High Scale Inflation: a Window to Quantum Gravity

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High Scale Inflation

Inflationary dynamics could lead to insight in fundamental properties of high energy physics

- sizeable tensor modes in microwave background
- probe high energies close to the Planck scale
- relevance for swampland discussion
- consistency of quantum gravity

We here consider axionic inflation, also known as natural inflation.

- shift symmetry of axionic inflation
- axions are abundant in string theory

Outline

Interest in theories with "observable" tensor modes in CMB

- Natural Inflation (Freese, Frieman, Olinto, 1990)
- trans-Planckian excursions (Lyth, 1996)
- single axion with a no-go theorem from string theory (Banks, Dine, Fox, Gorbatov, 2003)
- Aligned Axionic Inflation (Kim, Nilles, Peloso, 2005)
- Restrictions from the "Weak Gravity Conjecture" (Arkani-Hamed, Motl, Nicolis, Vafa, 2006)
- Modulated Natural Inflation (Kappl, Nilles, Winkler, 2016)
- Predictions for CMB data (Planck,BICEP/Keck,BAO) (Winkler, Gerbino, Benetti, 2019)

Axionic Inflation

The mechanism of inflation requires a “flat” potential.
We consider

- symmetry reason for flatness of potential
- slightly broken symmetry to move the inflaton

A natural candidate is axionic inflation

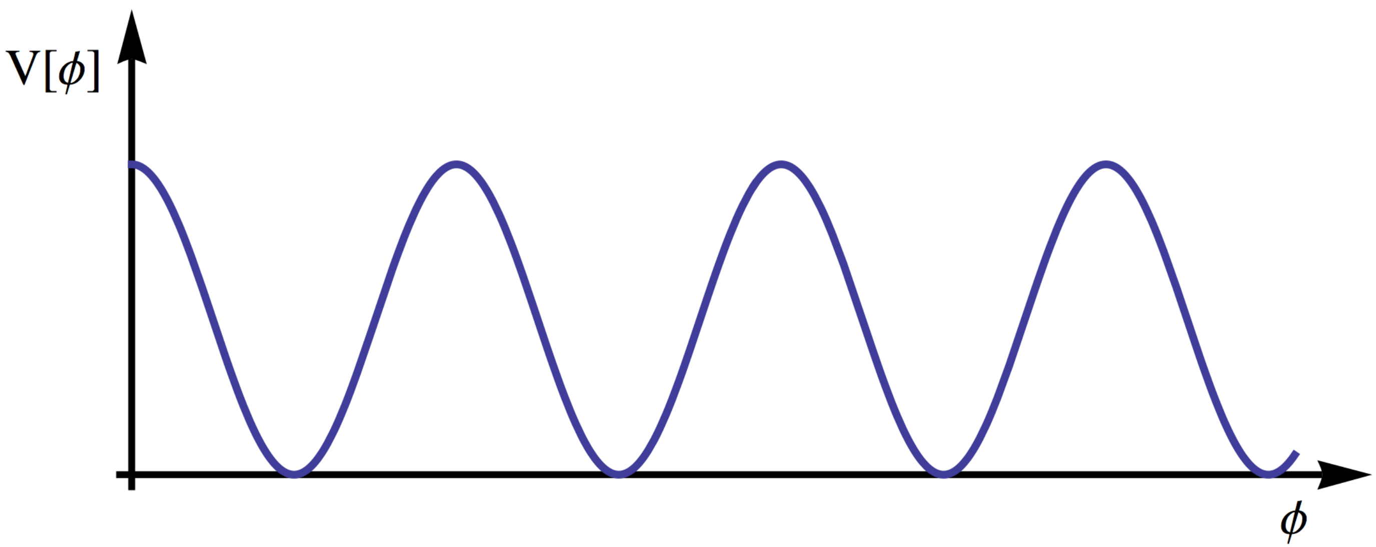
- axion has only derivative couplings to all orders in perturbation theory
- leads to a shift symmetry of the axion
- broken by non-perturbative effects (instantons)

(Freese, Frieman, Olinto, 1990)

The Axion Potential

The axion exhibits the shift symmetry $\phi \to \phi + c$

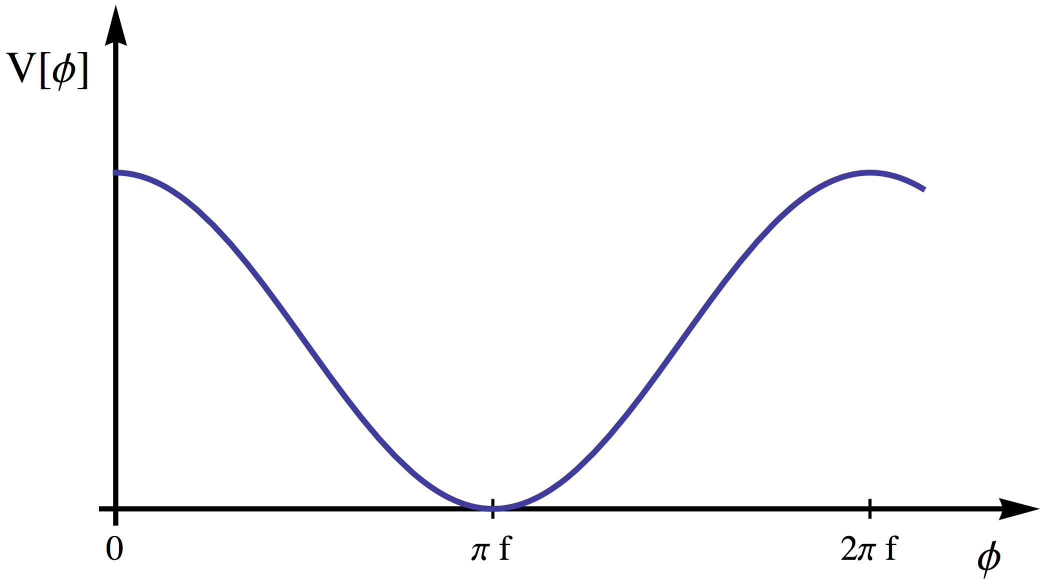
Nonperturbative effects break this symmetry to a remnant discrete shift symmetry



