seems plausible that under some circumstances the wiggles become important and

- spoil the flat direction,
- provide un upper limit on decay constant f_{eff}.

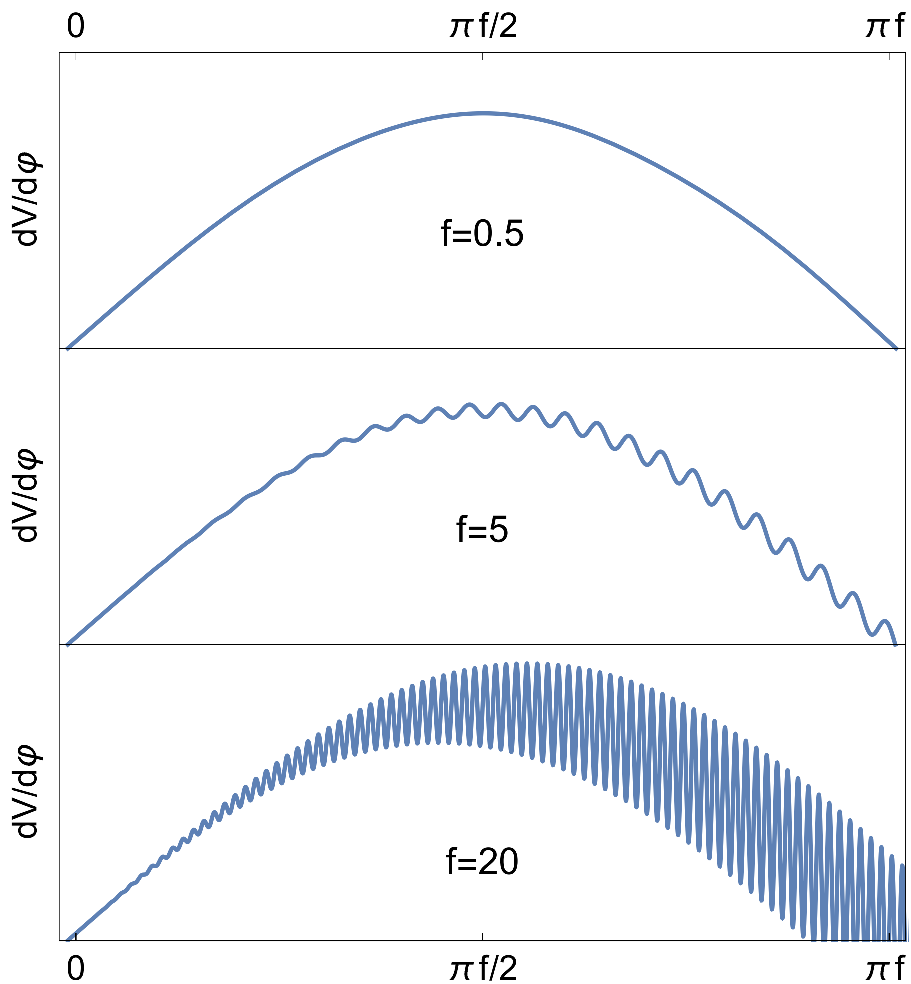
Explicit calculation are necessary to clarify the situation,

- but might be beyond our present capabilities;
- observational confirmation is extremely important.

Restrictions from WGC are satisfied here both in the aligned and non-aligned case.

(Kappl, Nilles, Winkler, 2015; Choi, Kim, 2015; Kobayashi, Nitta, Urakawa, 2016)

