

# Axions, Alignment and Clockworks

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# Axions

Axions have been first discussed in the framework of solutions to the strong CP problem. An axion is a

- pseudo-scalar particle  $a$
- pseudo-Goldstone boson of a global symmetry  $U(1)_A$
- $U(1)$  is anomalous
- coupling to gauge fields  $aF\tilde{F}$  via the anomaly

Relevant parameters include

- the axion decay constant  $f_a$ :  $\frac{a}{f_a}F\tilde{F}$
- its mass  $m_a$
- coupling to photons and gluons

# Use of Axions

Axions can play a role for

- the strong CP problem in QCD (Peccei, Quinn, 1977)
- the mechanism of inflation (Freese, Frieman, Olinto, 1990)
- the source of quintessence (Frieman, Hill, Stebbins, Waga, 1995)
- the relaxion (Graham, Kaplan, Rajendran, 2015)

Axions are abundant in string theory constructions, where they originate from various anti-symmetric tensor fields

- there is an opportunity for multi-axion systems
- that seems to be helpful for the consistency of some of the axionic models (**alignment of axions**)

# Scales and hierarchies

Usually the axions come with specific scales

- the QCD axion requires a specific window for  $f_a$  between  $10^9$  and  $10^{12}$  GeV (large compared to  $M_{\text{weak}}$ )
- trans-Planckian field values in axionic inflation
- quintessential axion requires extremely flat potential and small mass of  $10^{-33}$  eV
- extreme fine tunings for the relaxation

One of the challenges is to understand these scales within a consistent UV-complete theory

- in string theory the scale for  $f_a$  set by  $M_{\text{string}}$
- the relevance of **alignment** enters here

# Alignment

Alignment has been first suggested in natural inflation.

We shall discuss here

(Kim, Nilles, Peloso, 2005)

- axionic inflation
- tensor modes in CMB fluctuations
- trans-Planckian excursions of the (axionic) inflaton field
- **the alignment of axions**

Other application of multi-axion systems

- axionic domain walls for QCD axion (Choi, Kim, 1985)
- alignment of quintessential axions (Kaloper, Sorbo, 2006)
- the relaxion mechanism (Choi, Im, 2015)

# The Quest for Flatness

The mechanism of inflation requires a “flat” potential.  
We demand

- symmetry reason for flatness of the potential
- slightly broken symmetry to move the inflaton

The obvious candidate is **axionic inflation**

- axion has only derivative couplings to all orders in perturbation theory
- broken by non-perturbative effects (instantons)

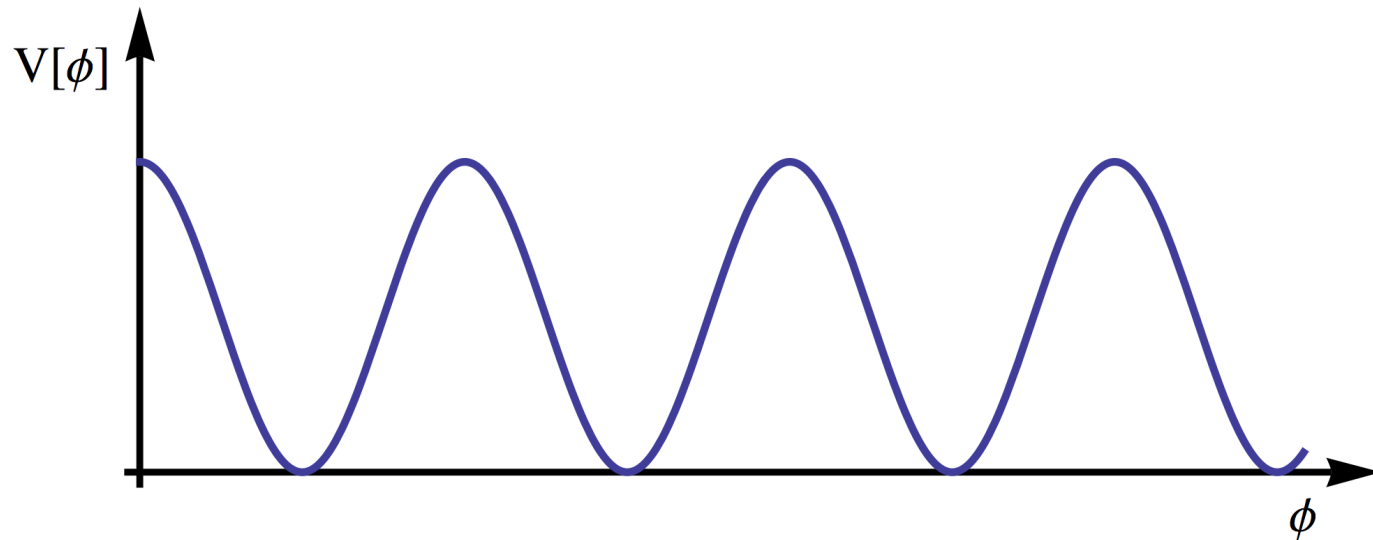
Motivated by the QCD axion

(Freese, Frieman, Olinto, 1990)

# The Axion Potential

The axion exhibits a shift symmetry  $\phi \rightarrow \phi + c$

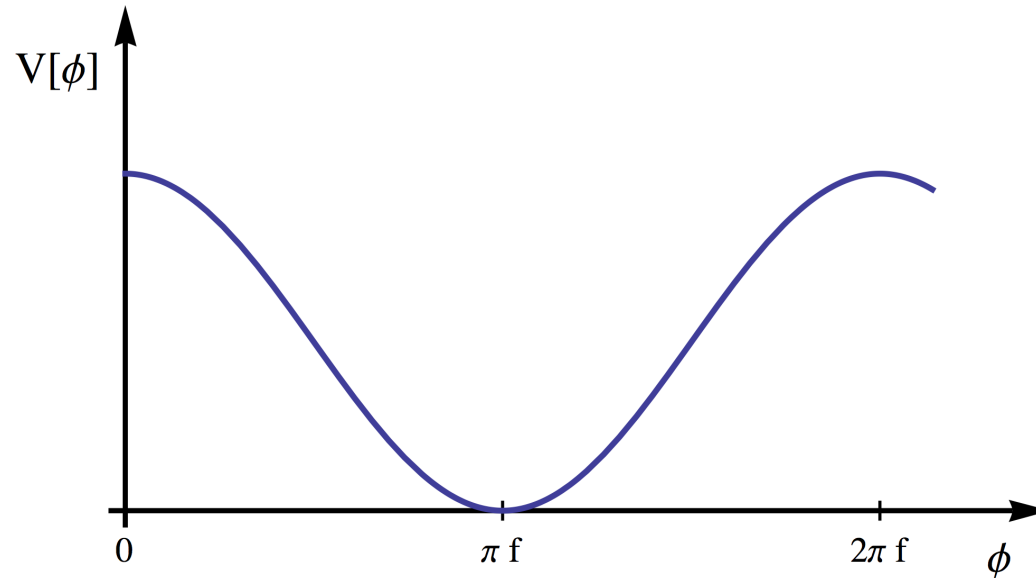
Nonperturbative effects break this symmetry to a remnant **discrete shift symmetry**



$$V(\phi) = \Lambda^4 \left[ 1 + \cos \left( \frac{2\pi\phi}{f} \right) \right]$$

# The Axion Potential

Discrete shift symmetry identifies  $\phi = \phi + 2\pi n f$

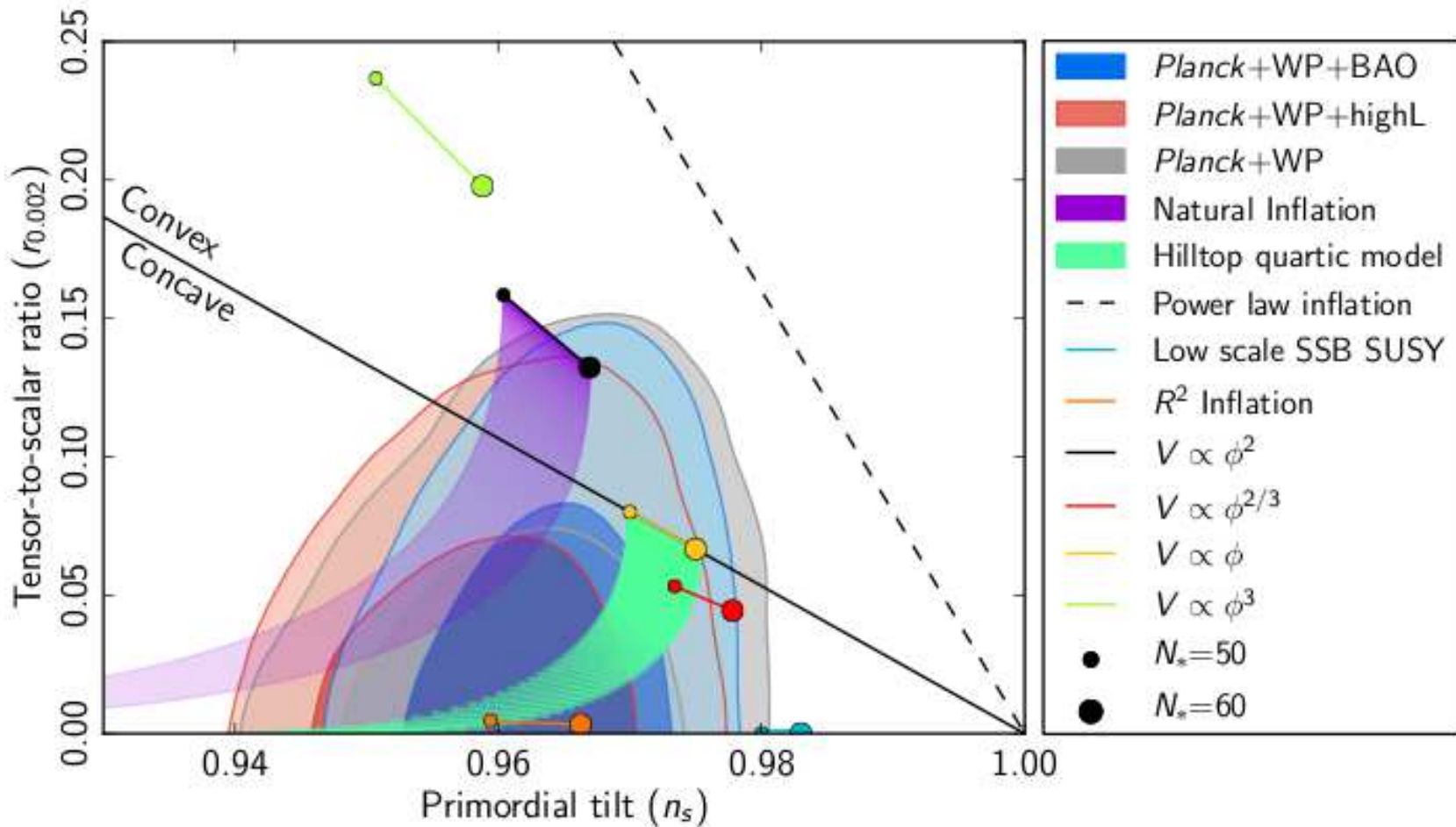


$$V(\phi) = \Lambda^4 \left[ 1 + \cos \left( \frac{2\pi\phi}{f} \right) \right]$$

$\phi$  confined to one fundamental domain



# Planck results (Spring 2013)



# BICEP2 (Spring 2014)

Tentatively large tensor modes of order  $r \sim 0.1$  had been announced by the BICEP collaboration

- large tensor modes brings us to scales of physics close to the Planck scale and the so-called “Lyth bound”
- potential  $V(\phi)$  of order of GUT scale  $\text{few} \times 10^{16}$  GeV
- trans-Planckian excursions of the inflaton field
- For a quadratic potential  $V(\phi) \sim m^2 \phi^2$  it implies  $\Delta\phi \sim 15M_{\text{P}}$  to obtain 60 e-folds of inflation

Axionic inflation, on the other hand, seems to require the decay constant to be limited:  $f \leq M_{\text{P}}$ .

So this might be problematic.

(Banks, Dine, Fox, Gorbatov, 2003)

# Aligned axions

A way out is the consideration of two (or more) fields.