$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{2\pi\phi}{f}\right)\right]$$

The Axion Potential

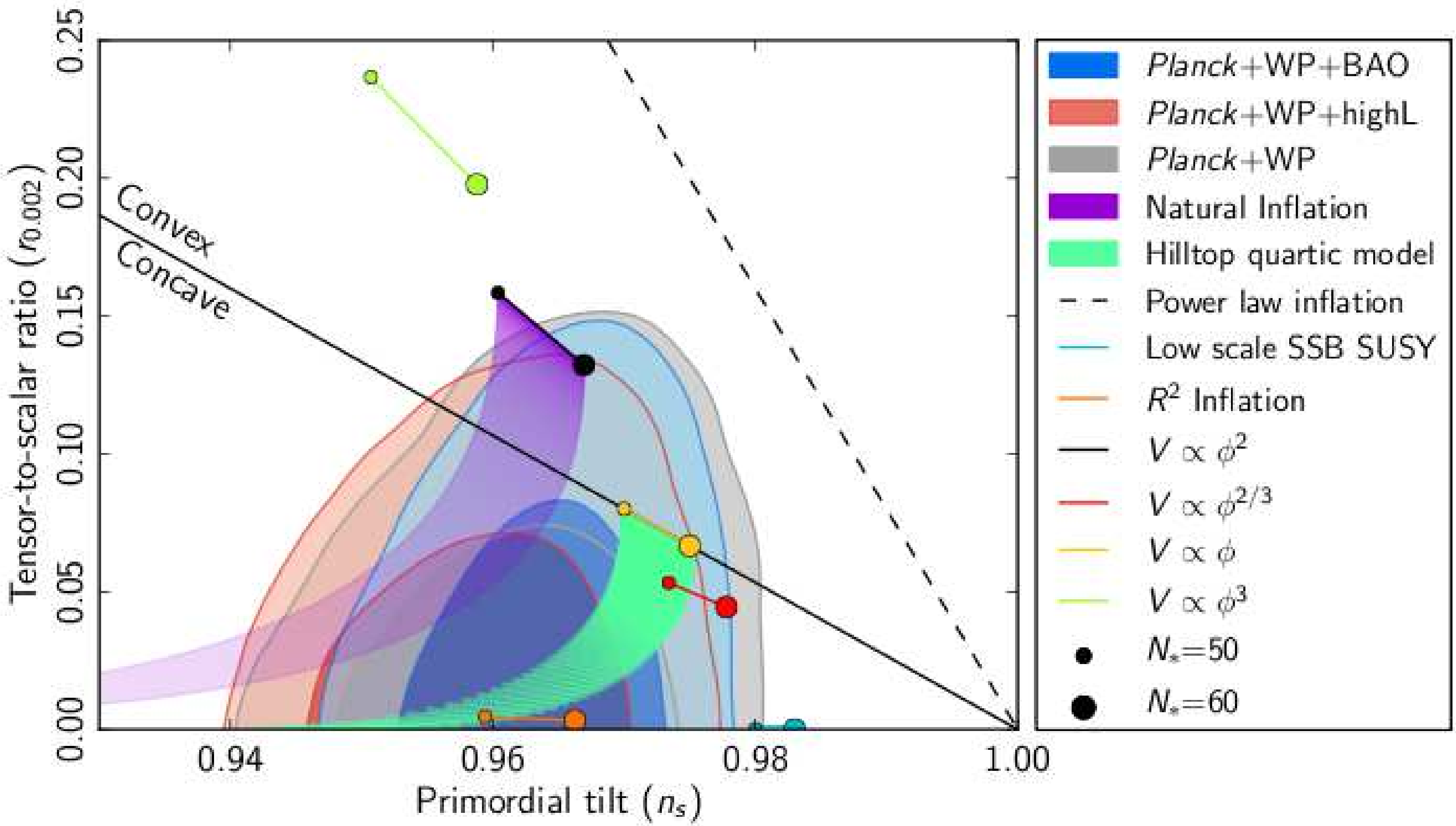
Discrete shift symmetry identifies $\phi = \phi + 2\pi n f$



$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{2\pi\phi}{f}\right)\right]$$

ϕ confined to one fundamental domain

Planck results (Spring 2013)



Tensor Modes in CMB

Tentatively large tensor modes of order $r \sim 0.05$ are still compatible with data

- large tensor modes brings us to scales of physics close to the Planck scale and the so-called "Lyth bound"
- potential $V(\phi)$ of order of GUT scale few $\times 10^{16}$ GeV
- trans-Planckian excursions of the inflaton field
- For a quadratic potential $V(\phi) \sim m^2\phi^2$ it implies $\triangle\phi \sim 15 M_{\rm P}$ to obtain 60 e-folds of inflation

In string theory we might expect constraints on the axion decay constant because of T-duality.

(Banks, Dine, Fox, Gorbatov, 2003)

$T-$Duality

String dualities give important constraints on the axion decay constants, especially $T-$duality $SL(2,Z)$:

$$T \rightarrow \frac{aT+b}{cT+d}$$

generated by an inversion and a shift

$$T \rightarrow -1/T, \qquad T \rightarrow T+1.$$

This leads to $f \rightarrow 1/f$ and $f \leq M_{\rm P}$ for a single axion.

The potential is given by combinations of modular forms. A sufficiently flat potential is only possible for

$f << M_{\rm p}$ (or $f >> M_{\rm p}$)

Aligned axions

A way out is the consideration of two (or more) fields.

(Kim, Nilles, Peloso, 2004)

- string theory favors a multi-axion picture
- requires $f \leq M_{\rm P}$ for the individual axions
- enhanced effective axion scale of aligned axion

The alignment prolongs the fundamental domain of the aligned axion to super-Planckian values, as the axionic inflaton spirals down in the potential of the second axion

This might solve the problem observed in the case of a single axion. We have various axions and one of them has a sufficiently flat potential.

