

# Unified String Theory of Flavor and CP

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# Flavor and CP symmetries

Flavor symmetries are important ingredients of the SM

- Yukawa interaction for various families
- masses and mixings for quarks and leptons
- the question of **CP-symmetry and its violation**

Complicated structure not well understood

- **different structures in quark and lepton sectors**
- CP-violation needs complexity
- Flavor symmetries are highly non-universal

**Question about the origin of flavor and CP**

# Flavor from String Theory

String theory provides a variety of (discrete) flavor symmetries. This comes from

- geometrical structure of extra dimensions
- string selection rules

We present a new and general method to determine the flavor symmetries of string theory

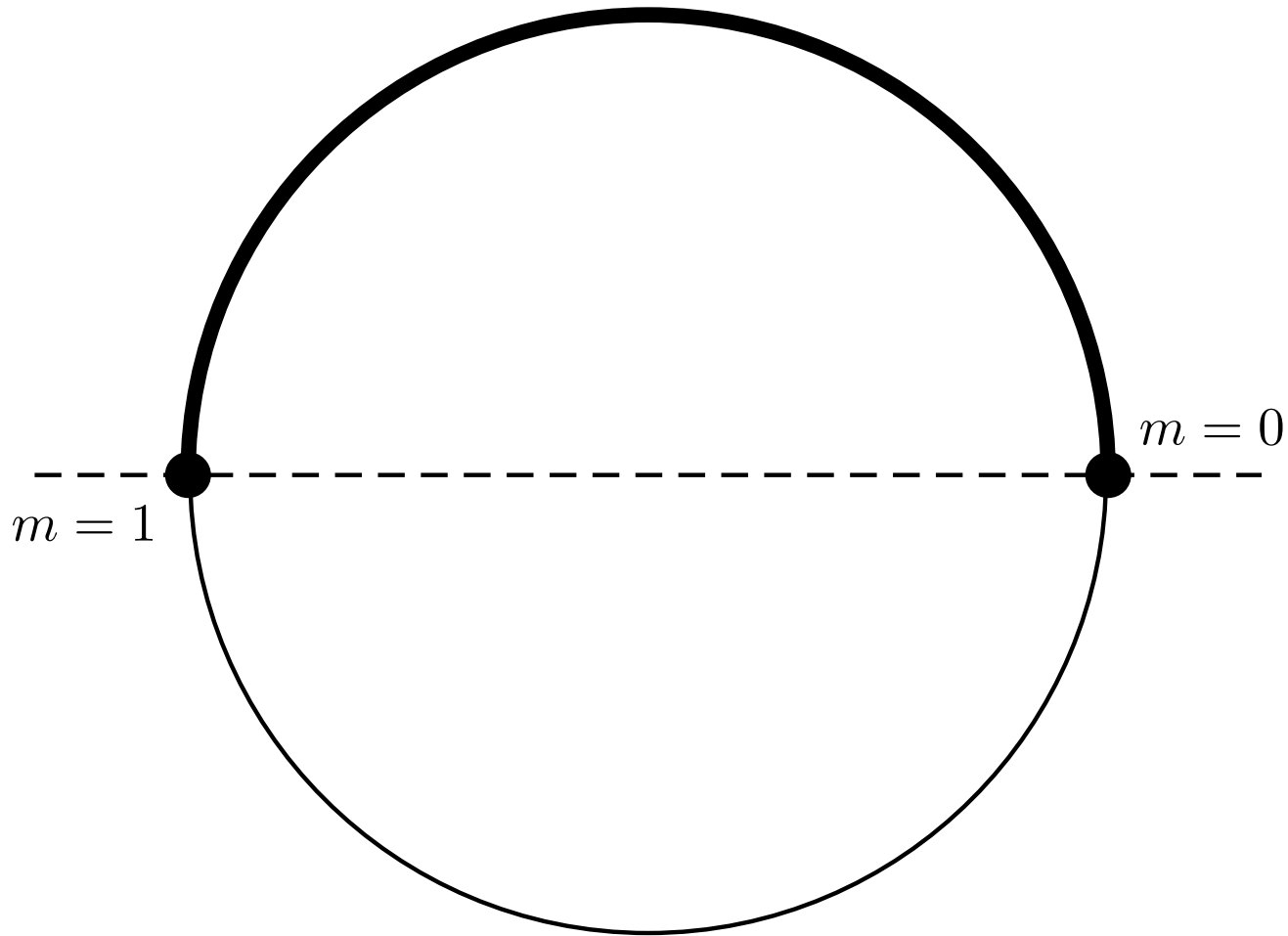
- it is based on an analysis of the Narain space group
- it unifies flavor and CP symmetries
- it includes modular symmetries in a nontrivial way

CP-transformation appears as a discrete gauge symmetry. Its origin are duality symmetries in string theory.