(Kim, Nilles, Peloso, 2004)

- top-down approach favours a multi-axion picture
- we require  $f \leq M_{\text{P}}$  for the individual axions

The alignment extends the fundamental domain of the aligned axion to super-Planckian values, as the axionic inflaton spirals down in the potential of the second axion

Alternative mechanisms, like e.g. "Axion Monodromy" give a similar qualitative picture

(McAllister, Silverstein, Westphal, 2008)

# The KNP set-up

We consider two axions

$$\mathcal{L}(\theta, \rho) = (\partial\theta)^2 + (\partial\rho)^2 - V(\rho, \theta)$$

with potential

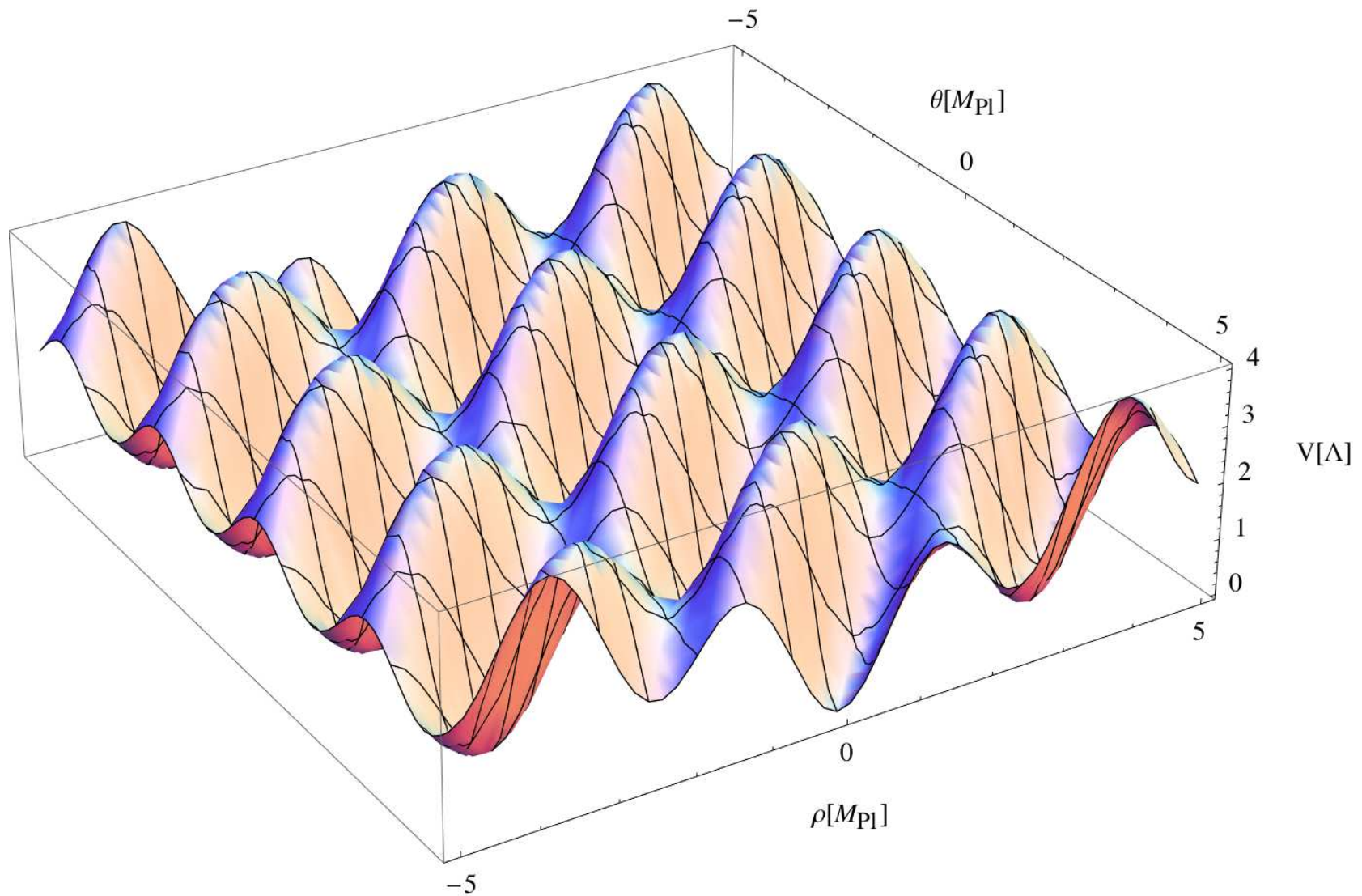
$$V(\theta, \rho) = \Lambda^4 \left( 2 - \cos \left( \frac{\theta}{f_1} + \frac{\rho}{g_1} \right) - \cos \left( \frac{\theta}{f_2} + \frac{\rho}{g_2} \right) \right)$$

This potential has a flat direction if  $\frac{f_1}{g_1} = \frac{f_2}{g_2}$

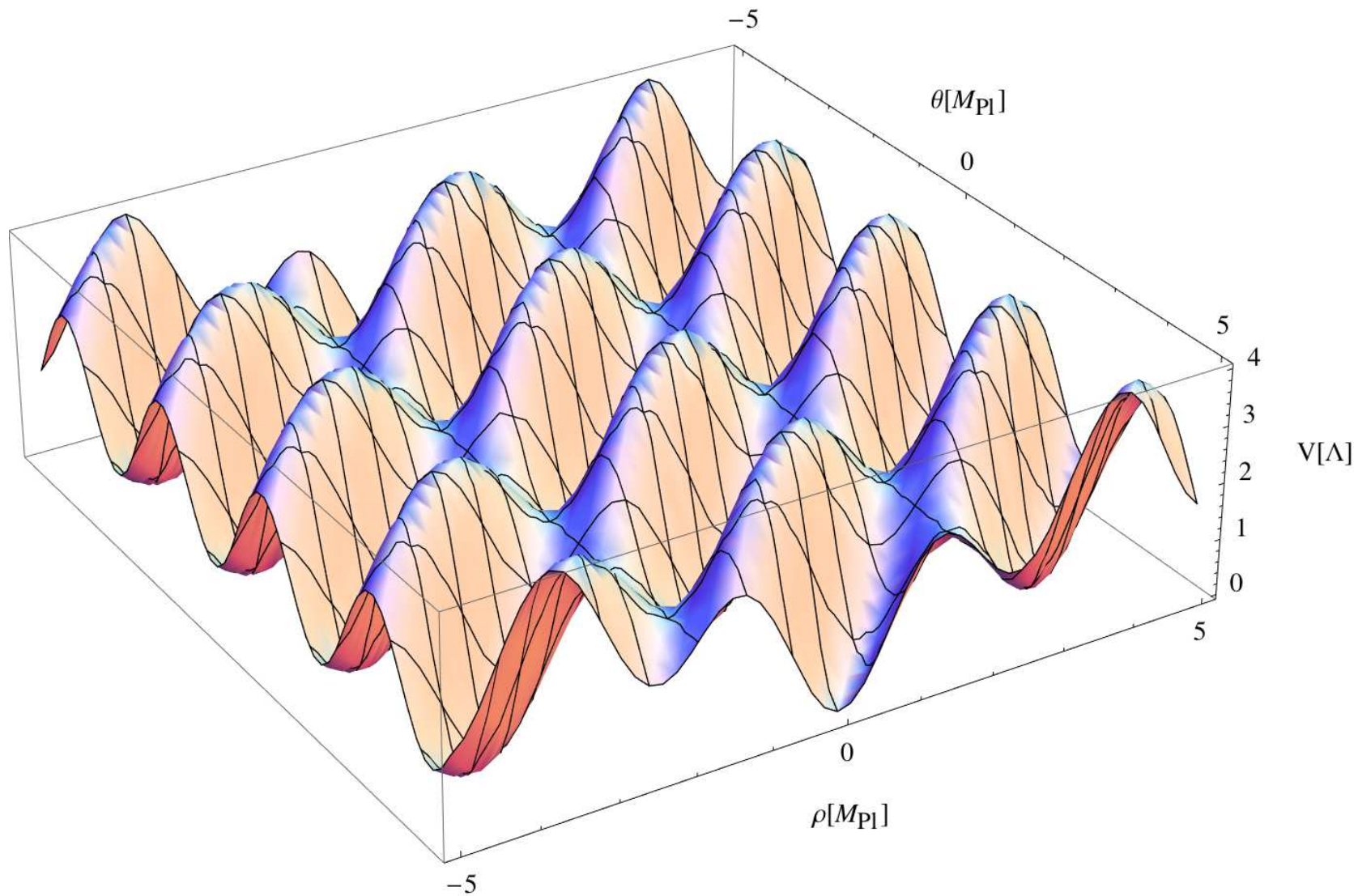
Alignment parameter defined through  $\alpha = g_2 - \frac{f_2}{f_1} g_1$

For  $\alpha = 0$  we have a massless field  $\xi$ .