The KNP set-up

We consider two axions

$$\mathcal{L}(\theta,\rho) = (\partial\theta)^2 + (\partial\rho)^2 - V(\rho,\theta)$$

with potential

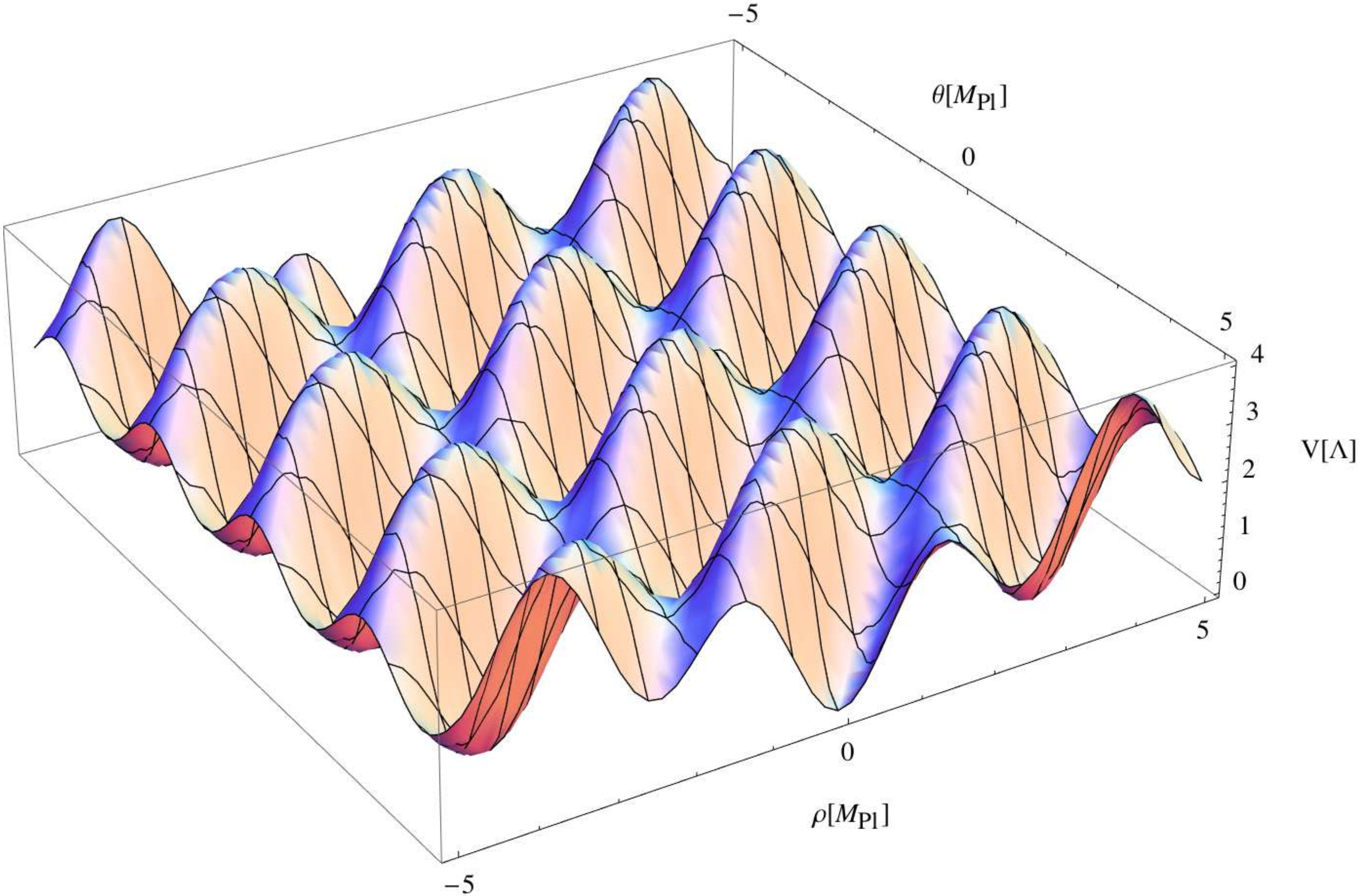
$$V(\theta,\rho) = \Lambda^4 \left(2 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1}\right) - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2}\right)\right)$$

This potential has a flat direction if $\frac{f_1}{g_1} = \frac{f_2}{g_2}$

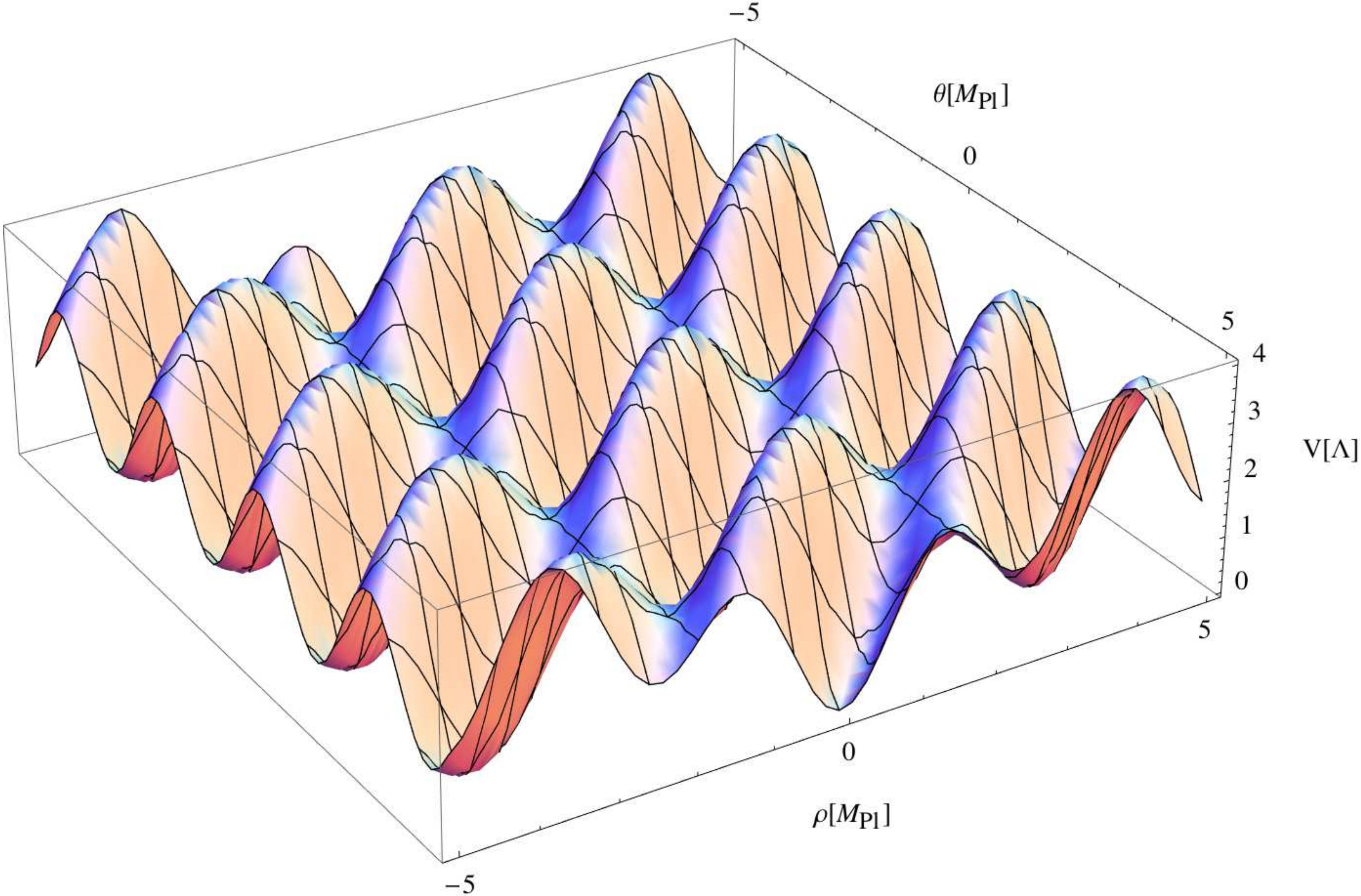
Alignment parameter defined through $\alpha = g_2 - \frac{f_2}{f_1} g_1$