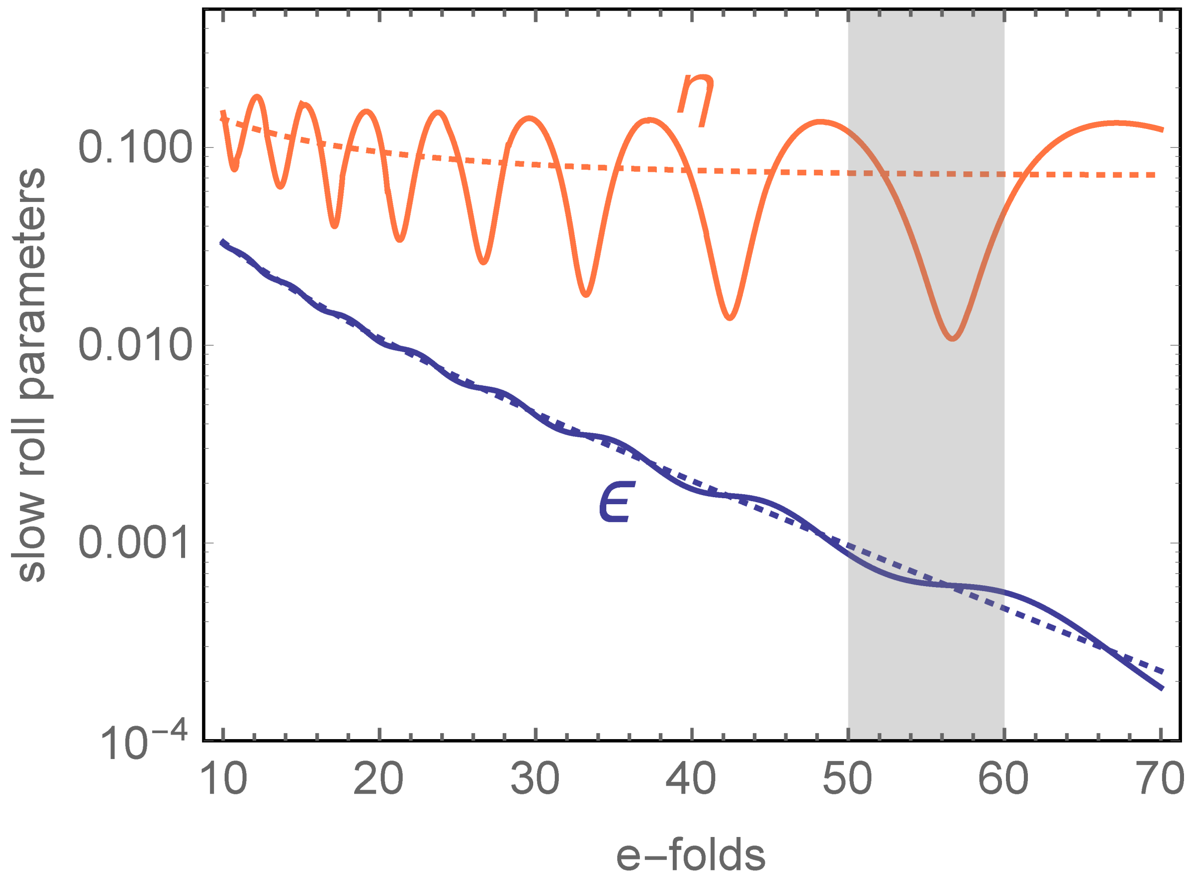
Slope of Potential



Wiggly structure becomes even more important

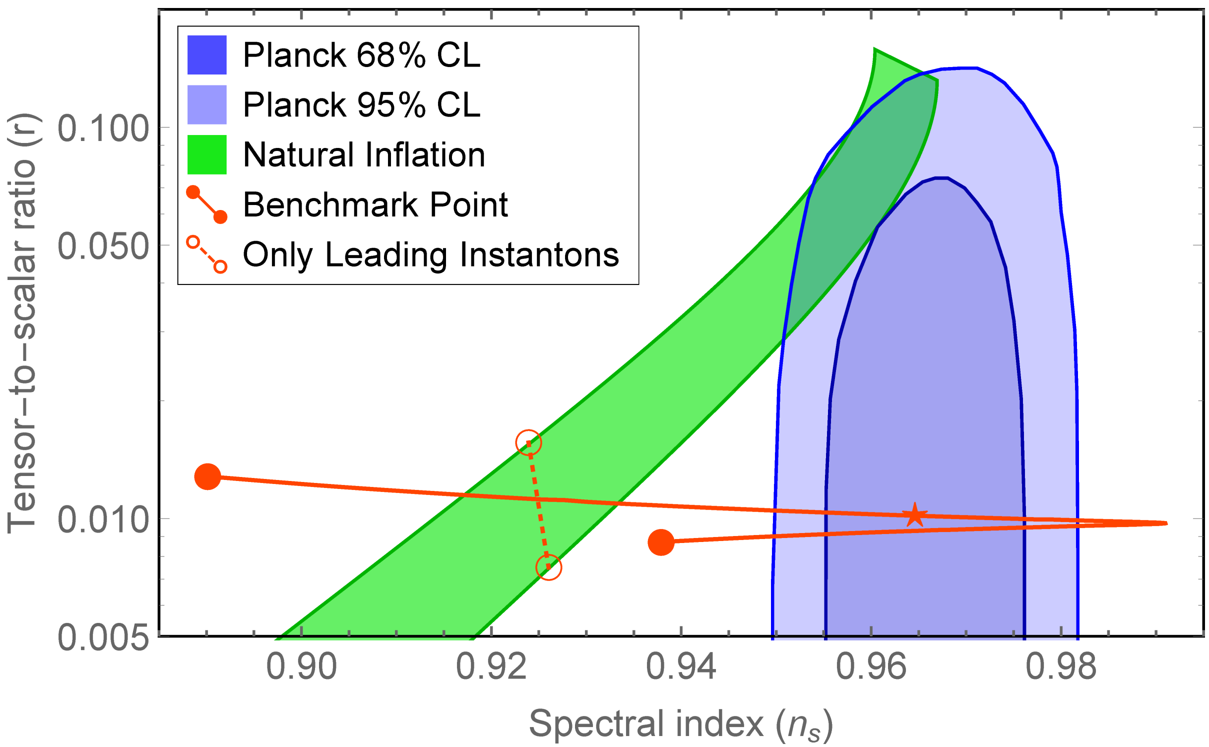
(Abe, Kobayashi, Otsuka, 2015; Kappl, Nilles, Winkler, 2015)