# Potential for $\alpha = 1.0$

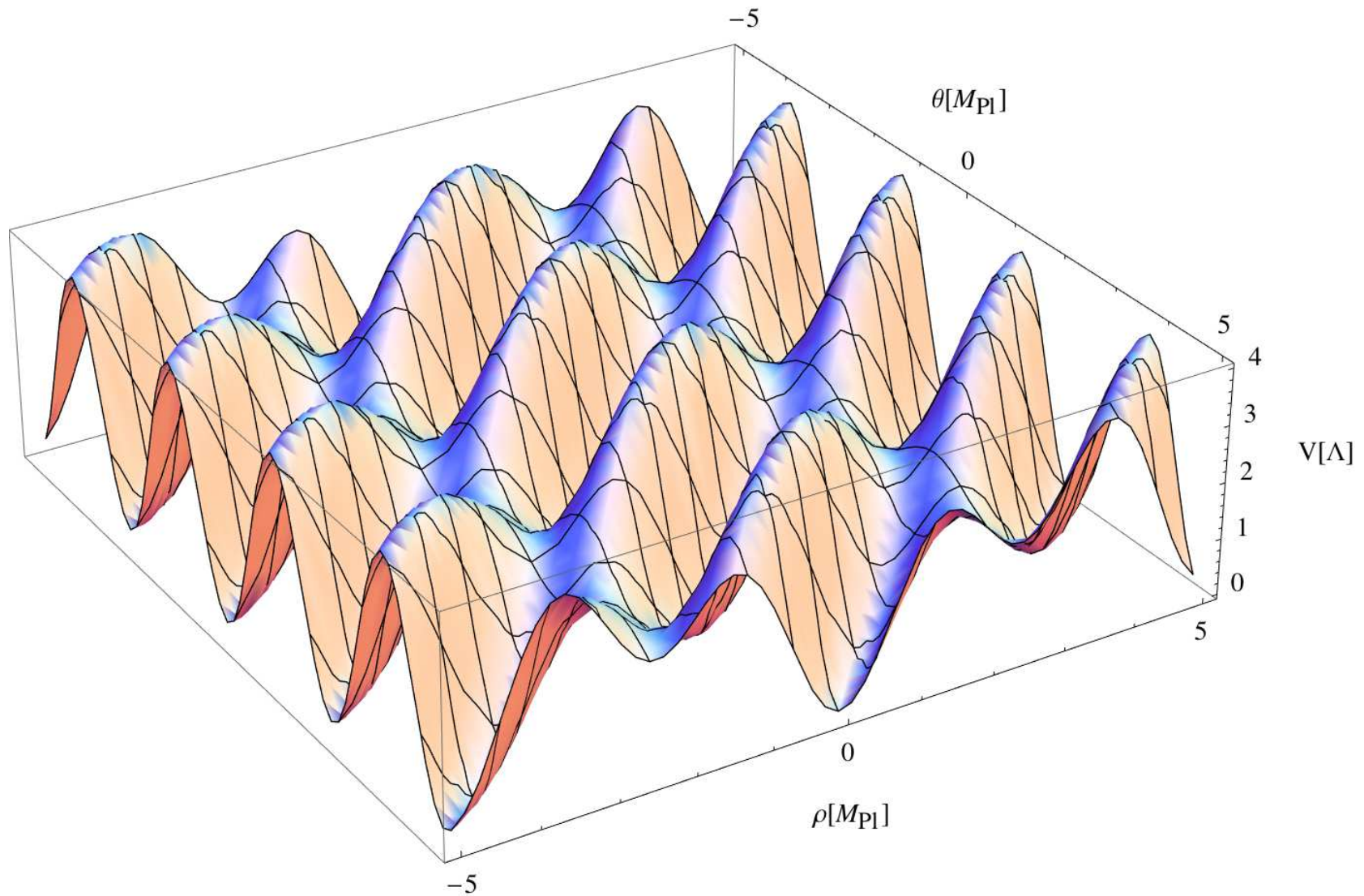


# Potential for $\alpha = 0.8$

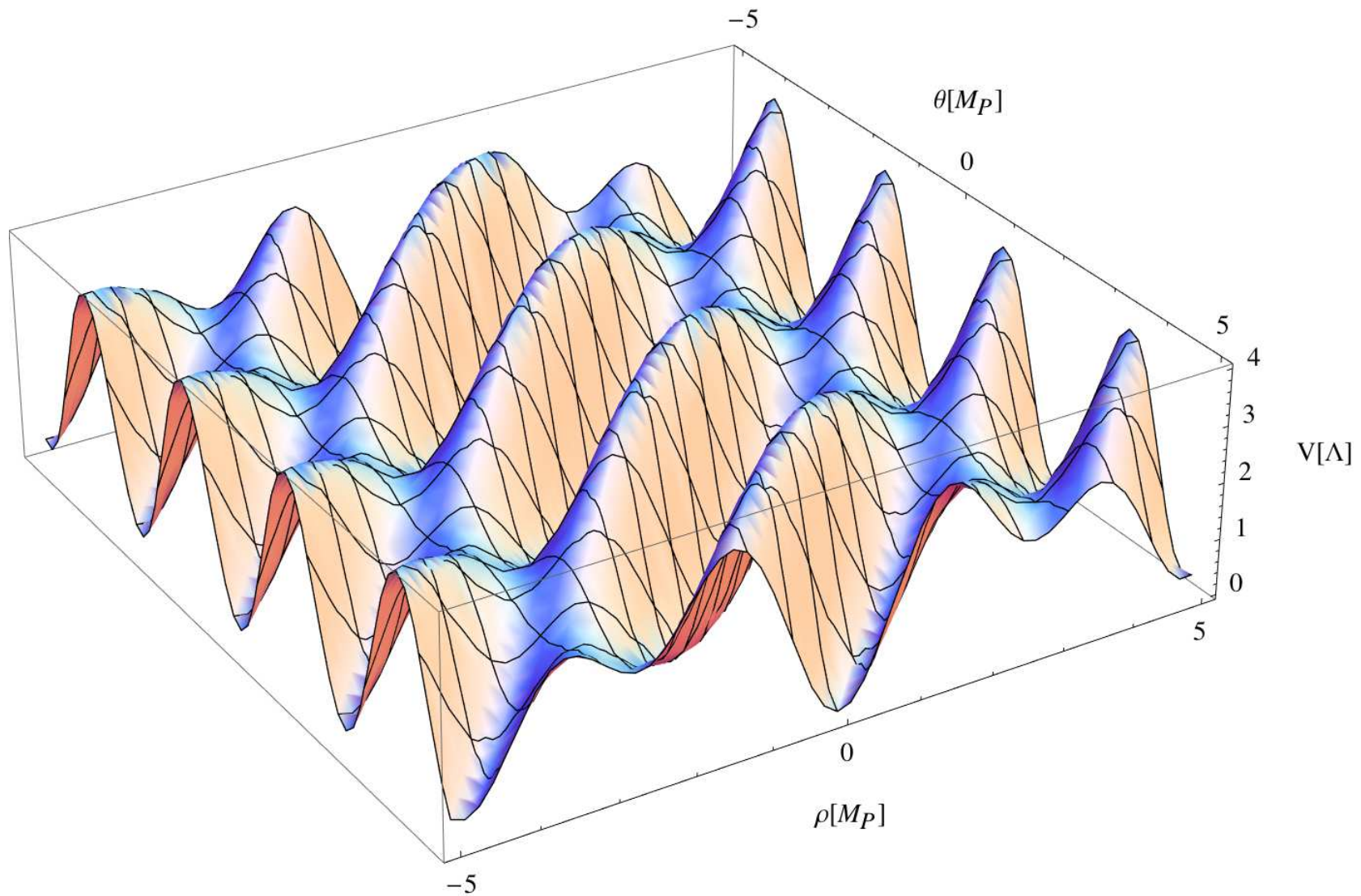




# Potential for $\alpha = 0.5$

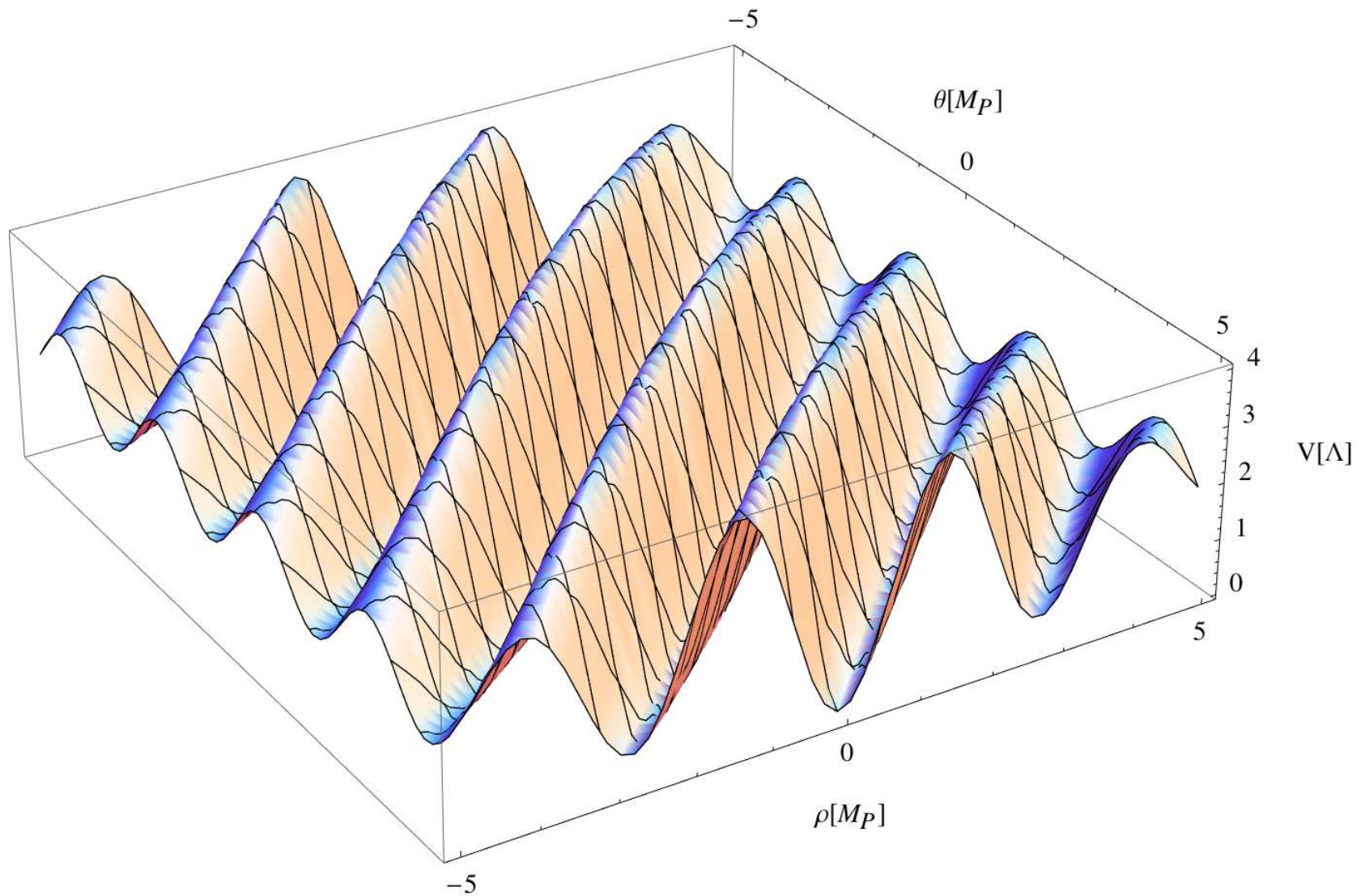


# Potential for $\alpha = 0.3$

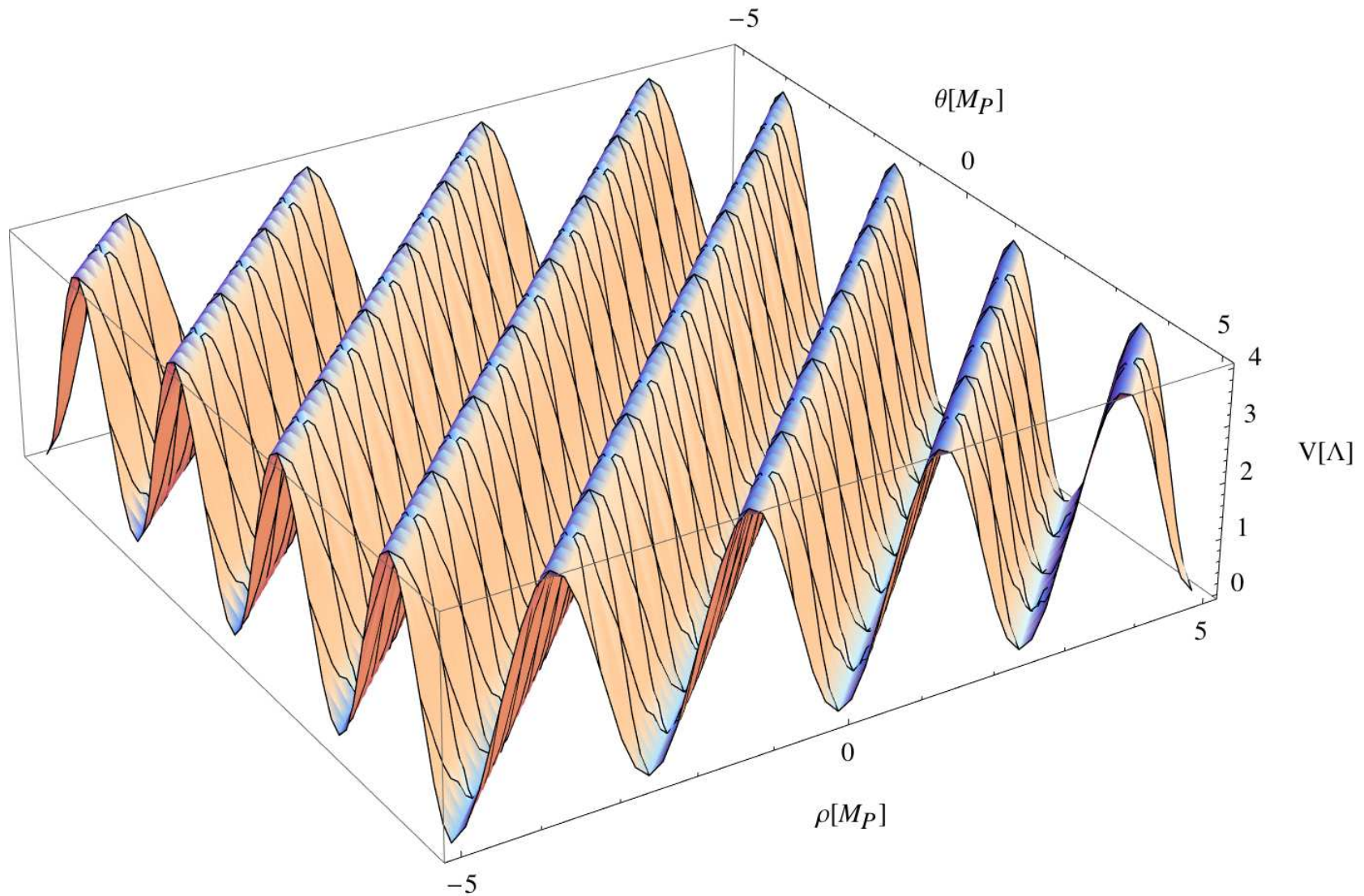




# Potential for $\alpha = 0.1$



# Potential for $\alpha = 0$



# The lightest axion

Mass eigenstates are denoted by  $(\xi, \psi)$ . The mass eigenvalues are

$$\lambda_{1/2} = F \pm \sqrt{F^2 + \frac{2g_1g_2f_1f_2 - f_2^2g_1^2 - f_1^2g_2^2}{f_1^2f_2^2f_1^2g_2^2}}$$

with 
$$F = \frac{g_1^2g_2^2(f_1^2 + f_2^2) + f_1^2f_2^2(g_1^2 + g_2^2)}{2f_1^2f_2^2g_1^2g_2^2}$$

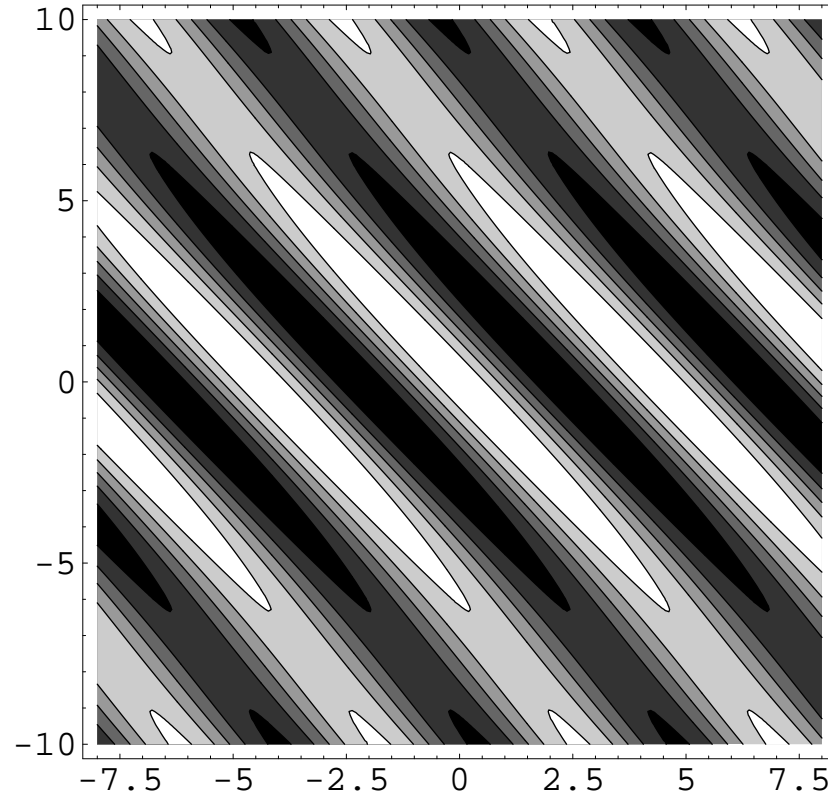
Lightest axion  $\xi$  has potential

$$V(\xi) = \Lambda^4 [2 - \cos(m_1(f_i, g_1, \alpha)\xi) - \cos(m_2(f_i, g_1, \alpha)\xi)]$$