# Outline

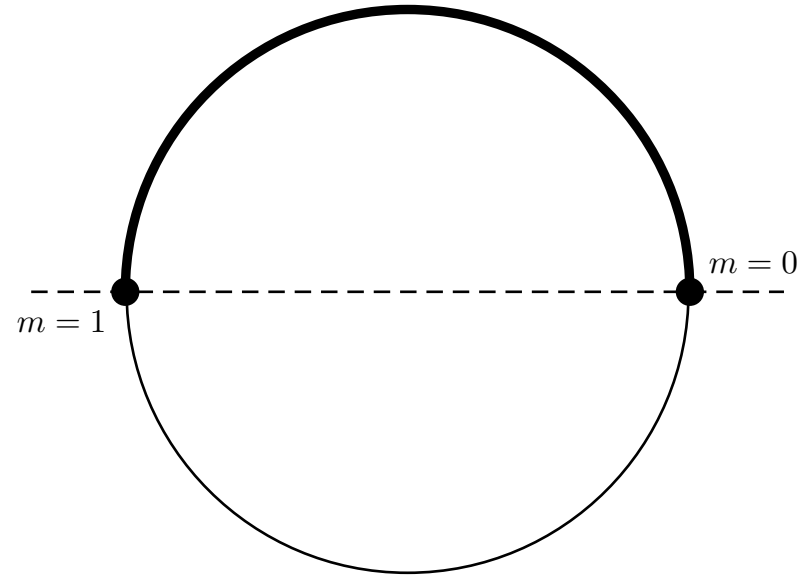
- the traditional approach to flavor symmetries via guesswork (Kobayashi, Nilles, Ploeger, Raby, Ratz, 2006)
- origin of CP and connection to flavor symmetries (Nilles, Ratz, Trautner, Vaudrevange, 2018)
- the Narain lattice and its outer automorphisms
- duality symmetries enhance "traditional" flavor symmetries (Baur, Nilles, Trautner, Vaudrevange, 2019)
- non-universal behaviour in moduli space
- explicit example of 2D  $Z_3$  orbifold and its "landscape" of flavor symmetries
- lessons from string theory for model building (Baur, Nilles, Trautner, Vaudrevange, to appear)