Slow roll parameters



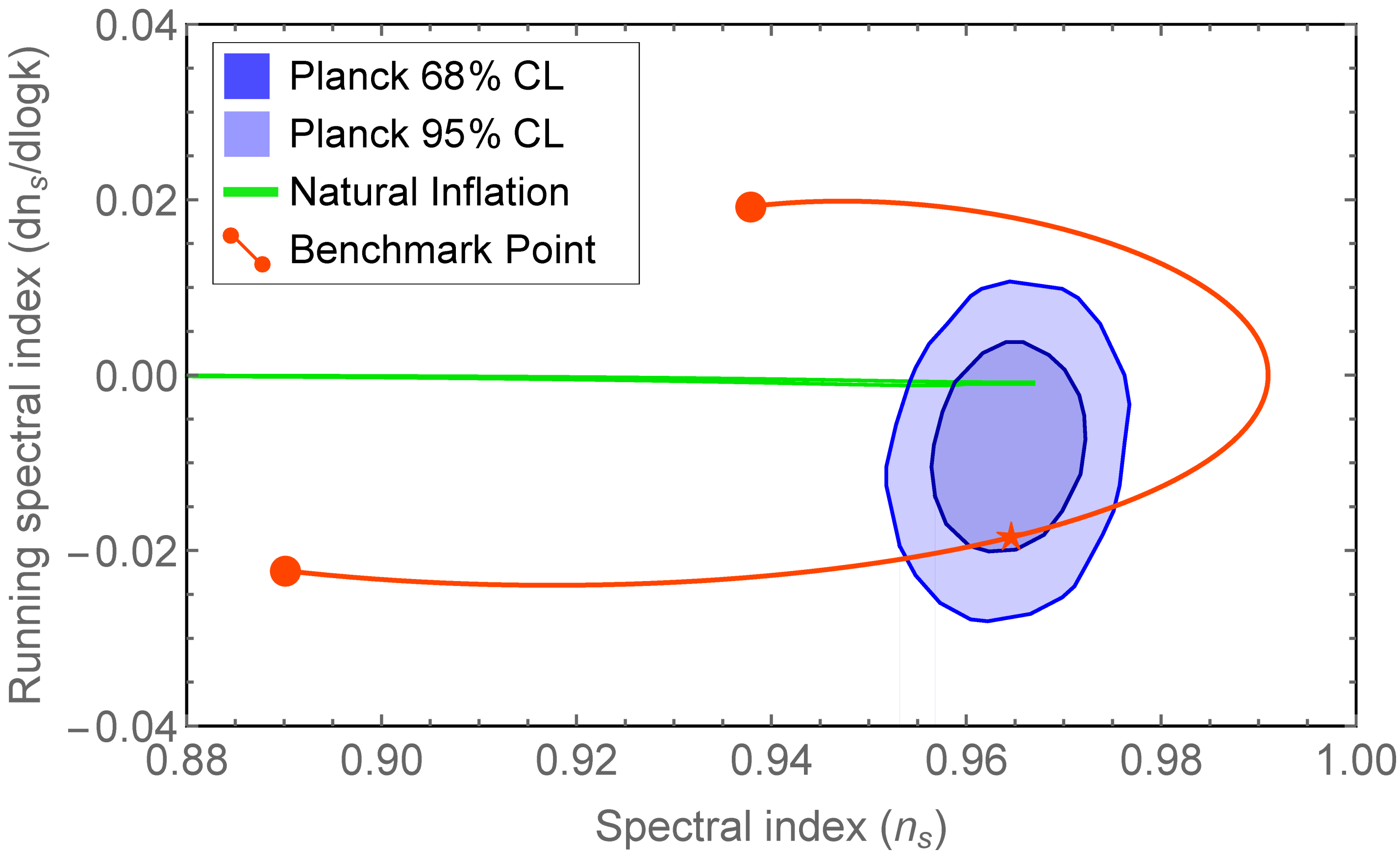
ϵ (η) depend on first (second) derivative

$n_s - r$ plane



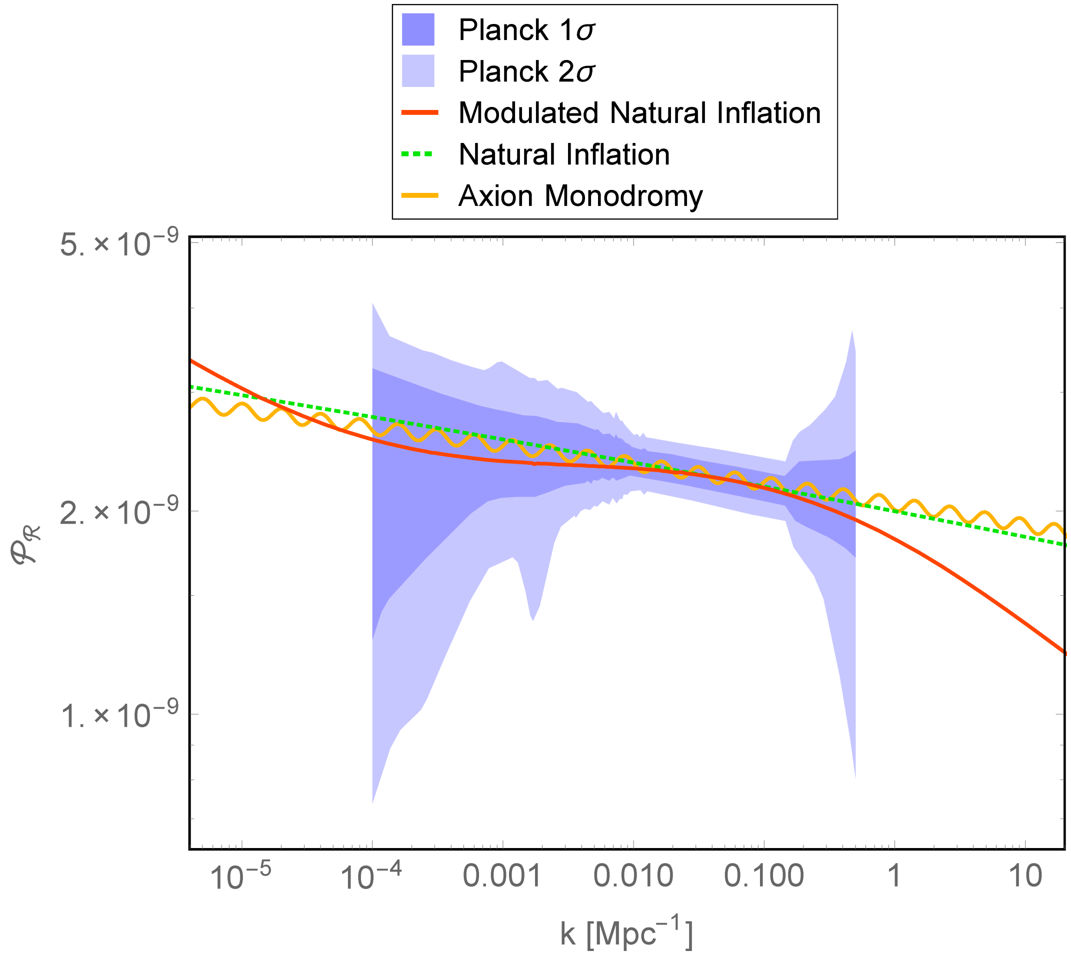
Strong variations of n_s on the number of e-folds