leading effectively to a **one-axion system**

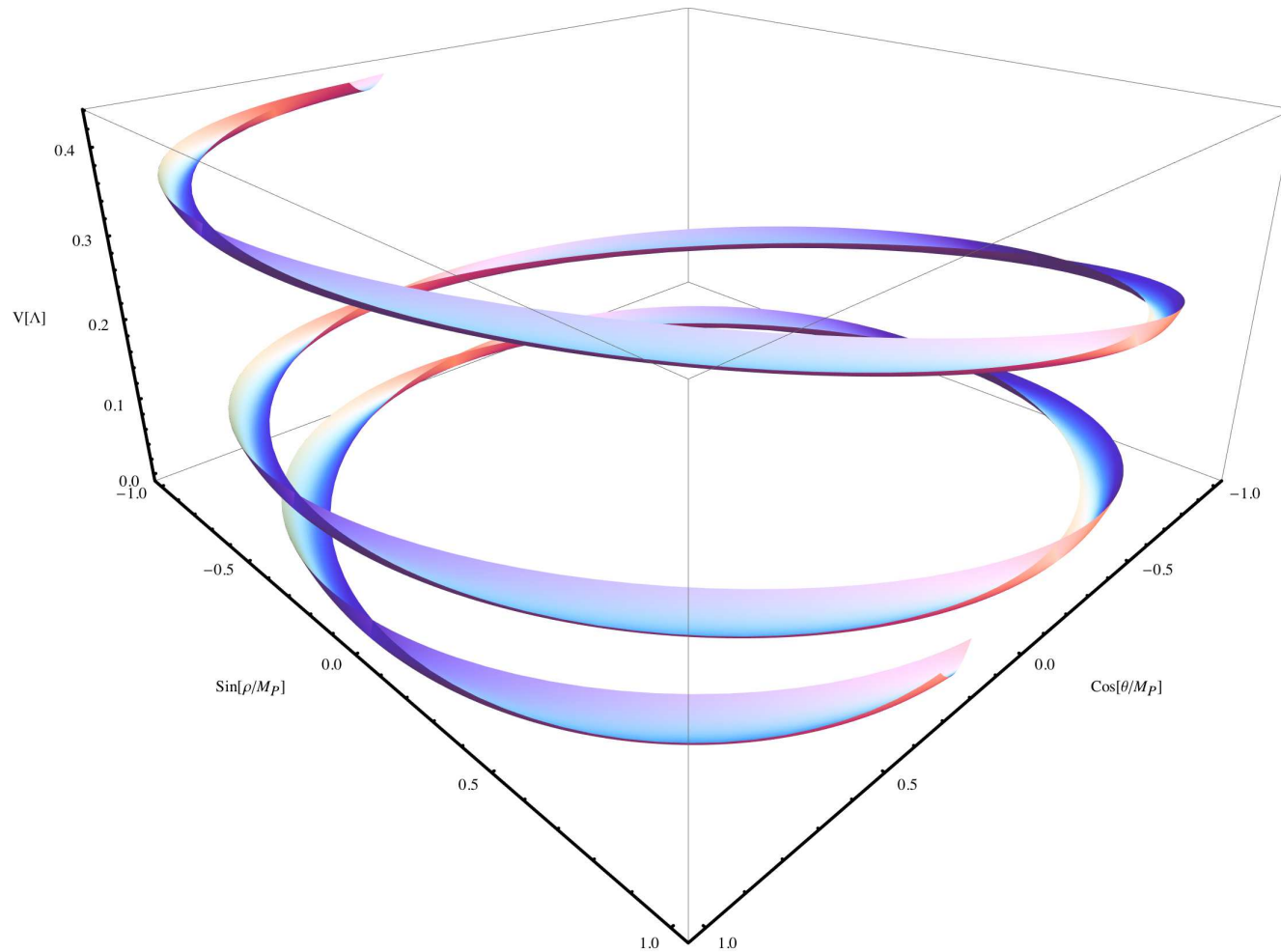
$$V(\xi) = \Lambda^4 \left[ 1 - \cos\left(\frac{\xi}{\tilde{f}}\right) \right] \quad \text{with} \quad \tilde{f} = \frac{f_2g_1\sqrt{(f_1^2 + f_2^2)(f_1^2 + g_1^2)}}{f_1^2\alpha}$$

# Axion landscape of KNP model



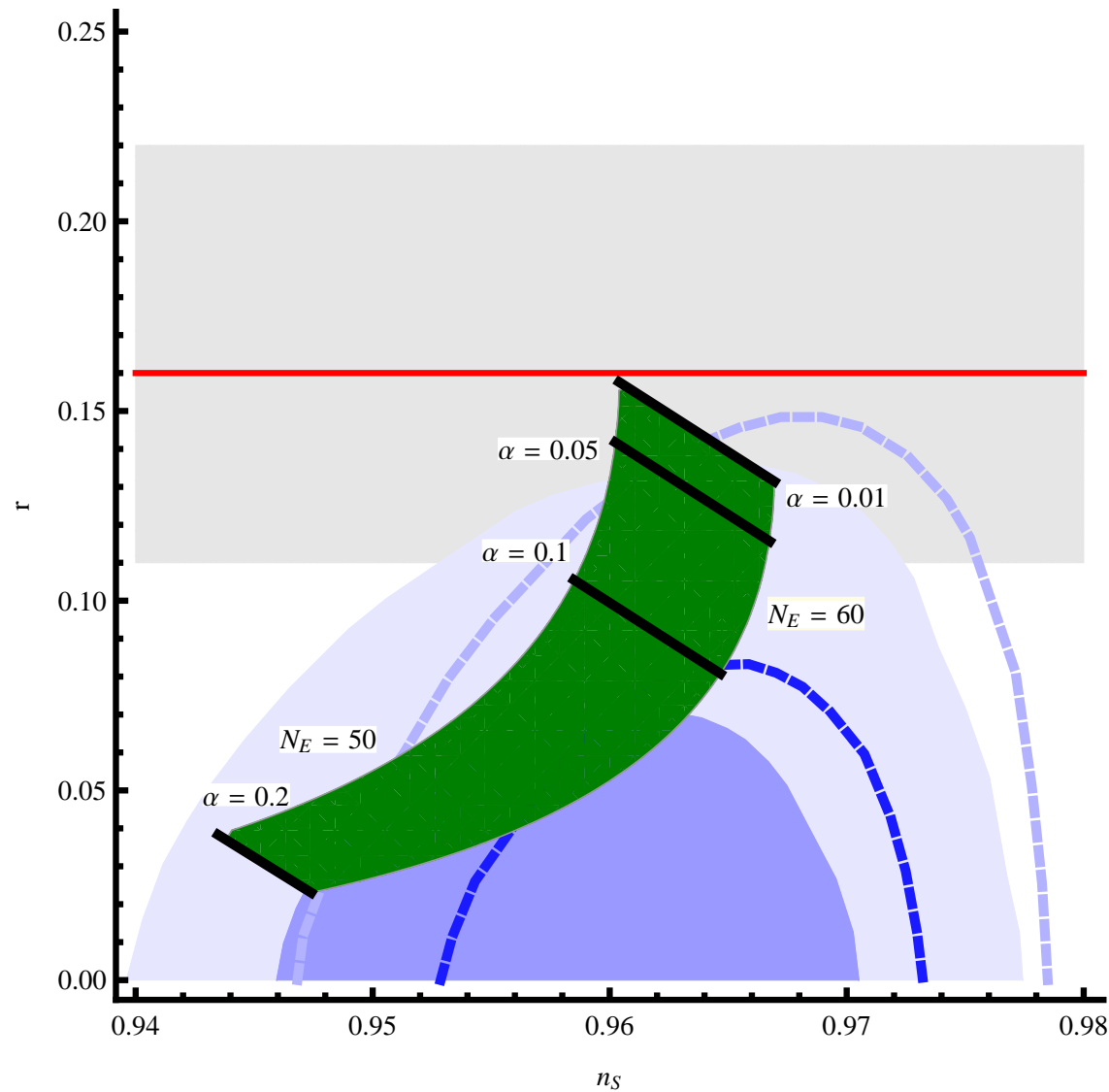
The field  $\xi$  rolls within the valley of  $\psi$ . The motion of  $\xi$  corresponds to a motion of  $\theta$  and  $\rho$  over **many cycles**. The system is still controlled by discrete symmetries.

# Monodromic Axion Motion



One axion spirals down in the valley of a second one.

# The “effective” one-axion system





# UV-Completion

Large tensor modes and  $\Lambda \sim 10^{16}$  GeV lead to theories at the “edge of control” and require a reliable UV-completion

- small radii
- large coupling constants
- light moduli might spoil the picture

# UV-Completion

Large tensor modes and  $\Lambda \sim 10^{16}$  GeV lead to theories at the “edge of control” and require a reliable UV-completion

- small radii
- large coupling constants
- light moduli might spoil the picture

So it is important to find reliable symmetries

- axions are abundant in string theory
- perturbative stability of “shift symmetry”
- broken by nonperturbative effects
- potential protection through supersymmetry



# Stability

We have a very flat direction and within the effective QFT we are at the “edge of control”

- is inflation perturbed by other effects?
- is there an upper limit on  $f_{\text{eff}}$ ?

Remember that in case of a single axion we had the limit

- $f_{\text{eff}} \leq M_{\text{string}}$  (Banks, Dine, Fox, Gorbatov, 2003)
- derived from dualities in string theory (e.g. T-duality)

In the multi-axion case these arguments are not directly applicable, but the question of trans-Planckian values should be tested in a given model

# Weak Gravity Conjecture (WGC)

It is based on prejudice about black hole properties and is formulated to constrain  $U(1)$  gauge interactions,

(Arkani-Hamed, Motl, Nicolis, Vafa, 2006)

- give limits on mass to charge ratio  $|q/m| > 1$
- “convex hull” restrictions in multi-field case.

But our “knowledge” on black hole properties (no-hair conjecture and information paradox) has changed recently,

- fuzzballs, (Mathur, 2009-2015)
- brick- and fire-walls. (Almheiri, Marolf, Polchinski, Sully, 2012)

The motivation for the WGC might thus be less convincing.

# WGC II

It is conjectured that the WGC (if true) might be applicable to axions, (Rudelius, 2015)

- based on a chain of string dualities,
- might give an upper limit on decay constants  $f_{\text{eff}}$ .

This might lead to a no-go theorem for large axion decay constants, but

- there are loop-holes in the presence of subleading instantons, (Brown, Cottrell, Shiu, Soler, 2015)
- computationally we are at the “edge of control”.

Needs to be clarified in explicit constructions.

(Kappl, Nilles, Winkler, 2015)

# $T$ –Duality

String dualities give important constraints on the axion decay constants, especially  $T$ –duality  $SL(2, Z)$ :

$$T \rightarrow \frac{aT - ib}{icT + d}$$

generated by an inversion and a shift

$$T \rightarrow 1/T, \quad T \rightarrow T + i.$$

$$G = K + \log |W|^2$$

must be invariant under  $T$ -duality.

# $T$ –Duality

$K$  and  $W$  might transform nontrivially. Consider e.g.

$$K = -3 \log (T + \bar{T}).$$

This Kähler potential transforms under  $SL(2, Z)$  as

$$K \rightarrow K + \log |icT + d|^6$$

and has to be compensated by a superpotential transforming as a **modular form of weight  $-3$** :

$$W \rightarrow (icT + d)^{-3} W .$$

# Explicit String Constructions

In string theory we do not just get cosine potentials, but obtain modular functions (e.g. Dedekind-functions) from

- world sheet instanton effects,
- gauge kinetic functions and gaugino condensates.

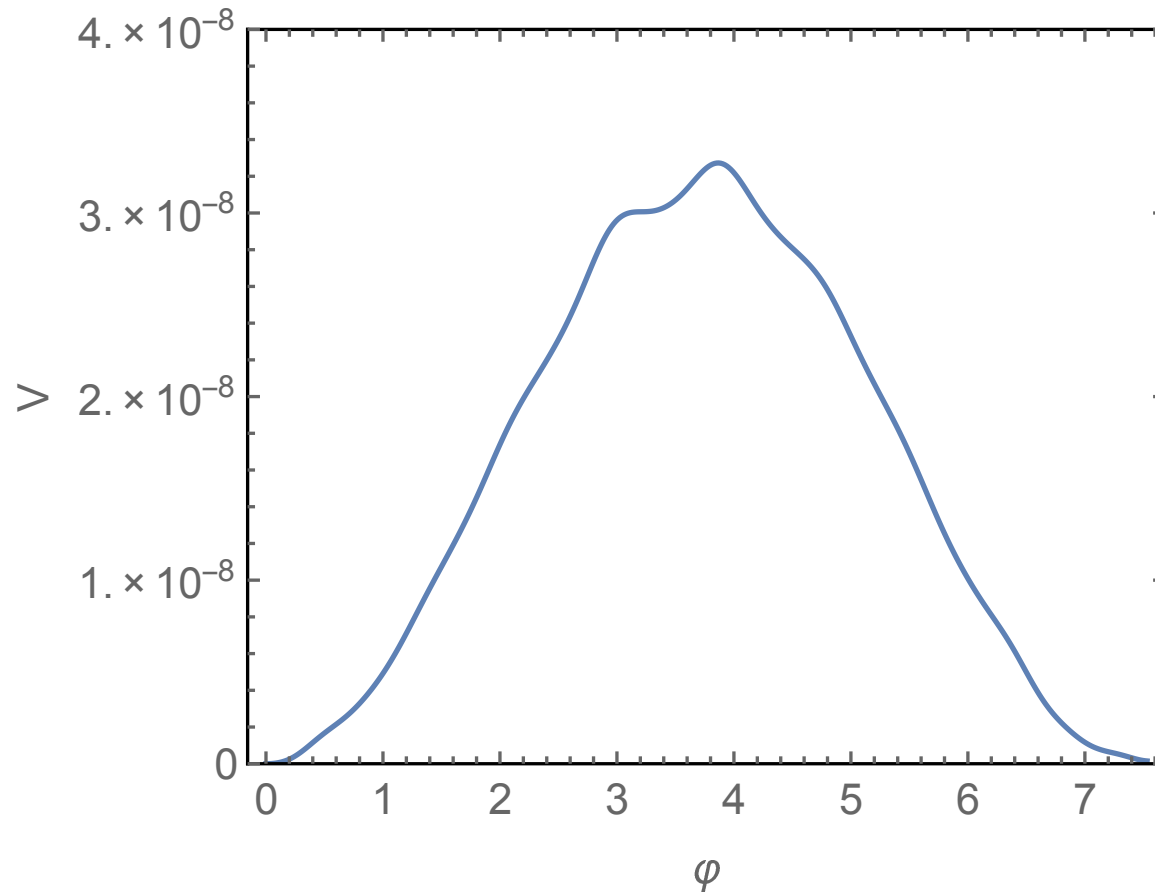
So we might consider instead

$$\eta(T) = e^{-\pi T/12} \times \prod_k (1 - e^{-2k\pi T})$$

a modular function of weight  $+1/2$ .