# Guessing symmetries: Interval $S_1/Z_2$



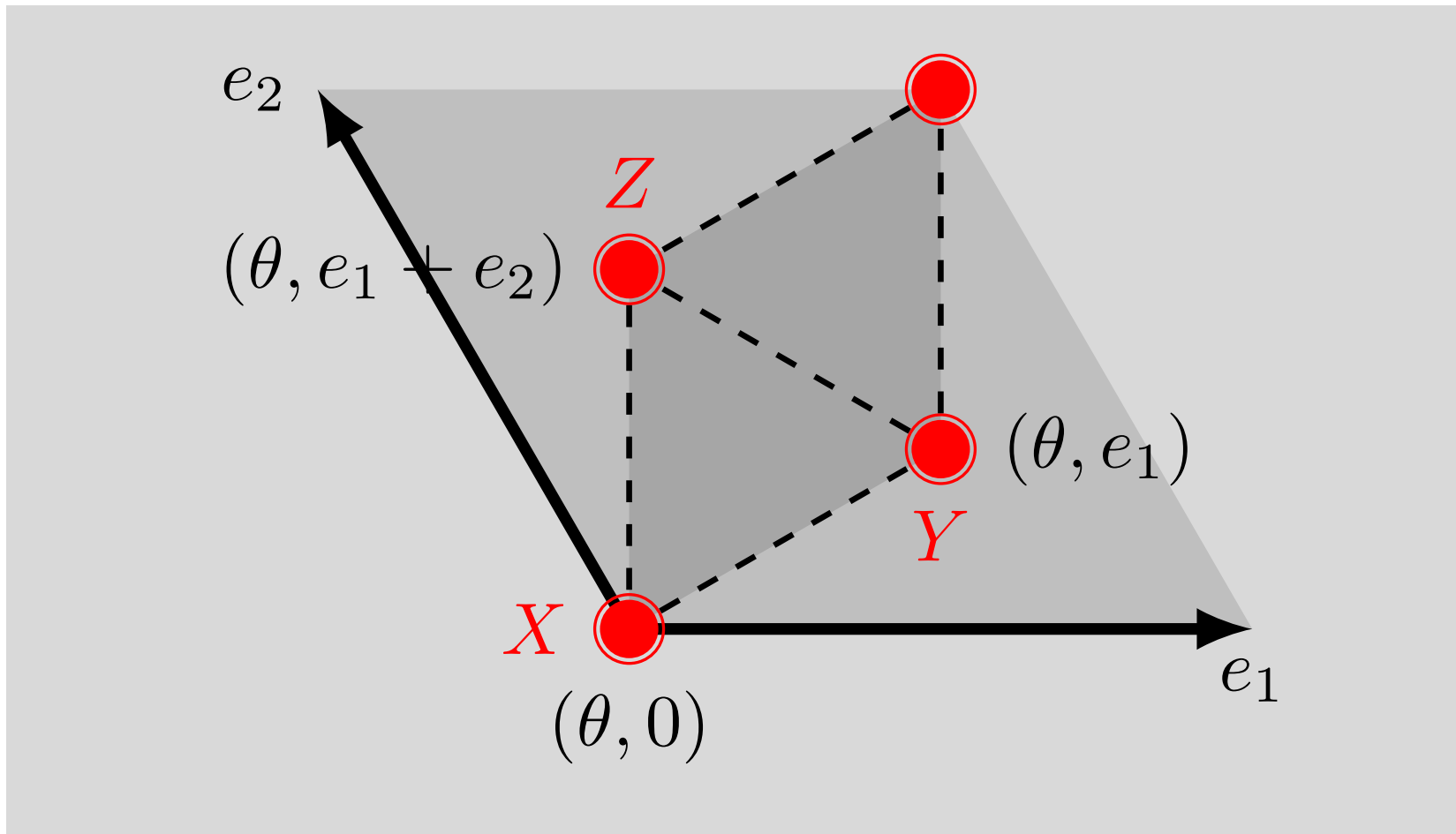
# Discrete symmetry $D_4$

- bulk and brane fields
- $S_2$  symmetry from interchange of fixed points
- $Z_2 \times Z_2$  symmetry from brane field selection rules
- $D_4$  as multiplicative closure of  $S_2$  and  $Z_2 \times Z_2$
- $D_4$  – a non-abelian subgroup of  $SU(2)_{\text{flavor}}$
- flavor symmetry for the two lightest families



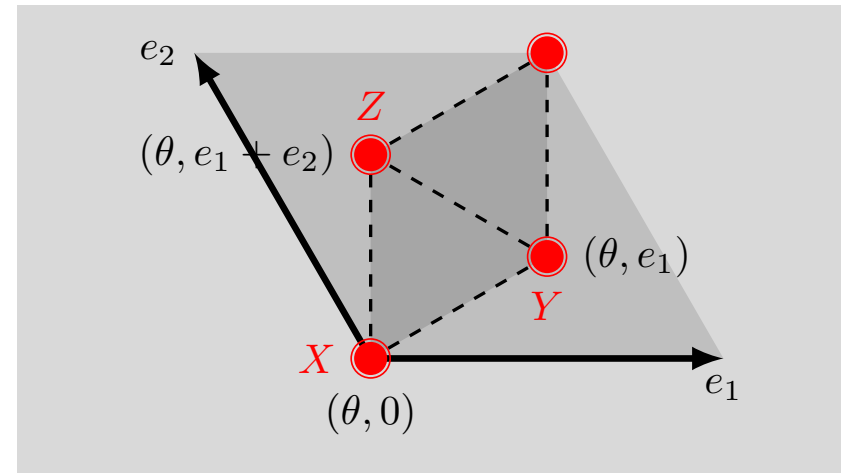
(Kobayashi, Nilles, Ploeger, Raby, Ratz, 2006)

# Orbifold $T_2/Z_3$



# Discrete symmetry $\Delta(54)$

- untwisted and twisted fields
- $S_3$  symmetry from interchange of fixed points
- $Z_3 \times Z_3$  symmetry from orbifold selection rules



- $\Delta(54)$  as multiplicative closure of  $S_3$  and  $Z_3 \times Z_3$
- $\Delta(54)$  – a non-abelian subgroup of  $SU(3)_{\text{flavor}}$
- flavor symmetry for three families of quarks and leptons

(Kobayashi, Nilles, Ploeger, Raby, Ratz, 2006)



# $\Delta(54)$ group theory

$\Delta(54)$  is a non-abelian group and has representations:

- one **trivial singlet**  $1_0$  and one **nontrivial singlet**  $1_-$
- two **triplets**  $3_1, 3_2$  and corresponding **anti-triplets**  $\bar{3}_1, \bar{3}_2$
- four **doublets**  $2_k$  ( $k = 1, 2, 3, 4$ )

Some relevant tensor products are:

- $3_1 \otimes \bar{3}_1 = 1_0 \oplus 2_1 \oplus 2_2 \oplus 2_3 \oplus 2_4$
- $2_k \otimes 2_k = 1_0 \oplus 1_- \oplus 2_k$

$\Delta(54)$  is a good candidate for a flavour symmetry.

**But where is CP?**

Is there any way to obtain CP from string theory?

# CP as outer automorphism

Outer automorphisms map the group to itself but are not group elements themselves

- $\Delta(54)$  has outer automorphism group  $S_4$
- CP could be one of the three  $Z_2$  subgroups of this  $S_4$
- Physical CP transforms  $(rep)$  to  $(rep)^*$

This gives an intimate relation of flavour and CP symmetry

- CP broken due to the presence of winding modes
- lepto-genesis through decay of winding modes
- CP-violation à la CKM via field dependent Yukawa couplings

(Nilles, Ratz, Trautner, Vaudrevange, 2018)

# Search for a general method

We have seen that even in simple systems we obtain sizeable flavor groups

- $D_4$  for the interval
- $\Delta(54)$  for the 2-dimensional  $Z_3$  orbifold

Did we find the complete flavor symmetry in these cases?

In reality we have even six compact dimensions

- variety of complicated group structures possible
- dependence on localisation of fields in extra dimensions

A general mechanism to find the complete set of flavor symmetries is based on the

outer automorphisms of the Narain space group.