Running of spectral index



Comparison of spectral index with Planck data

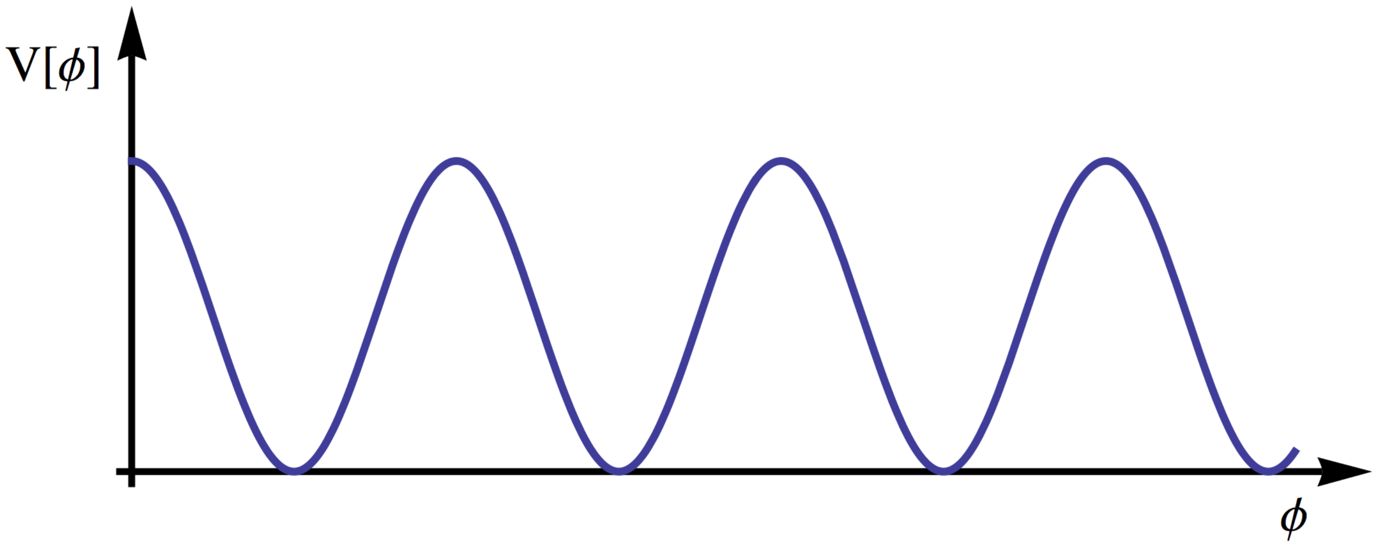
Scalar power spectrum



Comparison to Planck reconstructed power spectrum

Alignment and axionic domain walls

In general we have $a = a + 2\pi N f_a$ for $V \sim \cos(Na/f_a)$,



leading to N nontrivial degenerate vacua separated by maxima of the potential.

During the cosmic evolution this might lead to the production of potentially harmful axionic domain walls.

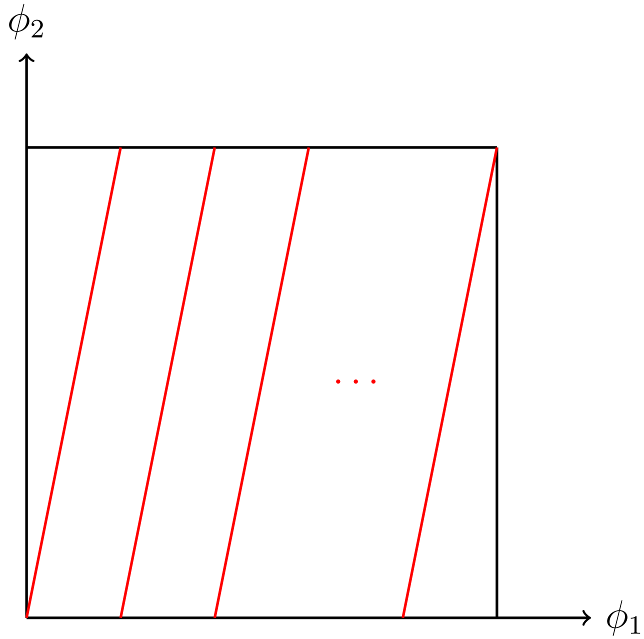
Two-Axion-model

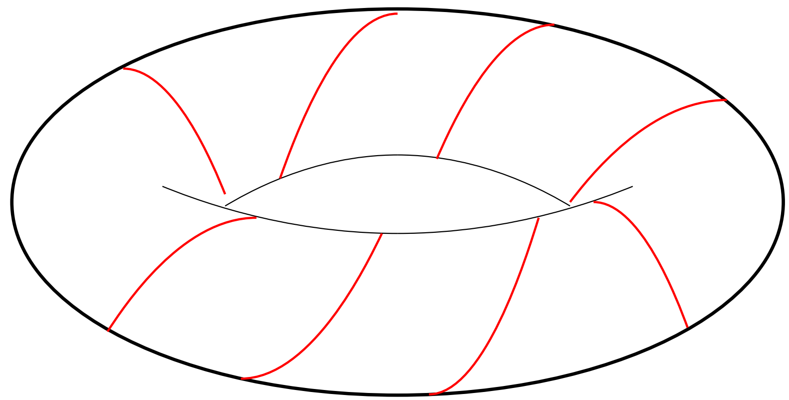
Consider a system with two axions

$$V \sim \Lambda_1^4 \cos\left(\frac{a_1}{f_1} + N\frac{a_2}{f_2}\right) + m\Lambda_2^3 \cos\left(\frac{a_2}{f_2}\right)$$