For $\alpha = 0$ we have a massless field ξ.

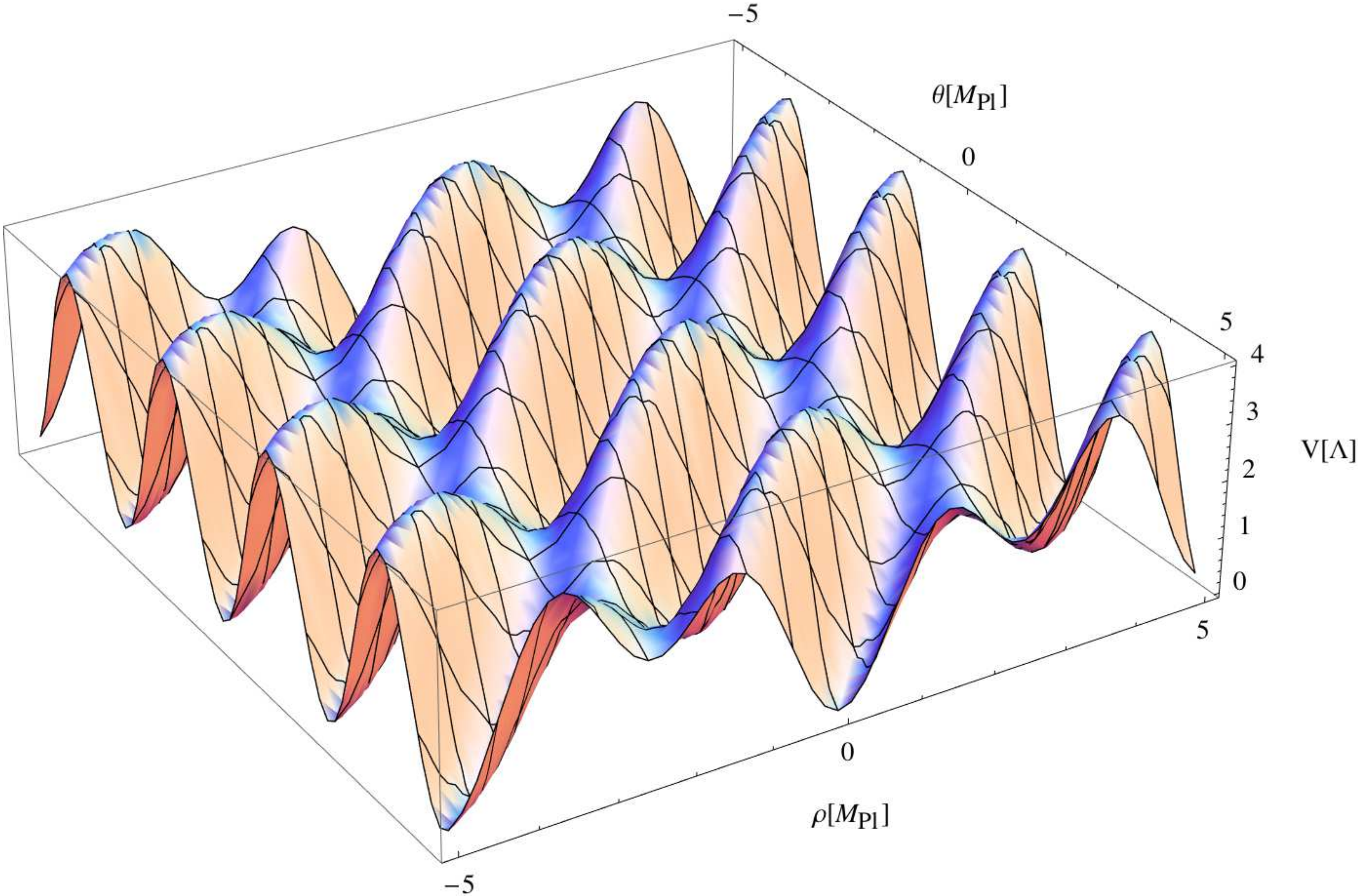
Potential for $\alpha = 1.0$



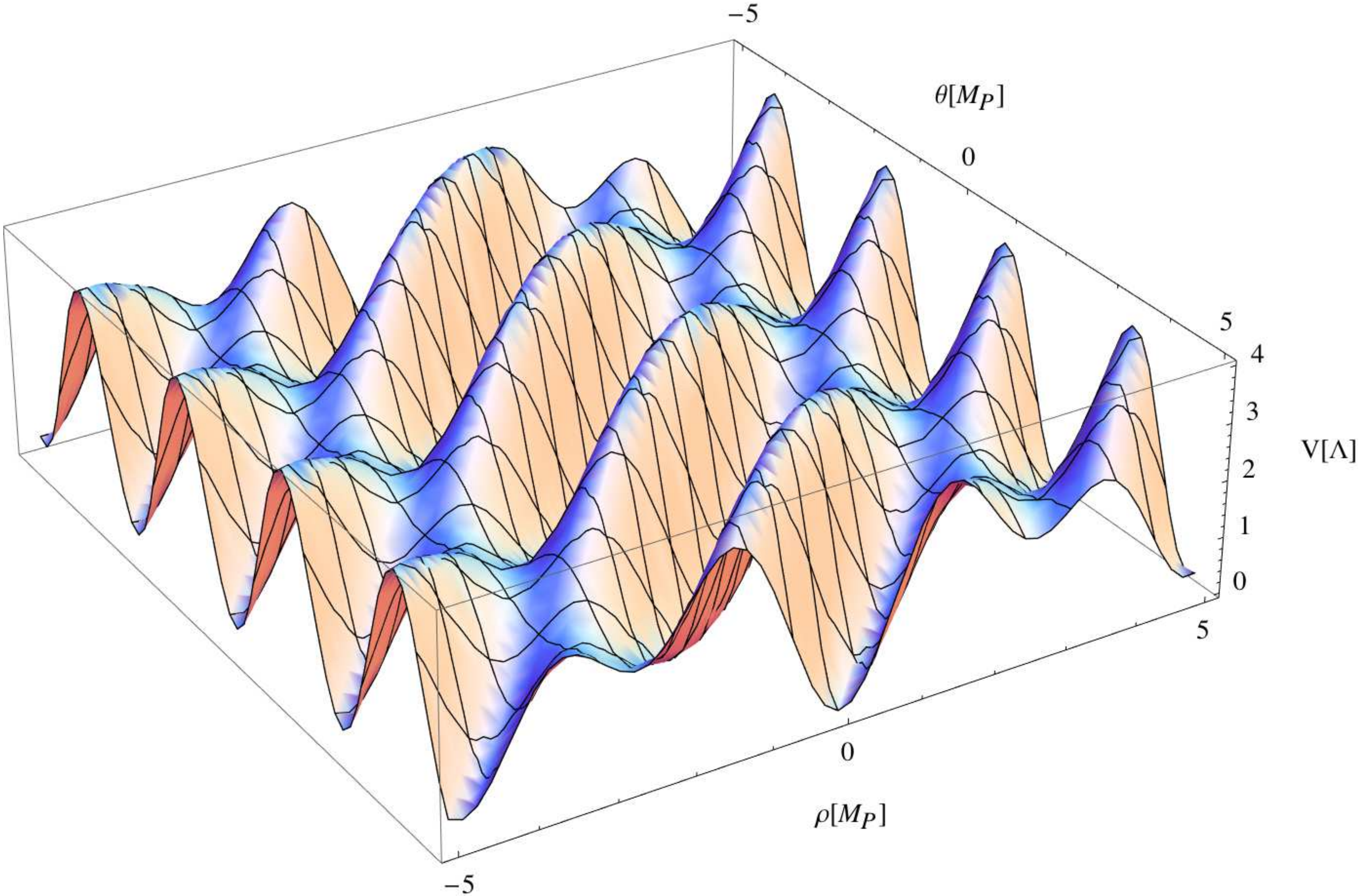
Potential for $\alpha = 0.8$



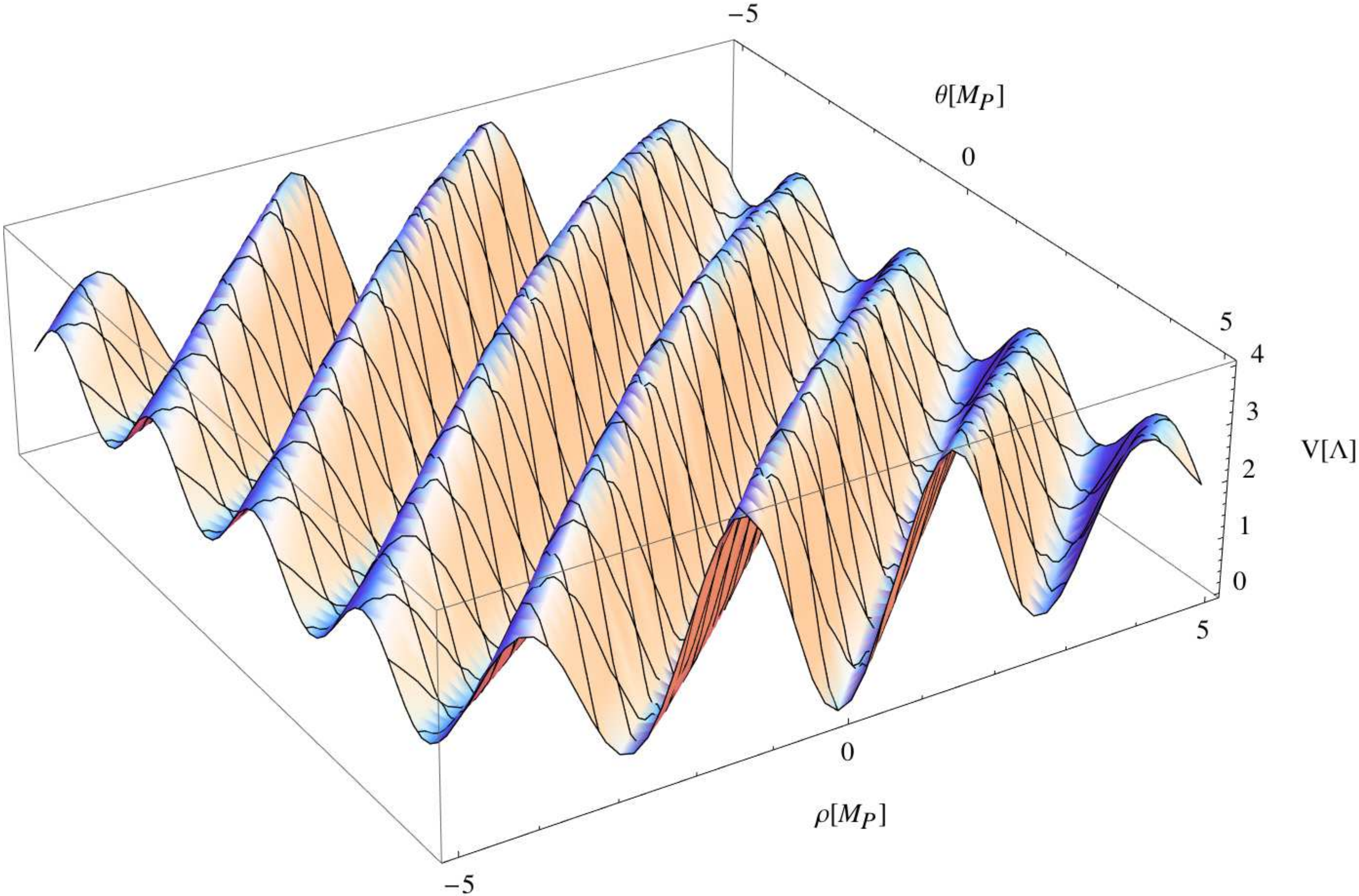