(Baur, Nilles, Trautner, Vaudrevange, 2019)

# The Narain Lattice

In the string there are  $D$  right- and  $D$  left-moving degrees of freedom  $Y = (y_R, y_L)$ .  $Y$  compactified on a  $2D$  torus

$$Y = \begin{pmatrix} y_R \\ y_L \end{pmatrix} \sim Y + E\hat{N} = \begin{pmatrix} y_R \\ y_L \end{pmatrix} + E \begin{pmatrix} n \\ m \end{pmatrix}$$

defines the **Narain lattice** with

- the string's winding and Kaluza-Klein quantum numbers  $n$  and  $m$
- the **Narain vielbein matrix**  $E$  that depends on the moduli of the torus: radii, angles and anti-symmetric tensor fields  $B$ .

# The Narain Space Group

A  $Z_K$  orbifold with twist  $\Theta$  leads to the identification

$$Y \sim \Theta^k Y + E\hat{N} \quad \text{where} \quad \Theta = \begin{pmatrix} \theta_R & 0 \\ 0 & \theta_L \end{pmatrix} \quad \text{and} \quad \Theta^K = 1$$

with  $\theta_L, \theta_R$  elements of  $SO(D)$ . For a symmetric orbifold  $\theta_L = \theta_R$  (we do not include roto-translations here).

The Narain space group  $g = (\Theta^k, E\hat{N})$  is then generated by

twists  $(\Theta, 0)$  and shifts  $(1, E_i)$  for  $i = 1 \dots 2D$

Outer automorphisms map the group to itself but are not elements of the group.

# Duality Transformations

Modular transformations (dualities) exchange windings and momenta and act nontrivially on the moduli of the torus.

In  $D = 2$  these transformations are connected to the group  $SL(2, Z)$  acting on Kähler and complex structure moduli.

The group  $SL(2, Z)$  is generated by two elements

$$S, T : \quad \text{with } S^4 = 1 \text{ and } S^2 = (ST)^3$$

On a modulus  $M$  we have the transformations

$$S : M \rightarrow -\frac{1}{M} \quad \text{and} \quad T : M \rightarrow M + 1$$

Further transformations might include  $M \rightarrow -\overline{M}$  and mirror symmetry between Kähler and complex structure moduli.