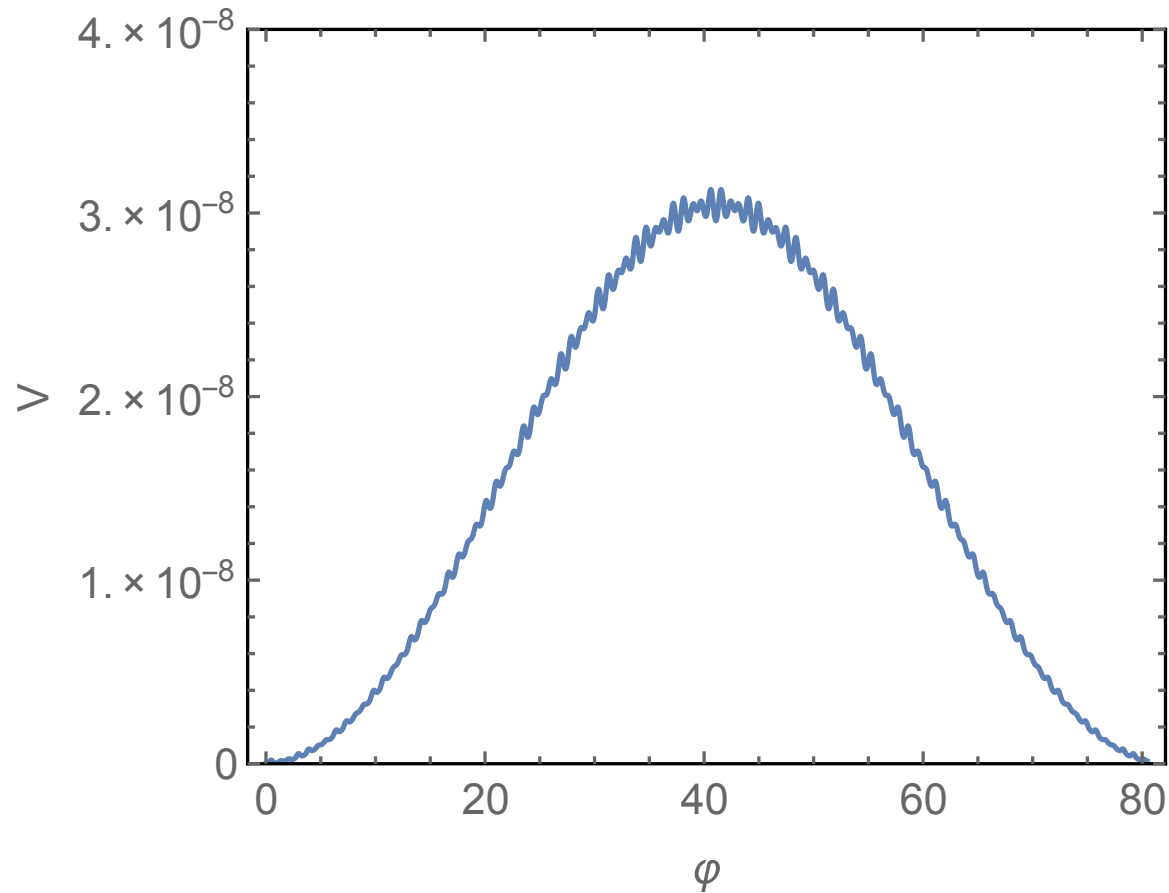
The higher harmonics give wiggles in the potential that perturb the flat direction and might stop inflation.

# Wiggles in the aligned potential



The wiggles in the case of weak alignment (small  $f$ )

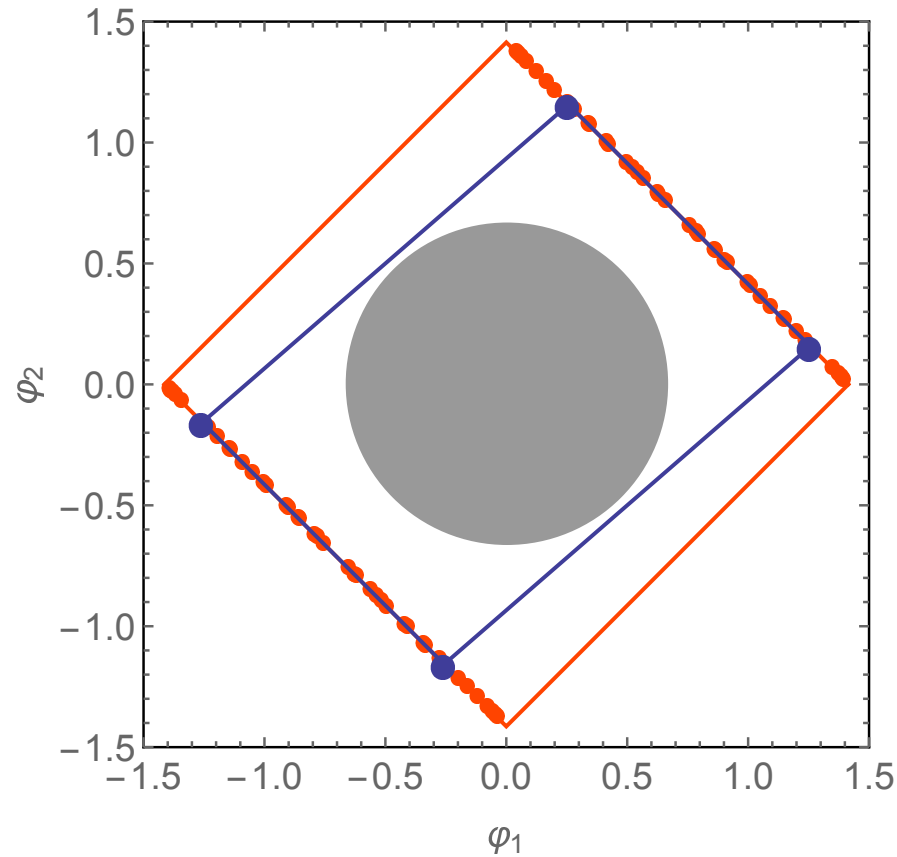
# Wiggles in the aligned potential



Strong alignment (potential superPlanckian  $f$ )

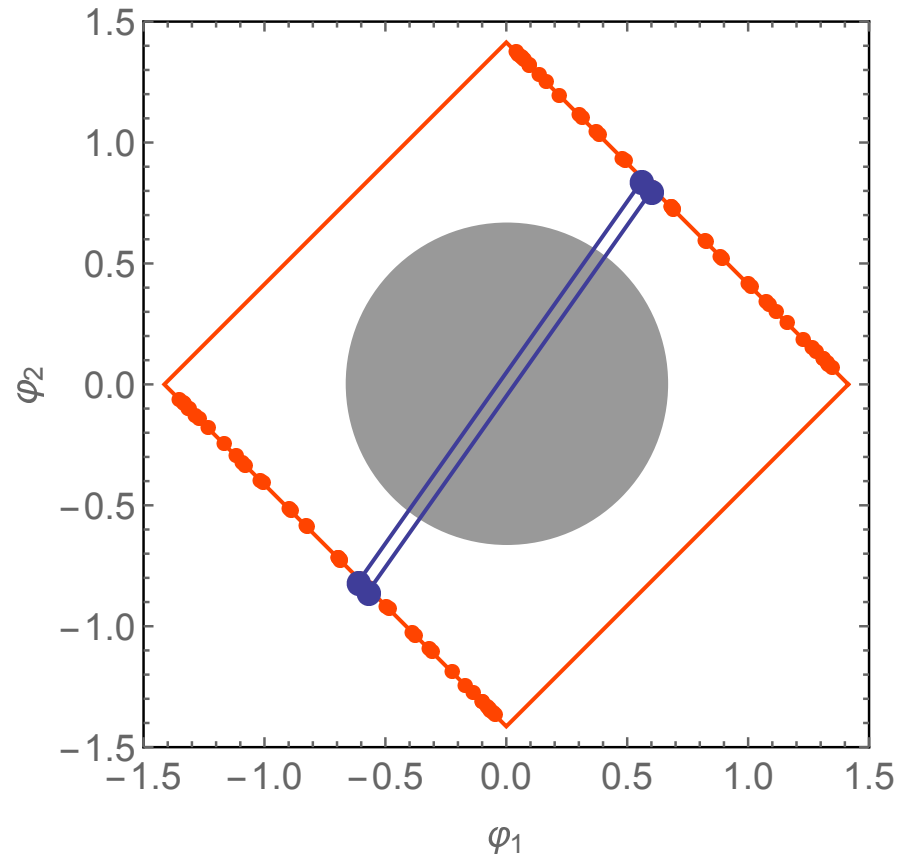


# Weak alignment



The convex hull restrictions are trivially satisfied

# Strong alignment



Subleading terms (red) satisfy the restrictions

# Modulated natural Inflation

It seems plausible that under some circumstances the wiggles become important and

- spoil the flat direction,
- provide an upper limit on decay constant  $f_{\text{eff}}$ .

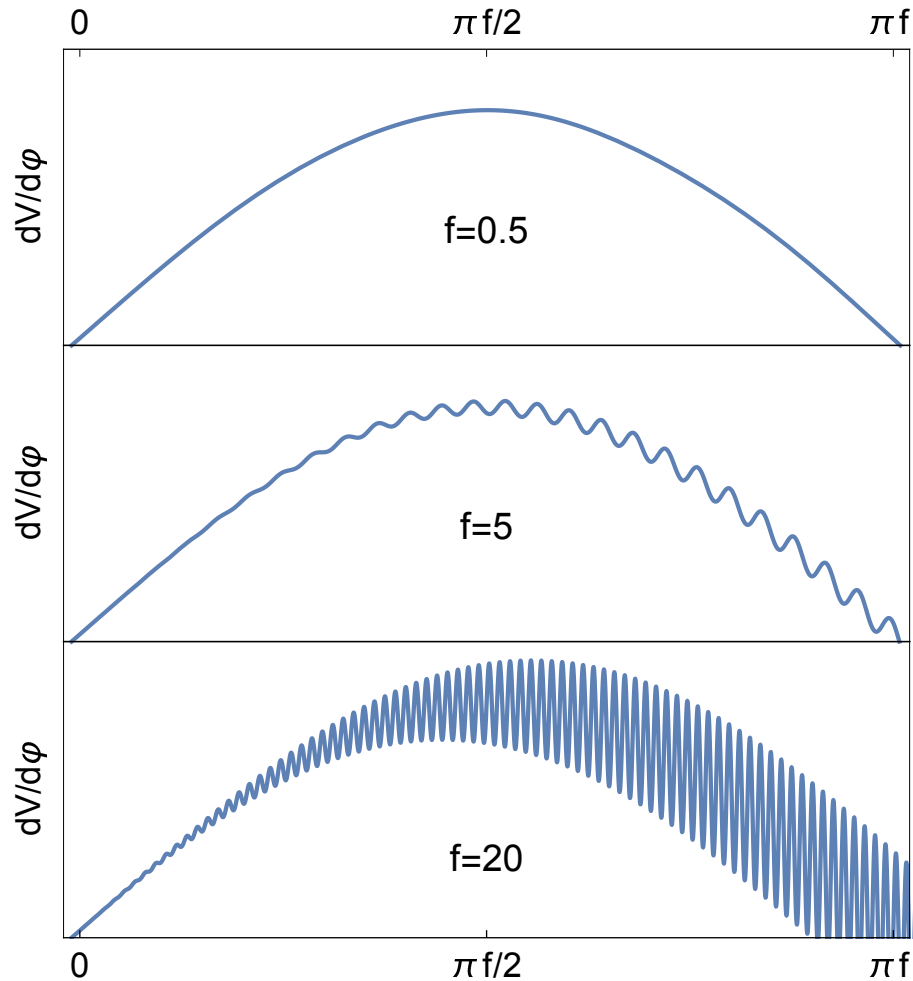
Explicit calculations are necessary to clarify the situation,

- but might be beyond our present capabilities;
- observational confirmation is extremely important.

Restrictions from WGC are satisfied here both in the aligned **and** non-aligned case.