Potential for $\alpha = 0.5$



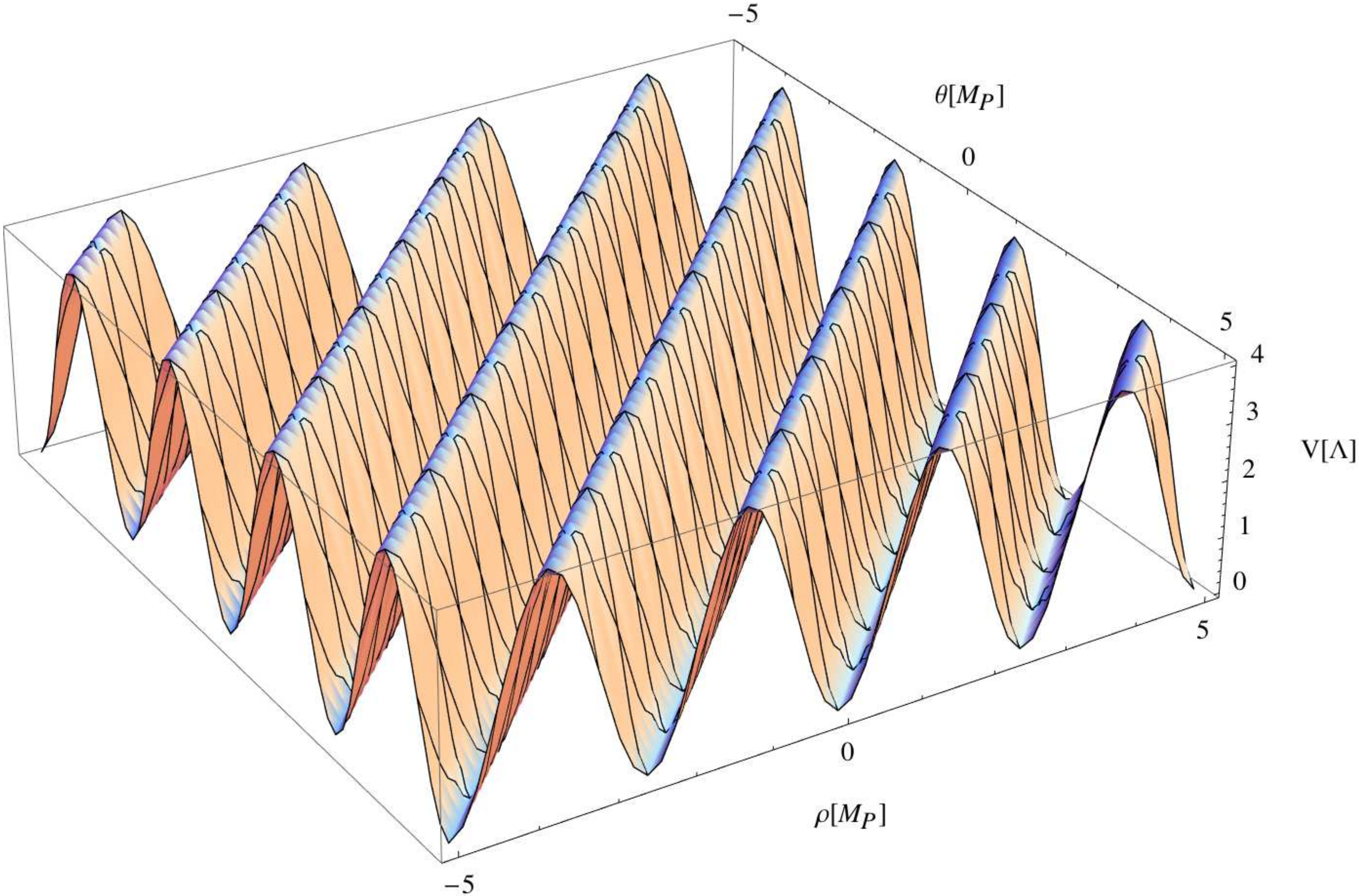
Potential for $\alpha = 0.3$



Potential for $\alpha = 0.1$



Potential for $\alpha = 0$



The lightest axion

Mass eigenstates are denoted by (ξ, ψ). The mass eigenvalues are

$$\lambda_{1/2} = F \pm \sqrt{F^2 + \frac{2g_1 g_2 f_1 f_2 - f_2^2 g_1^2 - f_1^2 g_2^2}{f_1^2 f_2^2 f_1^2 g_2^2}}$$

with $$F = \frac{g_1^2 g_2^2 (f_1^2 + f_2^2) + f_1^2 f_2^2 (g_1^2 + g_2^2)}{2 f_1^2 f_2^2 g_1^2 g_2^2}$$