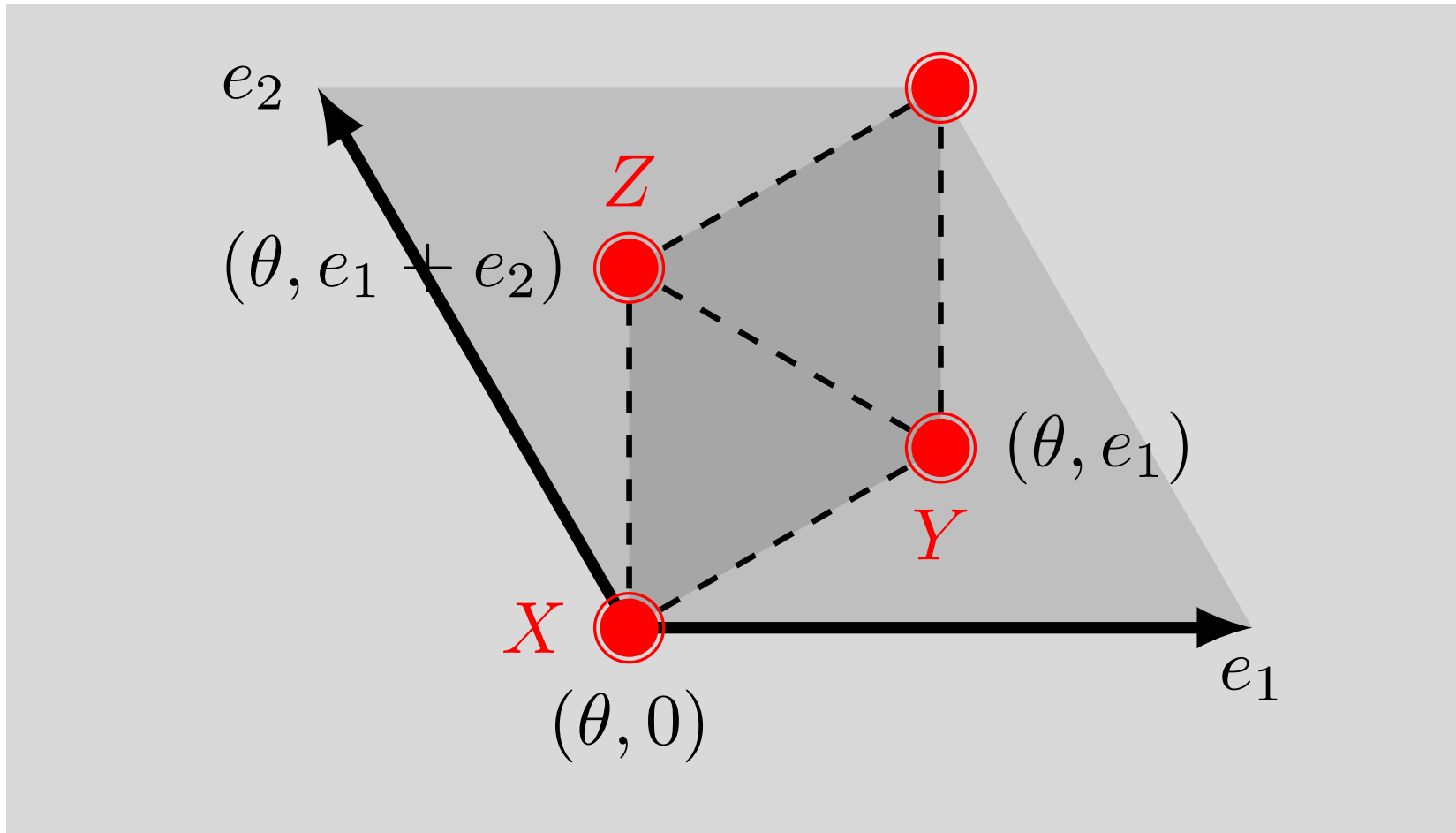
# Candidate symmetries

As outer automorphisms of the Narain space group we can identify

- traditional flavor symmetries which are **universal in moduli space**
- a subset of the modular **transformations** that act as **symmetries** at specific "points" in moduli space
- at these "points" we shall have an **enhanced symmetry** that combines the traditional flavor symmetry with some of the modular symmetries

**The full flavor symmetry is non-universal in moduli space.**  
At generic points in moduli space we have the universal traditional flavor symmetry.

# Orbifold $T_2/Z_3$





# Example: $T_2/Z_3$ Orbifold

On the orbifold some of the moduli are frozen

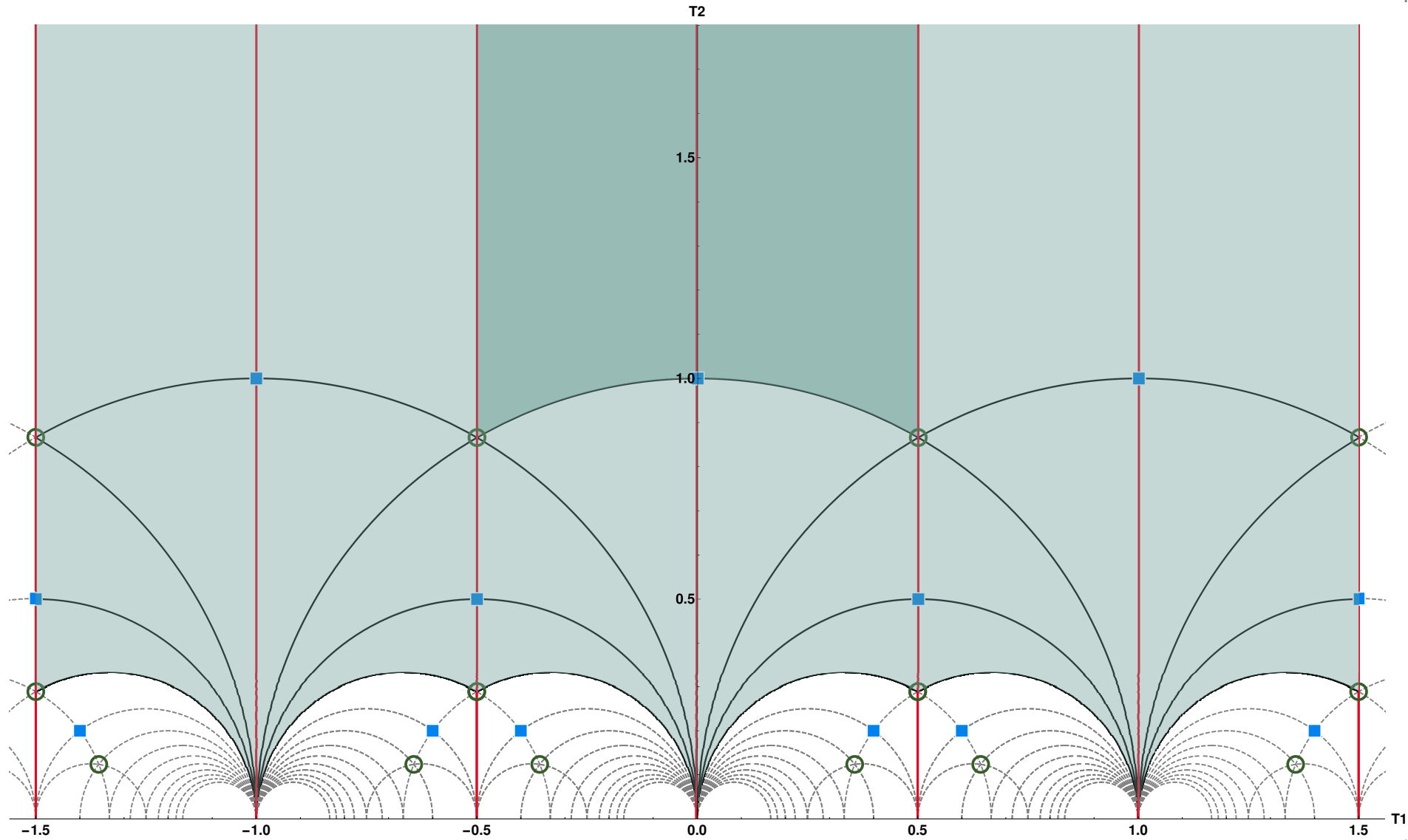
- lattice vectors  $e_1$  and  $e_2$  have the same length
- angle is 120 degrees