(Kappl, Nilles, Winkler, 2015; Choi, Kim, 2015; Kobayashi, Nitta, Urakawa, 2016)

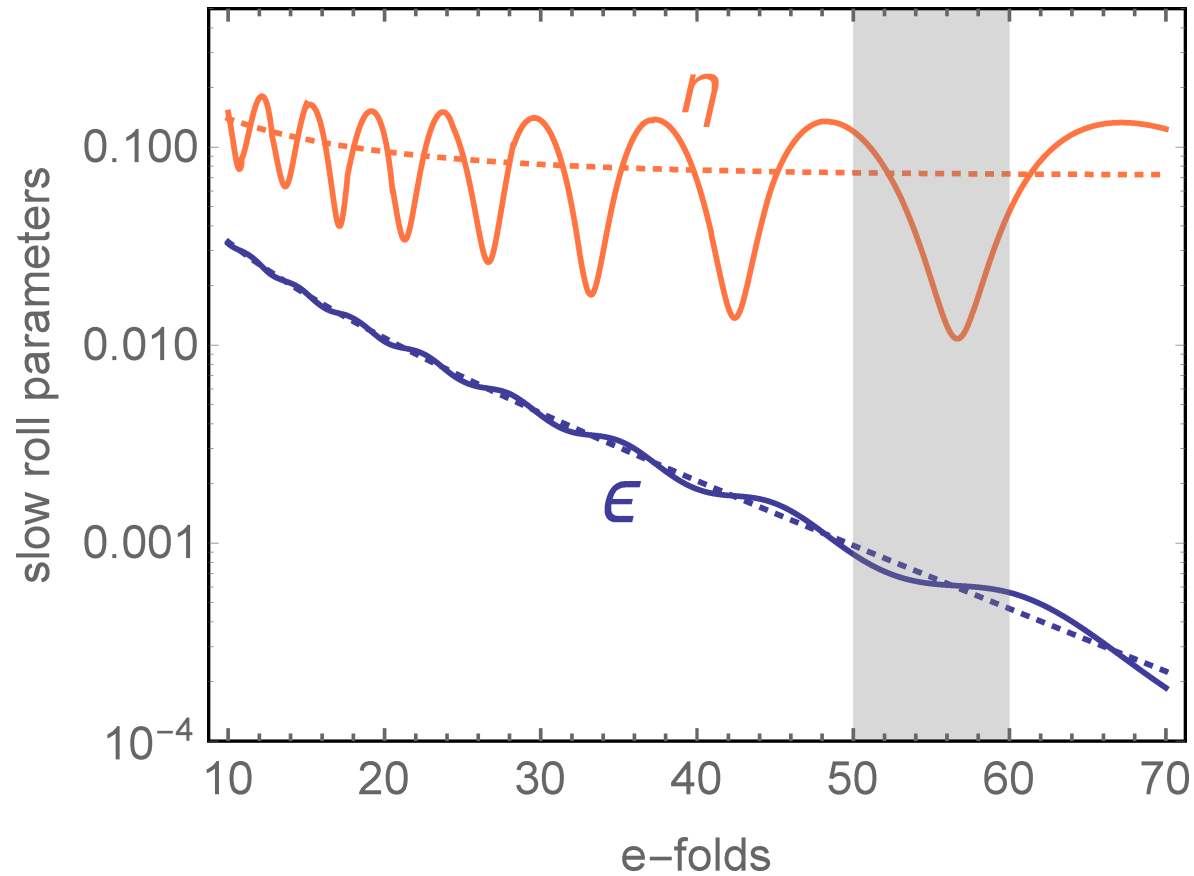
# Slope of Potential



Wiggly structure becomes even more important

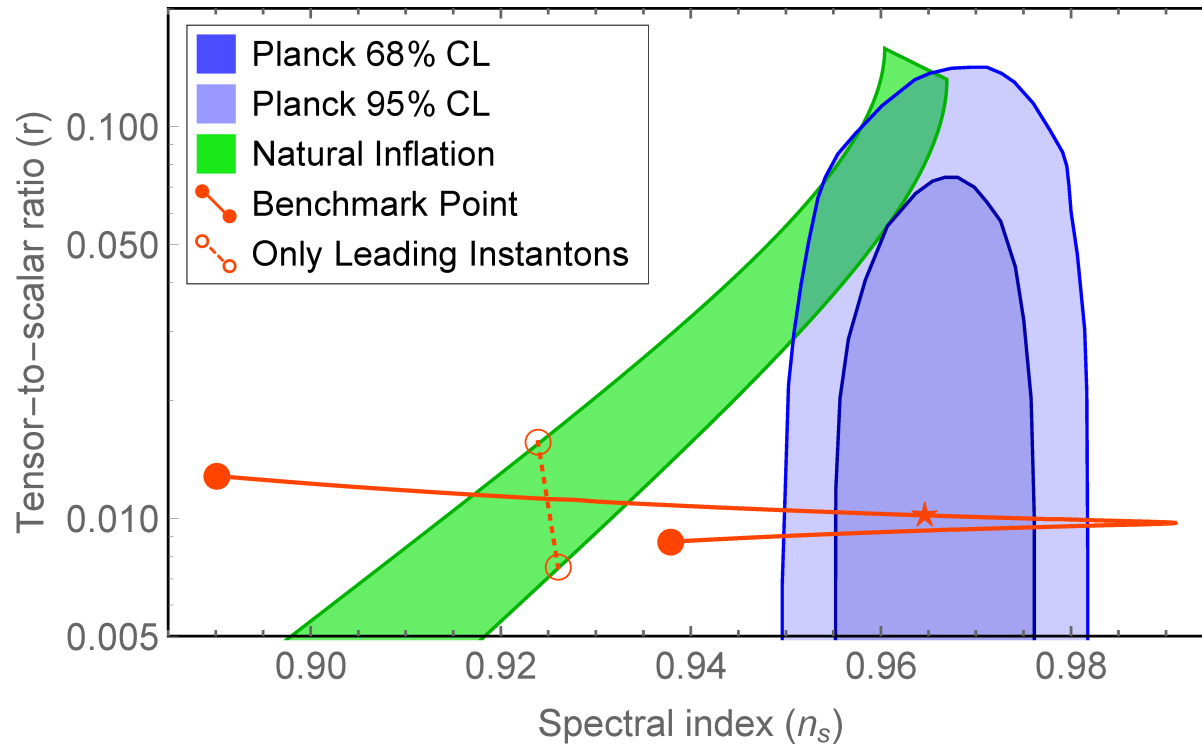
(Abe, Kobayashi, Otsuka, 2015; Kappl, Nilles, Winkler, 2015)

# Slow roll parameters



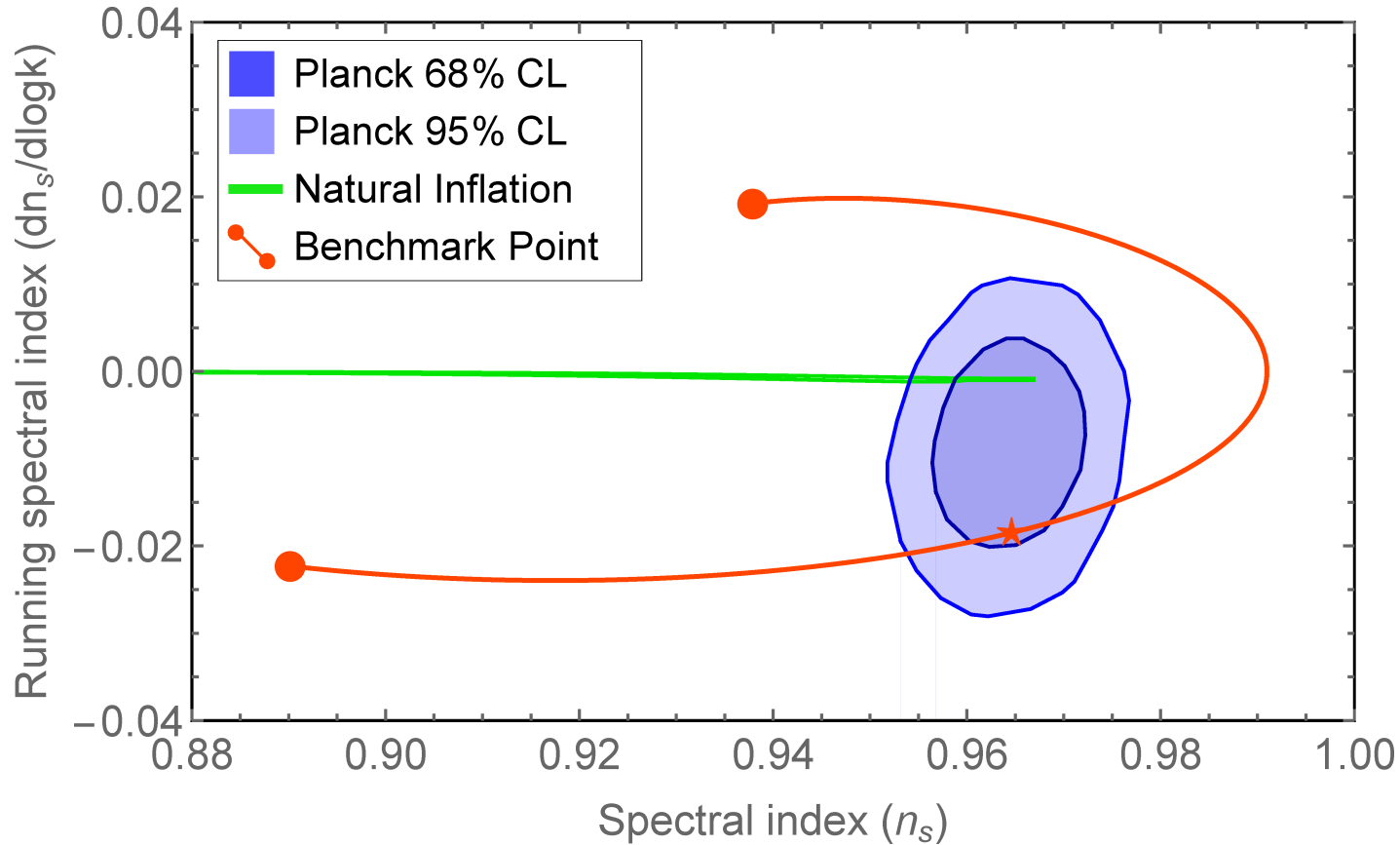
$\epsilon$  ( $\eta$ ) depend on first (second) derivative

# $n_s - r$ plane



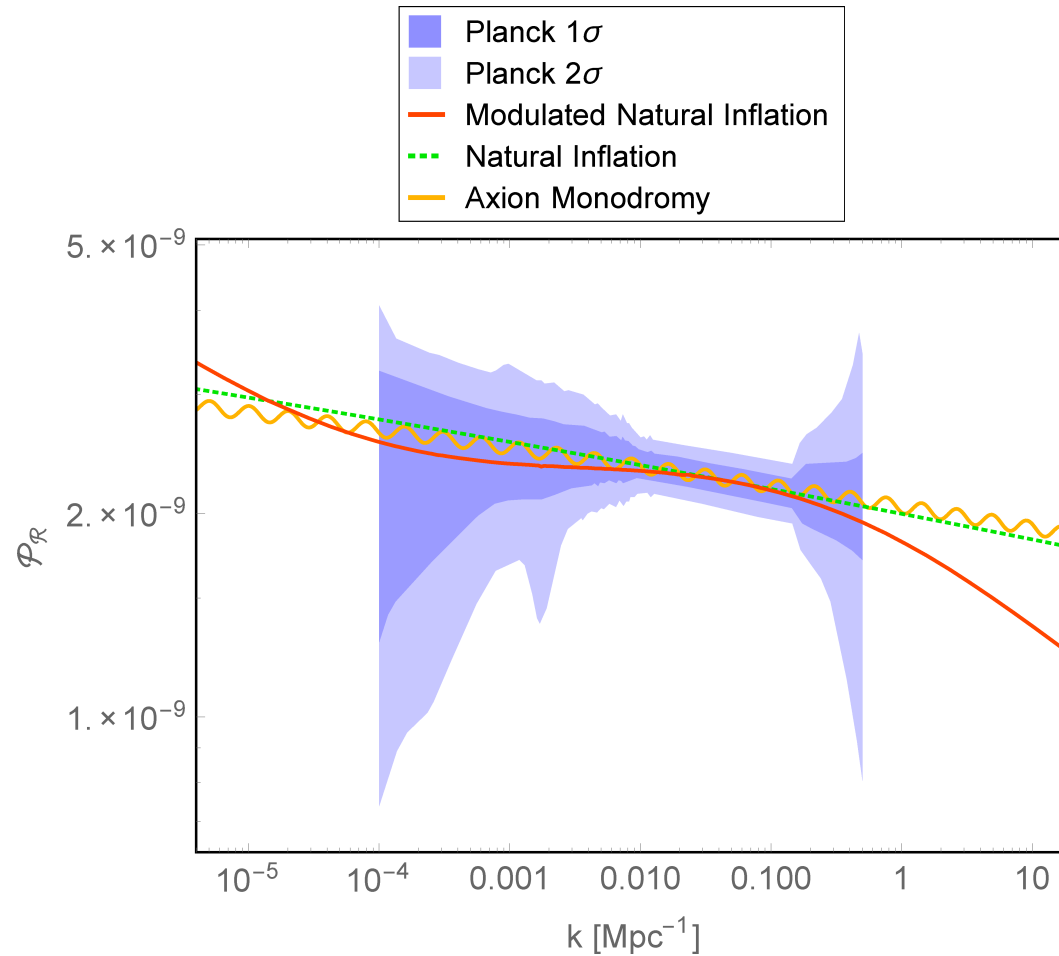
Strong variations of  $n_s$  on the number of e-folds