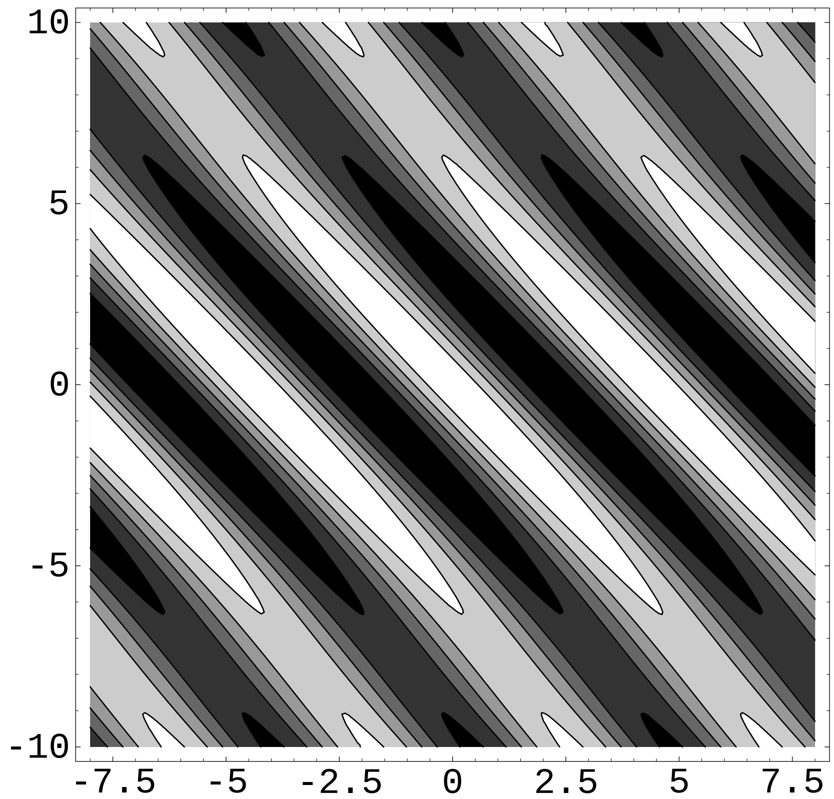
Lightest axion ξ has potential

$$V(\xi) = \Lambda^4 \left[2 - \cos\left(m_1(f_i, g_1, \alpha)\xi\right) - \cos\left(m_2(f_i, g_1, \alpha)\xi\right)\right]$$

leading effectively to a one-axion system

$$V(\xi) = \Lambda^4 \left[1 - \cos\left(\frac{\xi}{\tilde{f}_a}\right)\right]$$ with $$\tilde{f}_a = \frac{f_2 g_1 \sqrt{(f_1^2 + f_2^2)(f_1^2 + g_1^2)}}{f_1^2 \alpha}$$

Axion landscape of KNP model



The field ξ rolls within the valley of ψ. The motion of ξ corresponds to a motion of θ and ρ over many cycles. The system is still controlled by discrete symmetries.

UV-Completion

Large tensor modes and $\Lambda \sim 10^{16}$GeV lead to theories at the "edge of control" and require a reliable UV-completion

- small radii
- large coupling constants
- additional light moduli might spoil the picture

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- small radii
- large coupling constants
- additional light moduli might spoil the picture

So it is important to find systems with reliable symmetries

- axions are abundant in string theory
- perturbative stability of "shift symmetry"
- only broken by nonperturbative effects
- is there an upper limit on f? and thus lower limit on r?

Stability

We have a very flat direction and within the effective quantum field theory we might loose computational control:

- is inflation perturbed by other effects?
- is there an upper limit on $f_{\rm eff}$?

Remember that in case of a single axion we had limits

- $f_{\rm eff} \leq M_{\rm string}$ (Banks, Dine, Fox, Gorbatov, 2003)
- derived from dualities in string theory (e.g. T-duality)

In the multi-axion case these arguments are not directly applicable, but the question of trans-Planckian values should still be tested in a given model

Weak Gravity Conjecture (WGC)

It is based on prejudice about black hole properties and is formulated to constrain $U(1)$ gauge interactions,

(Arkani-Hamed, Motl, Nicolis, Vafa, 2006)

- give limits on mass to charge ratio $|q/m| > 1$
- "convex hull" restrictions in multi-field case.

It is conjectured that the WGC (if true) might be applicable to axions: (Rudelius, 2015 and many others)

- based on a chain of string dualities,
- might give restriction on decay constants $f_{\rm eff}$.

This might lead to a no-go theorem for large axion decay constants.

WGC II

The weak gravity conjecture comes in two versions

- strong version where the lightest state has to satisfy the convex hull condition
- weak version where an arbitrary state can satisfy the complex hull condition

The second case provides some loopholes

- due to subleading instantons,
- computationally we are at the “edge of control”.

Needs to be clarified in explicit constructions and experimental tests.

(Kappl, Nilles, Winkler, 2016)

Explicit Constructions

In general we do not just get cosine potentials, but obtain corrections due to higher harmonics from