Modular transformations form a subgroup of  $SL(2, Z)$

- $\Gamma(3)$  as a mod(3) subgroup of  $SL(2, Z)$ ; ( $\Gamma(3) = A_4$ )
- $\Gamma(3)$  acts on the moduli
- twisted fields transform under a **bigger group  $T'$** ,  
(similar to enhancement of  $SO(3)$  to  $SU(2)$  for spinors)  
(Lauer, Mas, Nilles, 1989; Lerche, Lüst, Warner, 1989)
- transformation  $M \rightarrow -\overline{M}$  completes the picture

**Full group is  $SG(48,29)$  with 48 elements**

# Moduli space of $\Gamma(3)$



# Flavor Symmetries I

Generic point in moduli space.

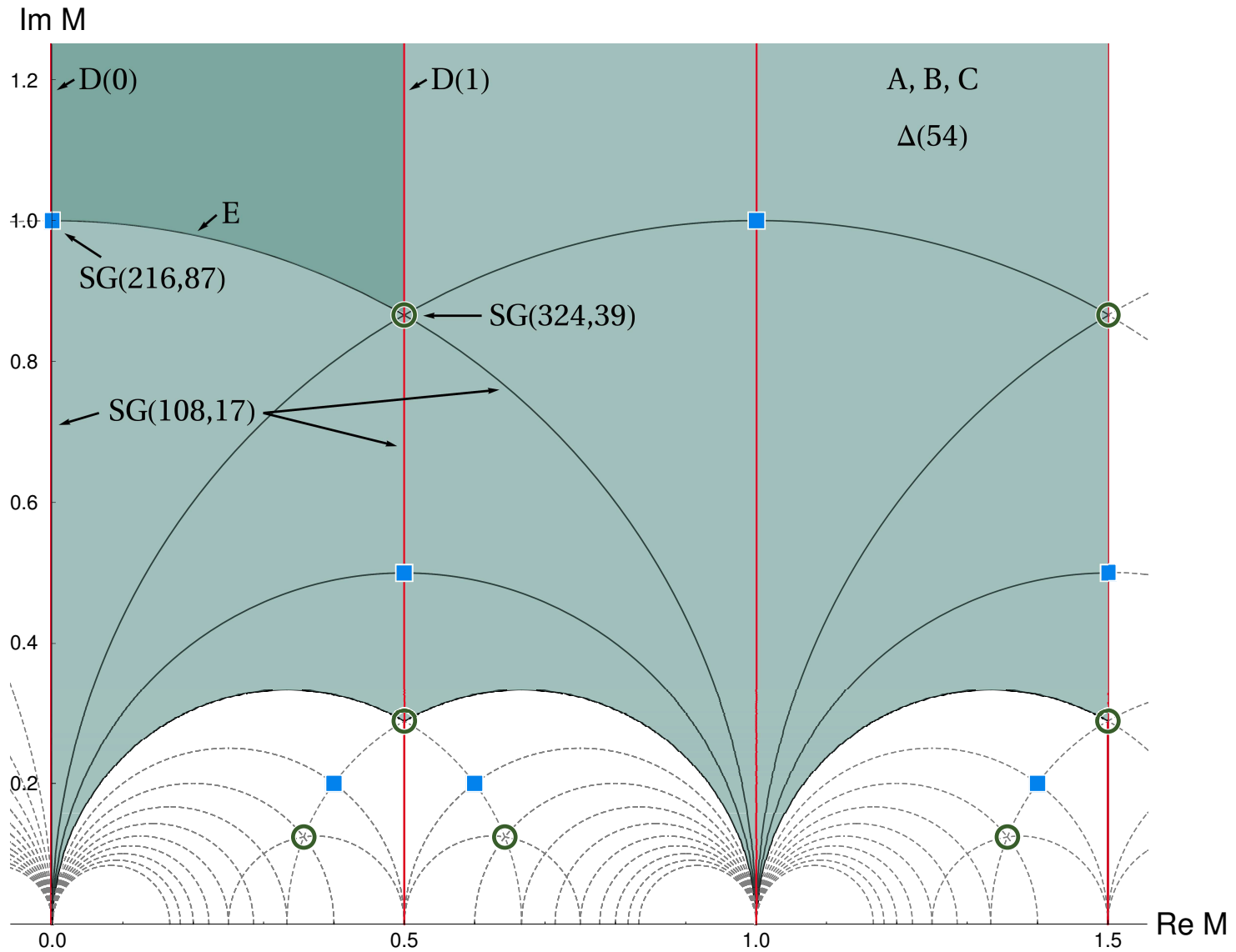
Outer automorphisms of the Narain space group are

- shift  $A = (1_4; \frac{1}{3}, \frac{2}{3}, 0, 0)$
- and shift  $B = (1_4; 0, 0, \frac{1}{3}, \frac{1}{3})$
- a left-right symmetric rotation  $C = (-1_4; 0, 0, 0, 0)$