# Running of spectral index



Comparison of spectral index with Planck data

# Scalar power spectrum

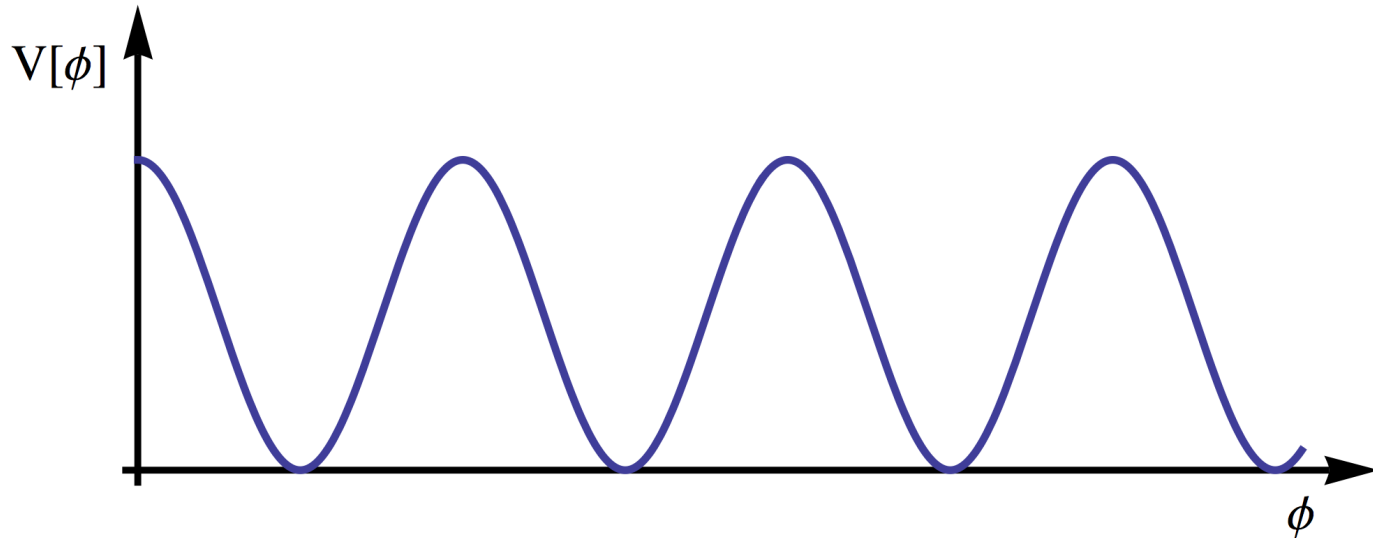


Comparison to Planck reconstructed power spectrum



# Alignment and axionic domain walls

In general we have  $a = a + 2\pi N f_a$  for  $V \sim \cos(Na/f_a)$ ,



leading to  $N$  nontrivial degenerate vacua separated by maxima of the potential.

During the cosmic evolution this might lead to the production of potentially harmful axionic domain walls.

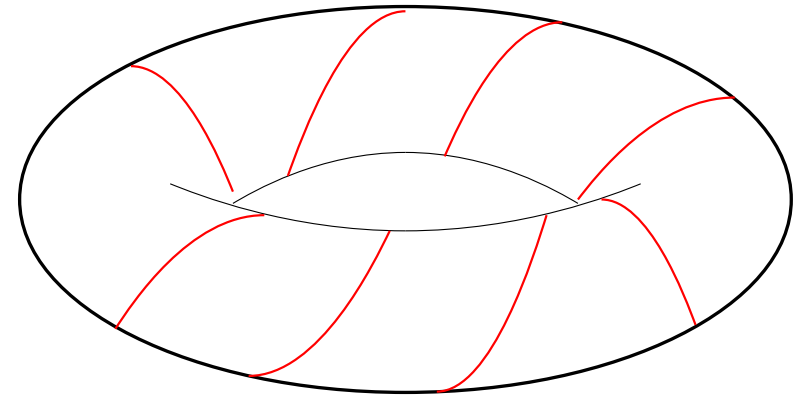
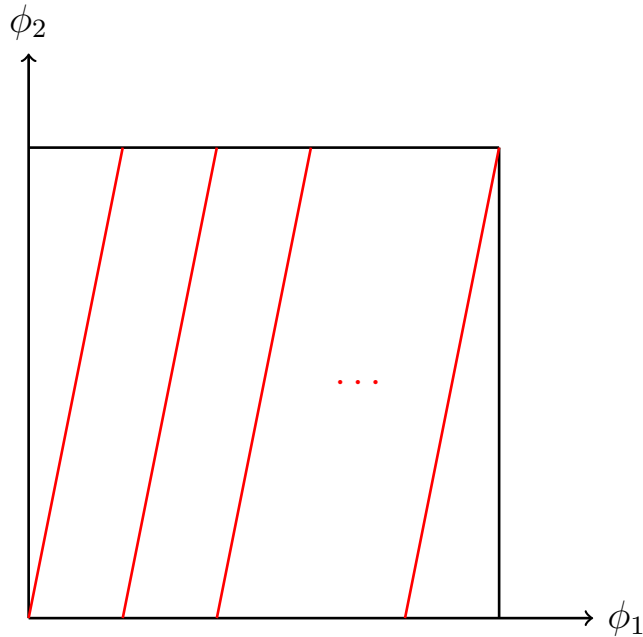
# Two-Axion-model

Consider a system with two axions

$$V \sim \Lambda_1^4 \cos \left( \frac{a_1}{f_1} + N \frac{a_2}{f_2} \right) + m \Lambda_2^3 \cos \left( \frac{a_2}{f_2} \right)$$

- For fixed  $a_1$  there are  $N$  nontrivial vacua and potentially  $N_{\text{DW}} = N$  domain walls
- for  $m = 0$  there is a Goldstone direction,
- and thus a continuous unique vacuum with effective domain wall number  $N_{\text{DW}} = 1$  (Choi, Kim, 1985)
- the Goldstone mode develops an axionic potential in the case  $m \neq 0$

# The axionic vacuum



(Choi, Kim, Yun, 2014)

- There is continuous unique vacuum with effective domain wall number is  $N_{\text{DW}} = 1$  (Choi, Kim, 1985)
- the Goldstone mode develops an axionic potential in the case  $m \neq 0$

# Quintessential axion alignment

Axions could be the source for dynamical dark energy

- in contrast to scalar quintessence, the axion has only derivative couplings and does not lead to a “fifth force”
- we need a slow roll field with  $\Lambda \sim 0.003 \text{ eV}$
- to act as dark energy today we need  $f_a \geq M_{\text{Planck}}$
- the quintaxion mass is  $m_a \sim \Lambda^2 / M_{\text{Planck}} \sim 10^{-33} \text{ eV}$

Again we need a trans-Planckian decay constant for a consistent description of the present stage of the universe

- the problem can be solved via aligned axions à la KNP

(Kaloper, Sorbo, 2006)

# The relaxion mechanism

Axions could be at the origin of mass hierarchies. This requires

(Graham, Kaplan, Rajendran, 2015)

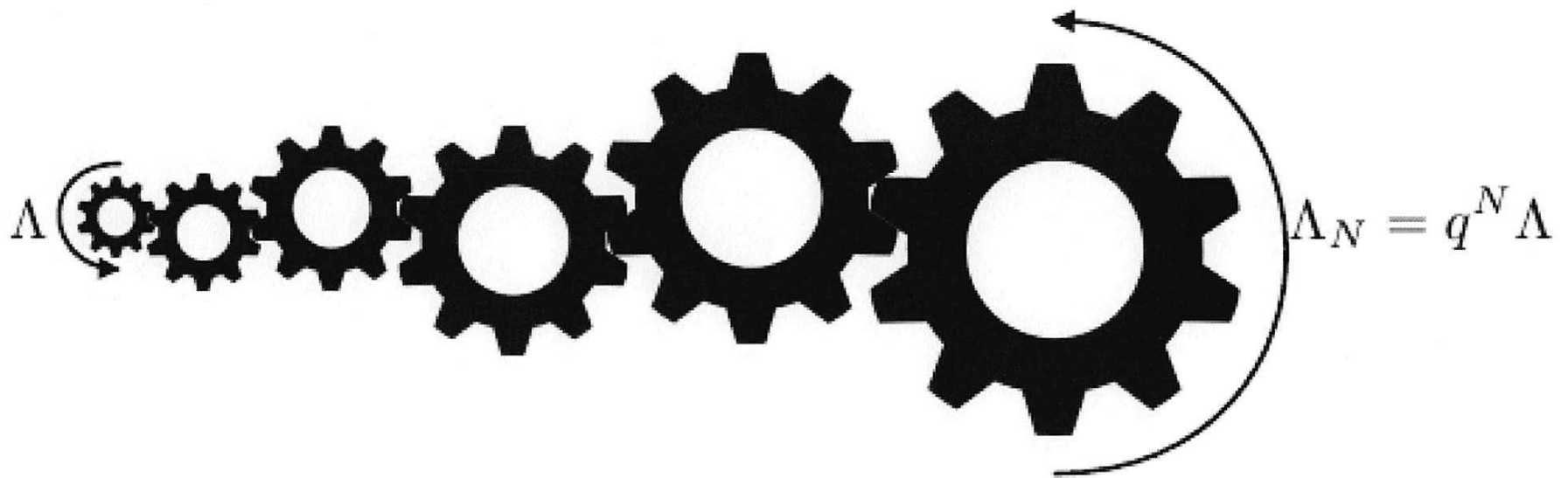
- a slowly rolling (relaxion) field,
- stopped by nonperturbative effects.
- Large mass hierarchies need a long time evolution of the relaxion field
- and an unconventional cosmological evolution.

Again we need a huge relaxion decay constant for a consistent description of the present stage of the universe:

- the problem can be solved via aligned axions à la KNP (called axion clock-work).

(Choi, Im, 2015; Kaplan, Rattazzi, 2015)

# Alignment with N axions



# Axion Clockworks

The step beyond towards  $N > 2$  axions simplifies the creation of hierarchies leading from  $(\text{small})^2$  to  $(\text{small})^N$  suppression. It has been applied to

- axionic inflation (Choi, Kim, Yun, 2014)
- the QCD axion (Higaki, Jeong, Kitajima, Takahashi, 2015)
- the relaxion (Choi, Im, 2015)

Towards large  $N$

- similarity to the "deconstruction" of extra dimensions (Arkani-Hamed, Cohen, Georgi, 2001; Hill, Pokorski, Wang, 2001)
- what about the continuum limit? (Giudice, McCullough, 2016)

# Continuous Clockwork

The transition from discrete to continuous clockwork leads to a warped extra dimension. (Giudice, McCullough, 2016)

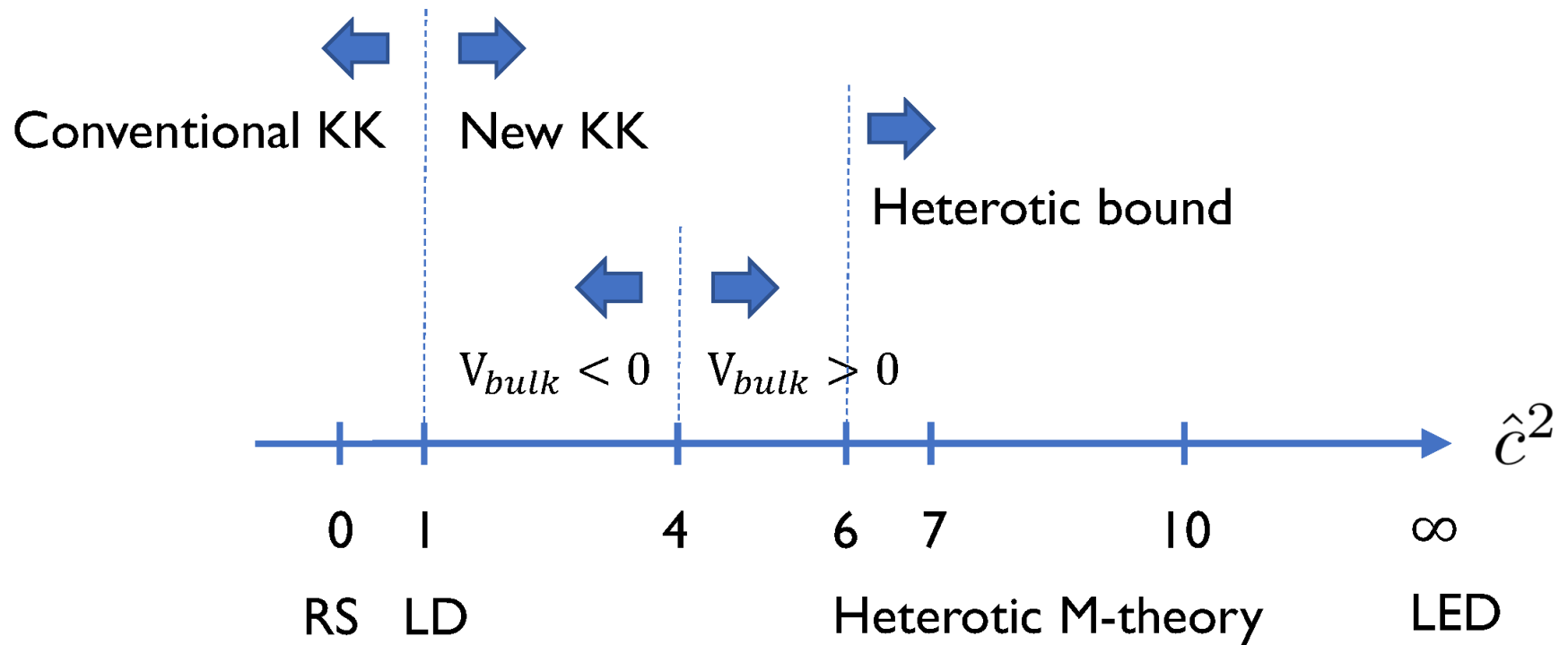
- simplest case corresponds to "Linear Dilaton Model"
- there are qualitative differences between the discrete and continuous case (Craig, Garcia Garcia, Sutherland, 2017)
- Generalized LD scenario includes RS and LED (Choi, Im, Shin, 2017)