- For fixed a_1 there are N nontrivial vacua and potentially $N_{\rm DW} = N$ domain walls
- for $m = 0$ there is a Goldstone direction,
- and thus a continuous unique vacuum with effective domain wall number $N_{\rm DW} = 1$ (Choi, Kim, 1985)
- the Goldstone mode develops an axionic potential in the case $m \neq 0$

The axionic vacuum





(Choi, Kim, Yun, 2014)

- There is continuous unique vacuum with effective domain wall number is $N_{\rm DW} = 1$ (Choi, Kim, 1985)
- the Goldstone mode develops an axionic potential in the case $m \neq 0$

Quintessential axion alignment

Axions could be the source for dynamical dark energy

- in contrast to scalar quintessence, the axion has only derivative couplings and does not lead to a “fifth force"
- we need a slow roll field with $\Lambda \sim 0.003$ eV
- to act as dark energy today we need $f_a \geq M_{\text{Planck}}$
- the quintaxion mass is $m_a \sim \Lambda^2/M_{\text{Planck}} \sim 10^{-33}$ eV

Again we need a trans-Planckian decay constant for a consistent description of the present stage of the universe

- the problem can be solved via aligned axions à la KNP

(Kaloper, Sorbo, 2006)

The relaxion mechanism

Axions could be at the origin of mass hierarchies. This requires (Graham, Kaplan, Rajendran, 2015)

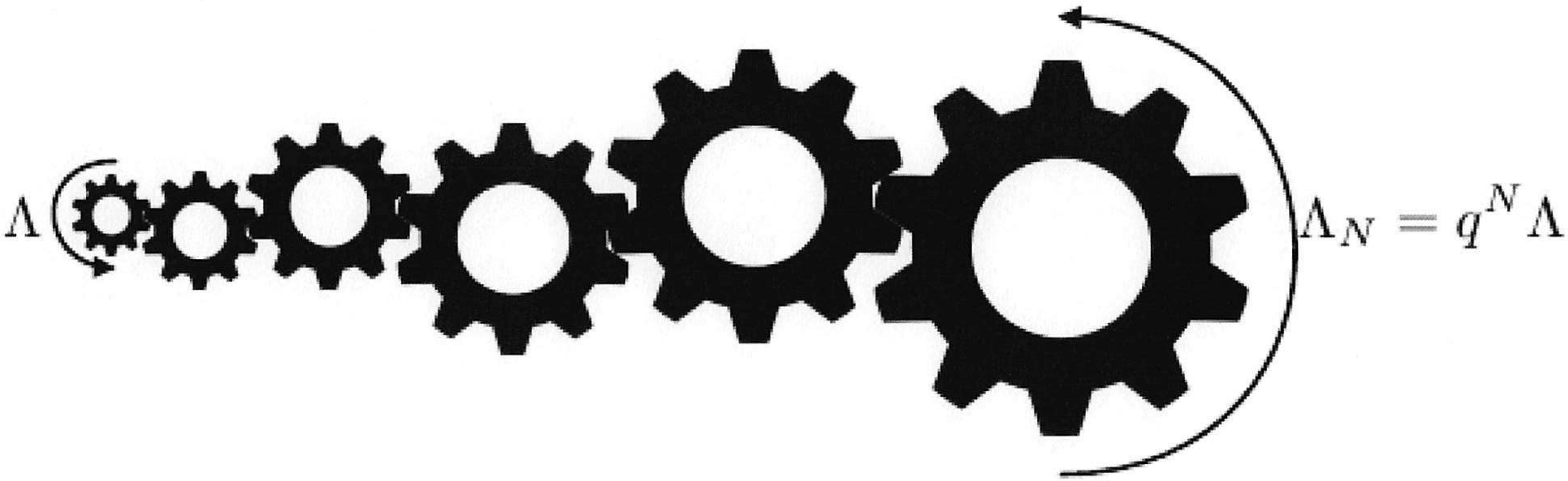
- a slowly rolling (relaxion) field,
- stopped by nonperturbative effects.
- Large mass hierarchies need a long time evolution of the relaxion field
- and an unconventional cosmological evolution.

Again we need a huge relaxion decay constant for a consistent description of the present stage of the universe:

- the problem can be solved via aligned axions à la KNP (called axion clock-work).

(Choi, Im, 2015; Kaplan, Rattazzi, 2015)

Alignment with N axions



Axion Clockworks

The step beyond towards $N > 2$ axions simplifies the creation of hierarchies leading from (small)^2 to (small)^N suppression. It has been applied to

- axionic inflation (Choi, Kim, Yun, 2014)
- the QCD axion (Higaki, Jeong, Kitajima, Takahashi, 2015)
- the relaxion (Choi, Im, 2015)

Towards large N

- similarity to the "deconstruction" of extra dimensions (Arkani-Hamed, Cohen, Georgi, 2001; Hill, Pokorski, Wang, 2001)
- what about the continuum limit? (Giudice, McCullough, 2016)