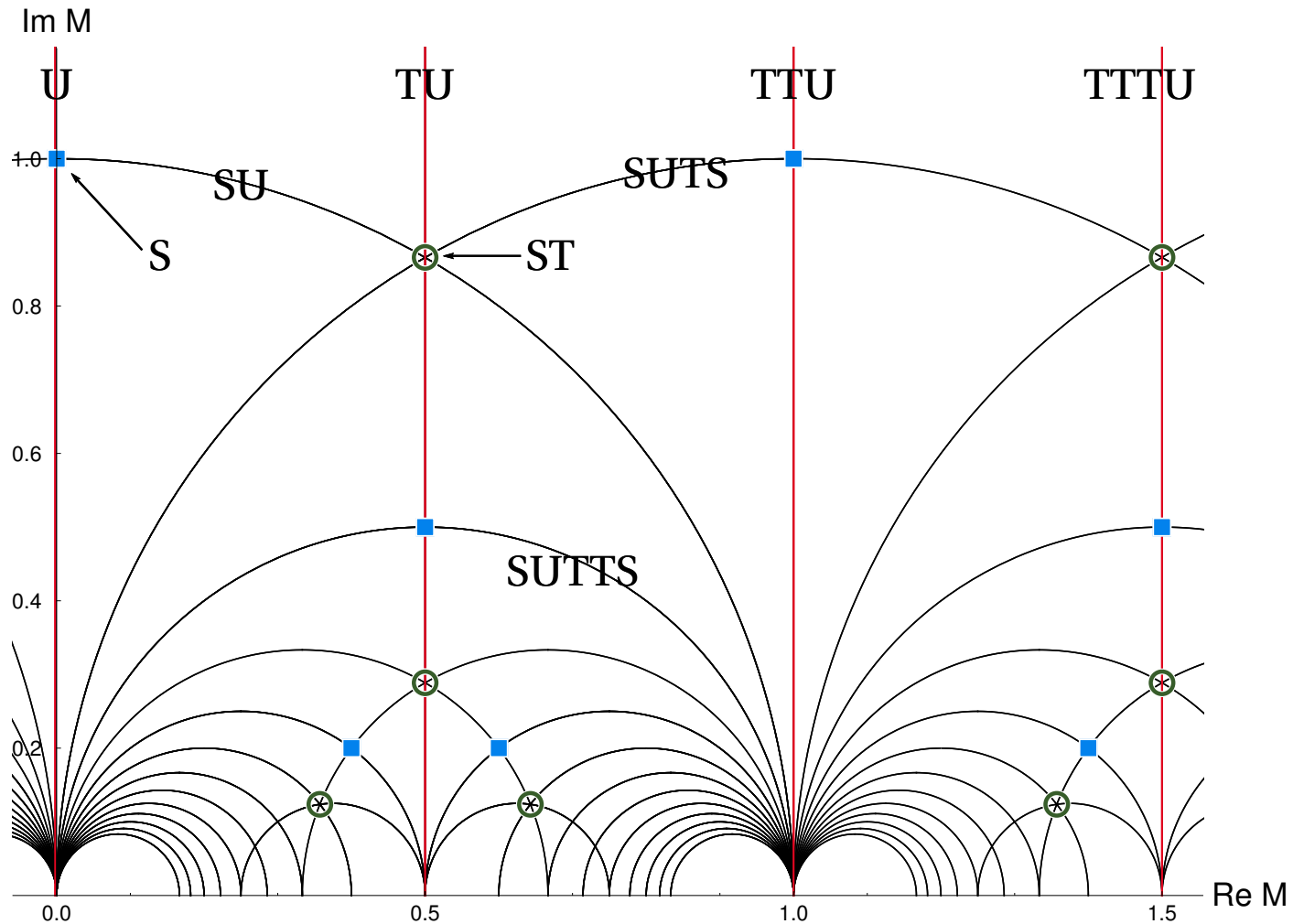
Multiplicative closure of  $A$ ,  $B$  and  $C$  leads to  $\Delta(54)$ .

- the earlier guesswork gave the correct result!
- but the new method produces the result automatically
- can be generalised easily to more complicated situations (like, e.g. six dimensions)

# Moduli space of flavour groups



# Fixed lines and points



$$S : M \rightarrow -\frac{1}{M}, \quad T : M \rightarrow M + 1 \quad \text{and} \quad U : M \rightarrow -\overline{M}$$

# Flavor Symmetries II

Duality transformations might become symmetries!

The red lines:

These are fixed lines under  $T$  and  $U$ . We have

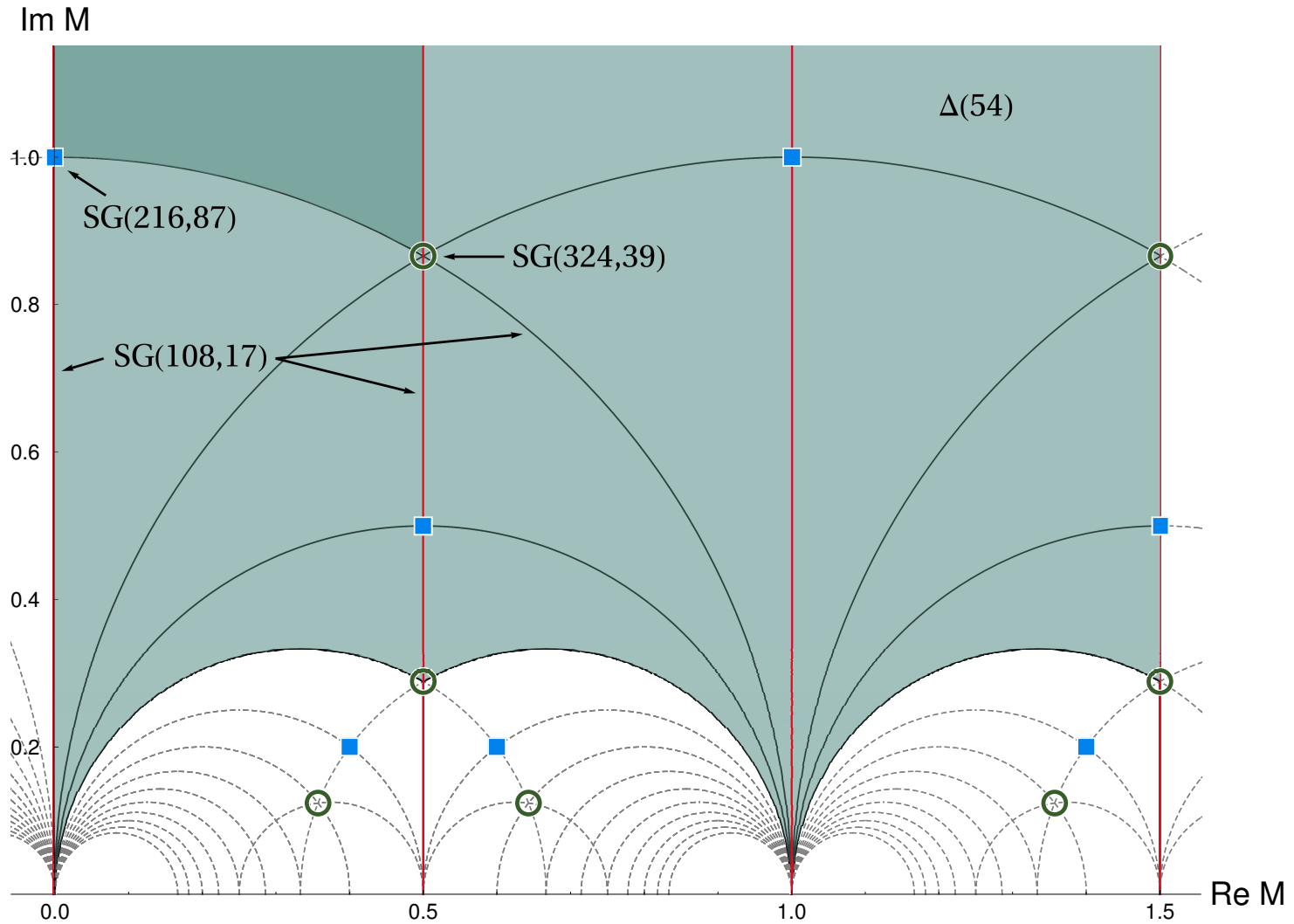
- again  $A, B, C$  and a left-right symmetric reflection  $D$

Multiplicative closure leads to  $SG(108, 17)$ . This includes the formerly discussed CP-transformation! Unification of flavor and CP (spontaneous breakdown away from the line).

The circles: e.g. fixed lines under  $S$  and  $U$

- new asymmetric reflection  $E$  (instead of  $D$ )
- again  $SG(108, 17)$  but differently aligned
- enhanced with different  $Z_2$  from  $S_4 = \text{Out}(\Delta(54))$

# Moduli space of flavour groups



# Flavor Symmetries III

Blue squares: two lines meet

- enhancement to  $SG(216, 87)$

The small circles: three lines meet

- maximum enhancement to  $SG(324, 39)$

The modular group  $T'$  has 24 elements, but not all of them lead to an enhancement of the flavor group  $\Delta(54)$ .

Only the elements within  $S_4$  of the outer automorphisms of  $\Delta(54)$  are relevant

- this leads to unification of flavour and CP
- CP exact at those fixed lines and points



# Messages

We have designed a generic method to find all flavor symmetries (based on the Narain space group)

- unification of traditional (discrete) flavor, CP and modular symmetries
- traditional flavor symmetry is universal in moduli space
- there are non-universal enhancements (including CP at some places (broken in generic moduli space))
- CP is a consequence of the duality symmetries of string theory
- the potential flavor groups are large and non-universal (in our example already up to  $SG(324, 39)$  for two extra dimensions)

# Consequences

This opens a new arena for flavor model building

- a new look at CP as discrete gauge symmetry  
(Nilles, Ratz, Trautner, Vaudrevange, 2018)
- modular symmetries for flavor (Altarelli, Feruglio, 2006; Feruglio, 2017)
- groups are large and allow for flexibility (Hagedorn, König, 2018)
- the concept of local flavor symmetries allows different flavor groups for different sectors of the theory  
(Baur, Nilles, Trautner, Vaudrevange, 2019)
- non-universal structure from duality symmetries  
(there is still the traditional universal flavor group)
- different flavor symmetries for quarks and leptons are no surprise