The quest for an ultraviolet (UV) completion

- heterotic M-theory with "unconventional" warping (Im, Nilles, Olechowski, 2018)
- discrete set of warped clockwork solutions allow for a consistent UV-completion

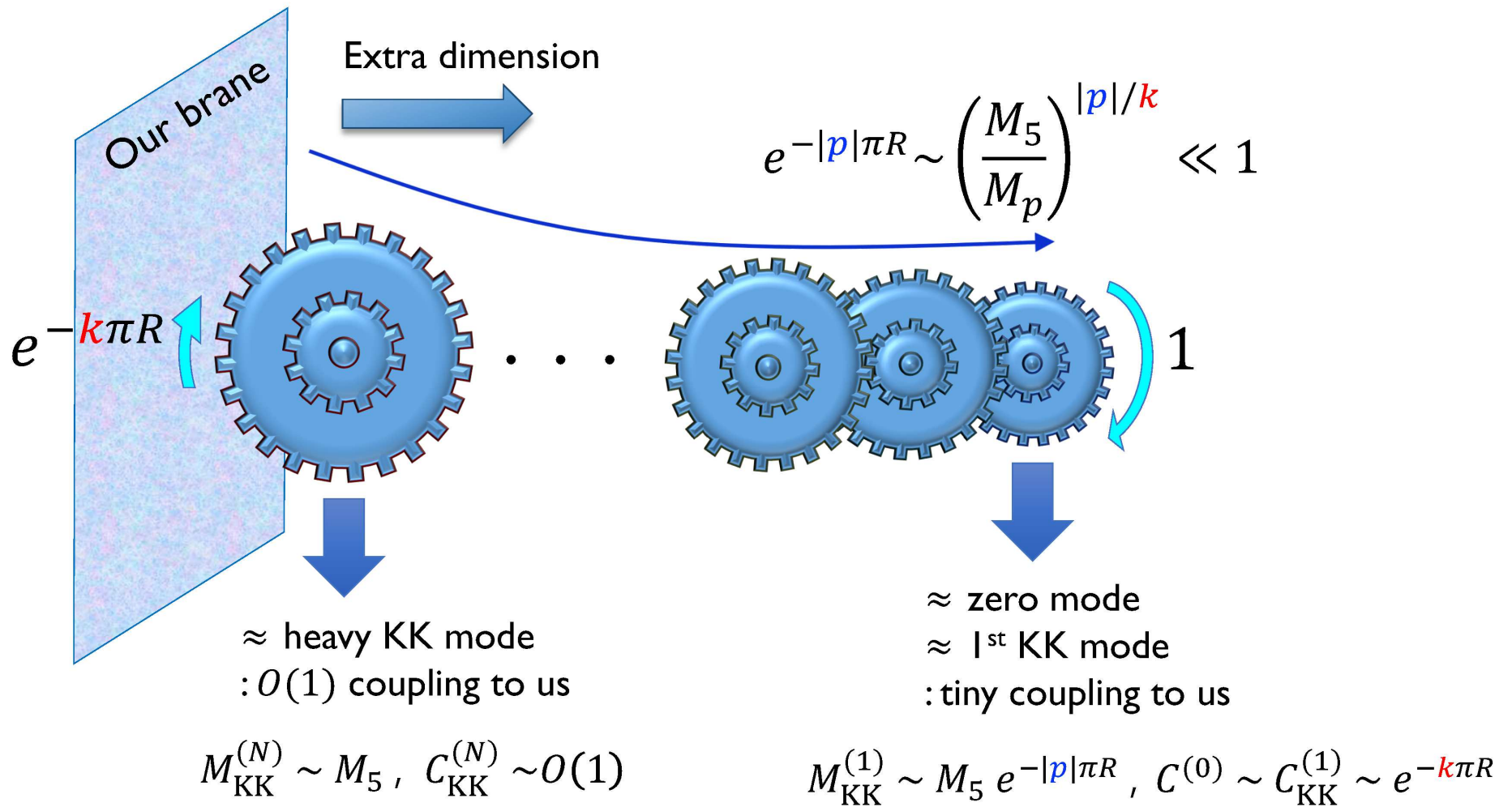


# A discrete set of solutions

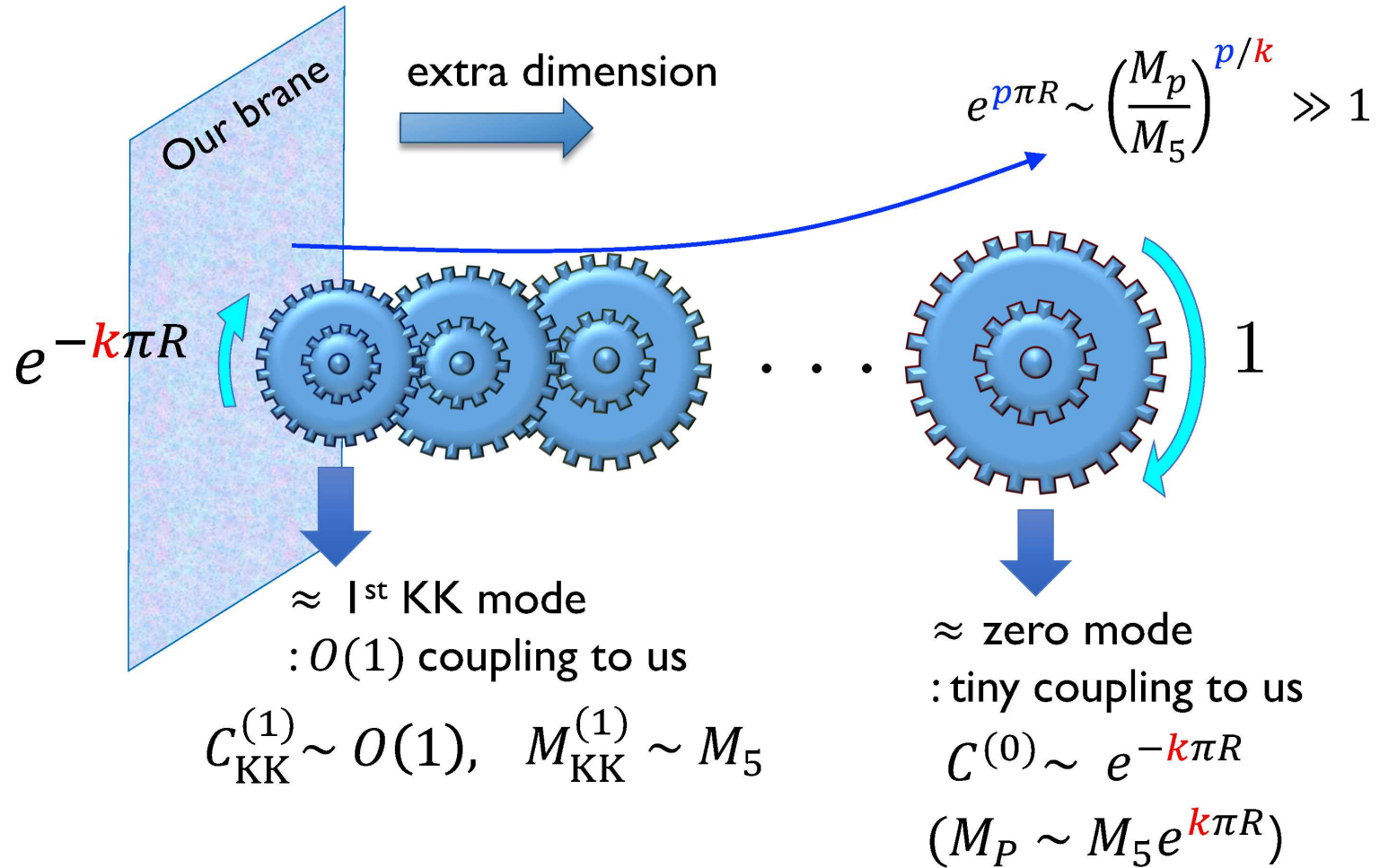


$$ds^2 = e^{2k_1 y} dx^2 + e^{2k_2 y} dy^2 \quad p = k_1 - k_2 \quad k_2 = \hat{c}^2 k_1$$

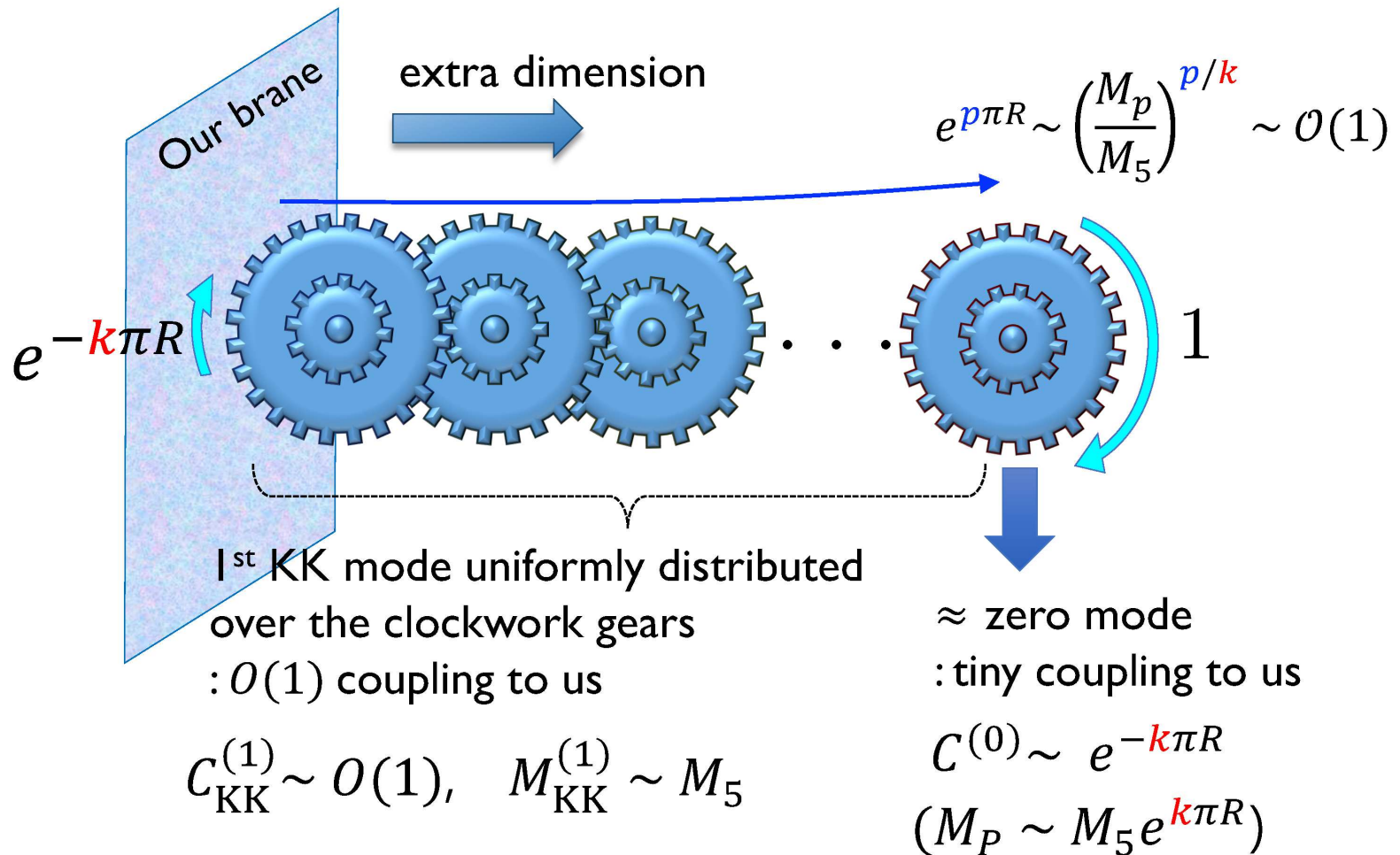
# Large $\hat{c}^2$



# Small $\hat{c}^2$



# Linear Dilaton: $\hat{c}^2 = 1$



# QCD axion from heterotic M-theory

The same mechanism allows a hierarchy between the string scale and the axion scale  $f_a$  (Im, Nilles, Olechowski, 2019)

- various combinations of model-independent and model-dependent axions
- allow a unique implementation of a QCD axion
- with  $f_a \sim 0.1M_{11}$  in the axion window (via warping)

It appears in heterotic M-theory with "unconventional" warping. The QCD axion is accompanied by

- an **ultra-light axion** on the hidden brane
- with decay constant  $f_{\text{UL}} \sim 0.1M_{11}\sqrt{\frac{M_{11}}{M_P}} < f_a$