Continuous Clockwork

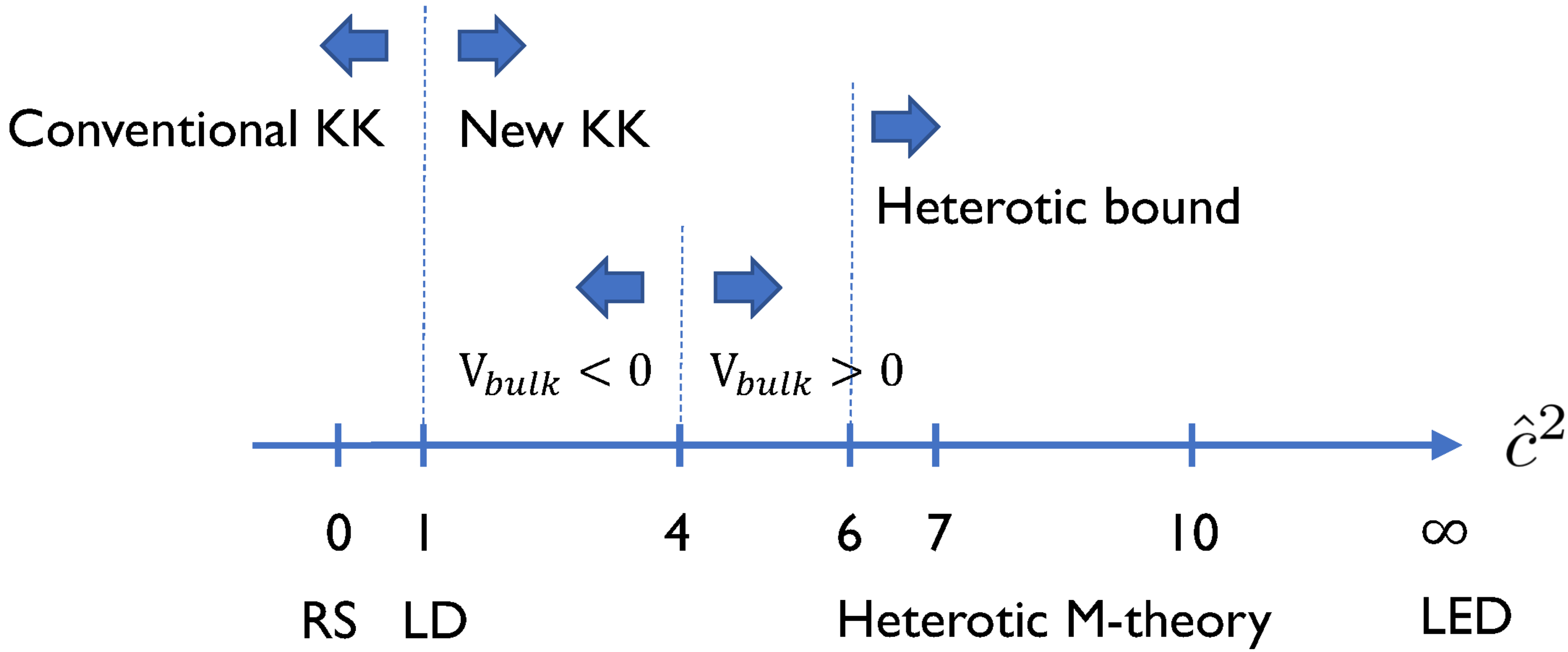
The transition from discrete to continous clockwork leads to a warped extra dimension. (Giudice, McCullough, 2016)

- simplest case corresponds to "Linear Dilaton Model"
- there are qualitative differences between the discrete and continuous case (Craig, Garcia Garcia, Sutherland, 2017)
- Generalized LD scenario includes RS and LED (Choi, Im, Shin, 2017)

The quest for an ultraviolet (UV) completion

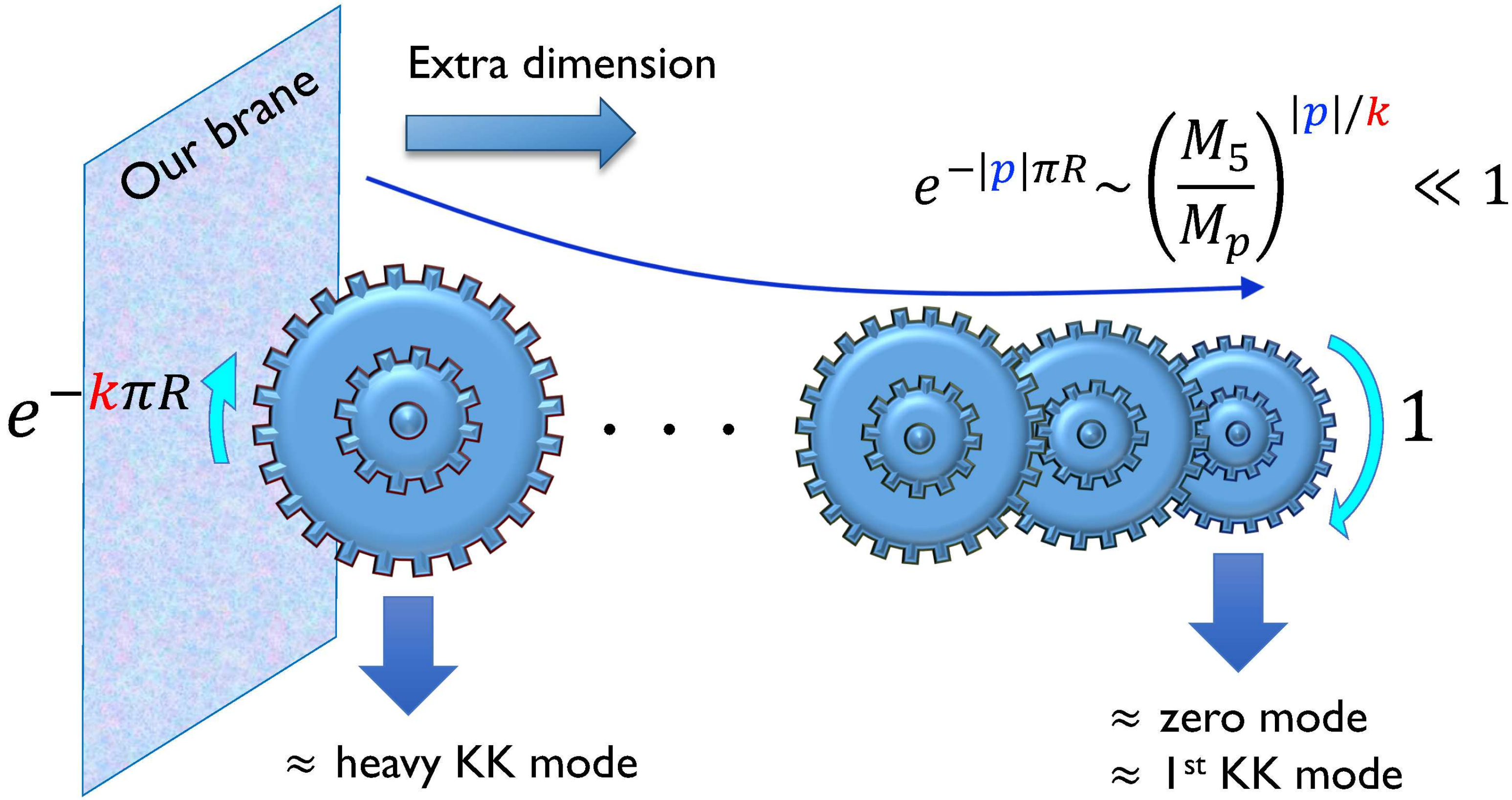
- heterotic M-theory with "unconventional" warping (Im, Nilles, Olechowski, 2018)
- discrete set of warped clockwork solutions allow for a consistent UV-completion

A discrete set of solutions



$$ds^2 = e^{2k_1 y} dx^2 + e^{2k_2 y} dy^2 \qquad p = k_1 - k_2 \qquad k_2 = \hat{c}^2 k_1$$

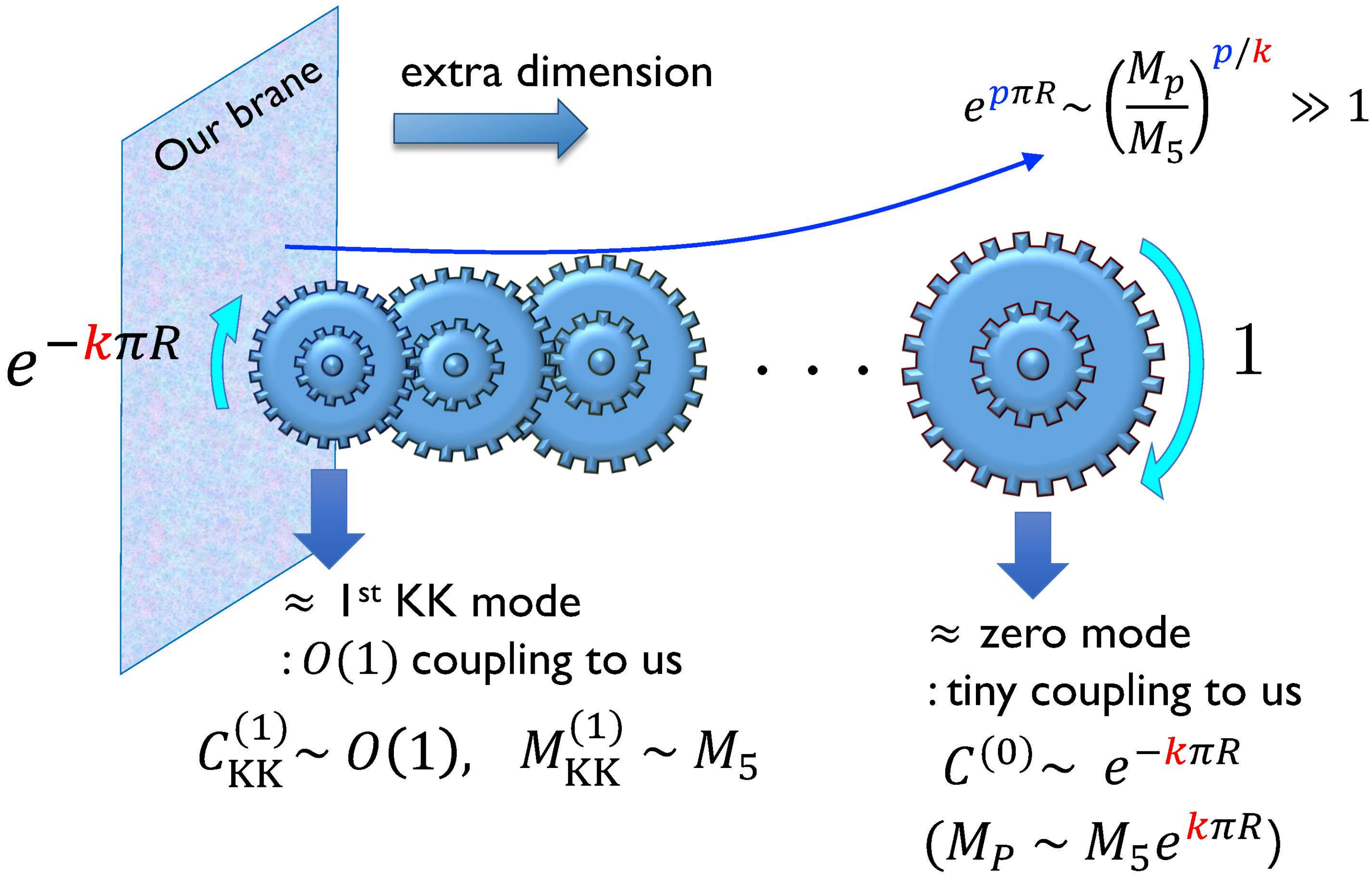
Large $\hat{c}^2$



$M_{\rm KK}^{(N)} \sim M_5\,,\ C_{\rm KK}^{(N)} \sim O(1)$

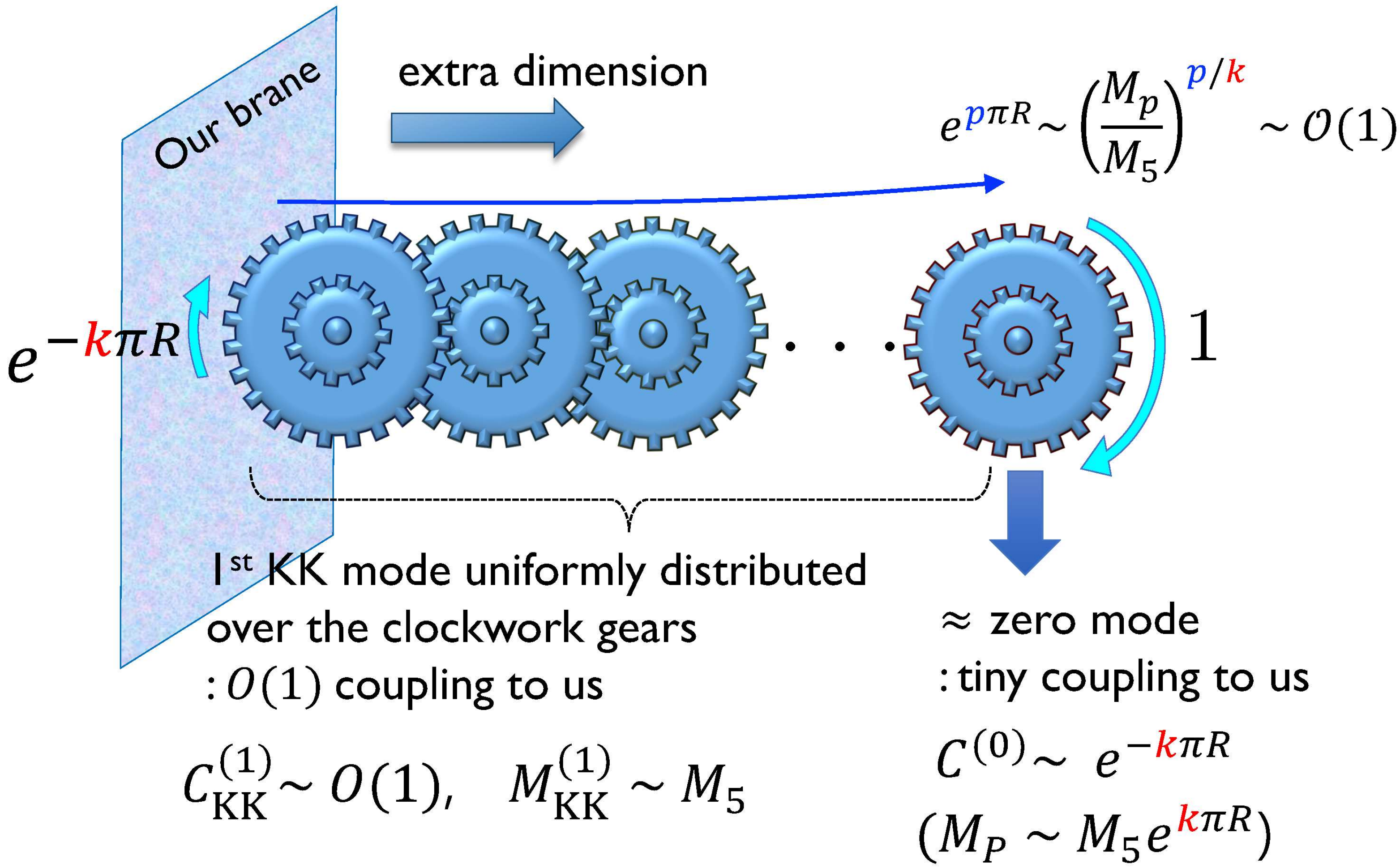
$M_{\rm KK}^{(1)} \sim M_5\, e^{-|p|\pi R}\,,\ C^{(0)} \sim C_{\rm KK}^{(1)} \sim e^{-k\pi R}$

Small $\hat{c}^2$



Our brane
extra dimension
$e^{p\pi R} \sim \left(\frac{M_p}{M_5}\right)^{p/k} \gg 1$
$e^{-k\pi R}$
1
≈ 1st KK mode
: $O(1)$ coupling to us
$C_{KK}^{(1)} \sim O(1), \quad M_{KK}^{(1)} \sim M_5$
≈ zero mode
: tiny coupling to us
$C^{(0)} \sim e^{-k\pi R}$
$(M_P \sim M_5 e^{k\pi R})$

Linear Dilaton: $\hat{c}^2 = 1$



QCD axion from heterotic M-theory

The same mechanism allows a hierarchy between the string scale and the axion scale f_a (Im, Nilles, Olechowski, 2019)

- various combinations of model-independent and model-dependent axions
- allow a unique implementation of a QCD axion
- with $f_a \sim M_{11}$ in the axion window (via warping)

It appears in heterotic M-theory with "unconventional" warping. The QCD axion is accompanied by

- an ultra-light axion on the hidden brane
- with decay constant $f_{\rm UL} \sim M_{11}\sqrt{\frac{M_{11}}{M_P}} < f_a$