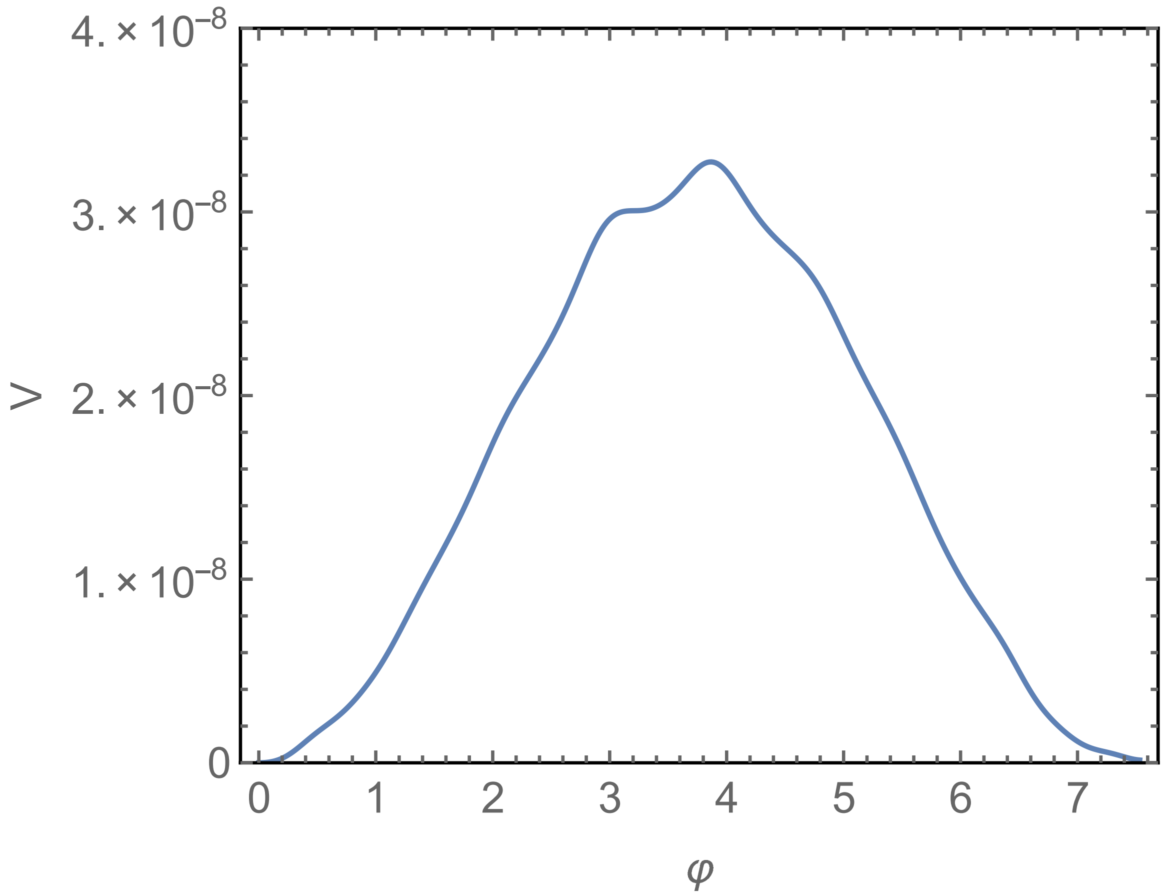
- multi-instanton effects
- restrictions from modular invariance (T-duality)

In string theory there appear modular functions like

$$\eta(T) = e^{-\pi T/12} \times \prod_k \left(1 - e^{-2k\pi T}\right)$$

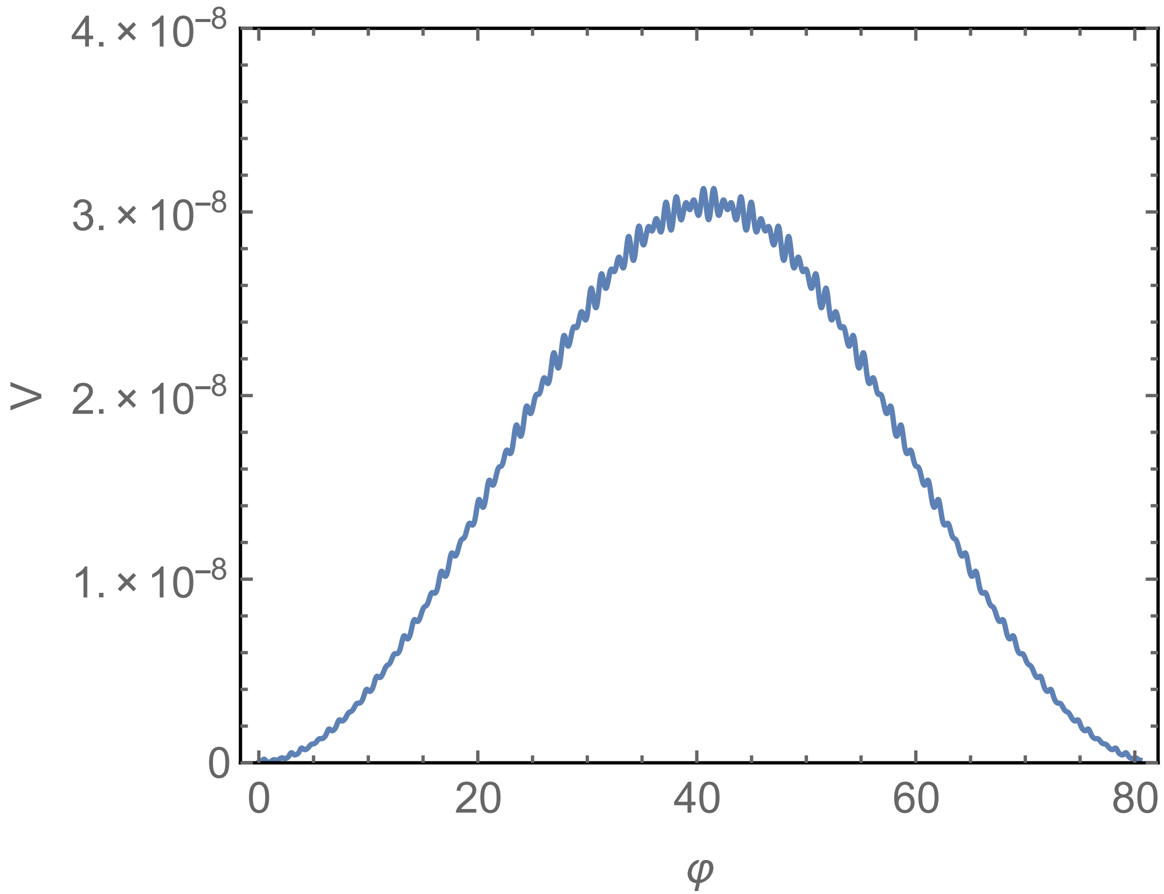
Higher harmonics might satisfy the WGC (in its weak form). This leads to wiggles in the potential that perturb the flat direction and might stop inflation.

Wiggles in the potential



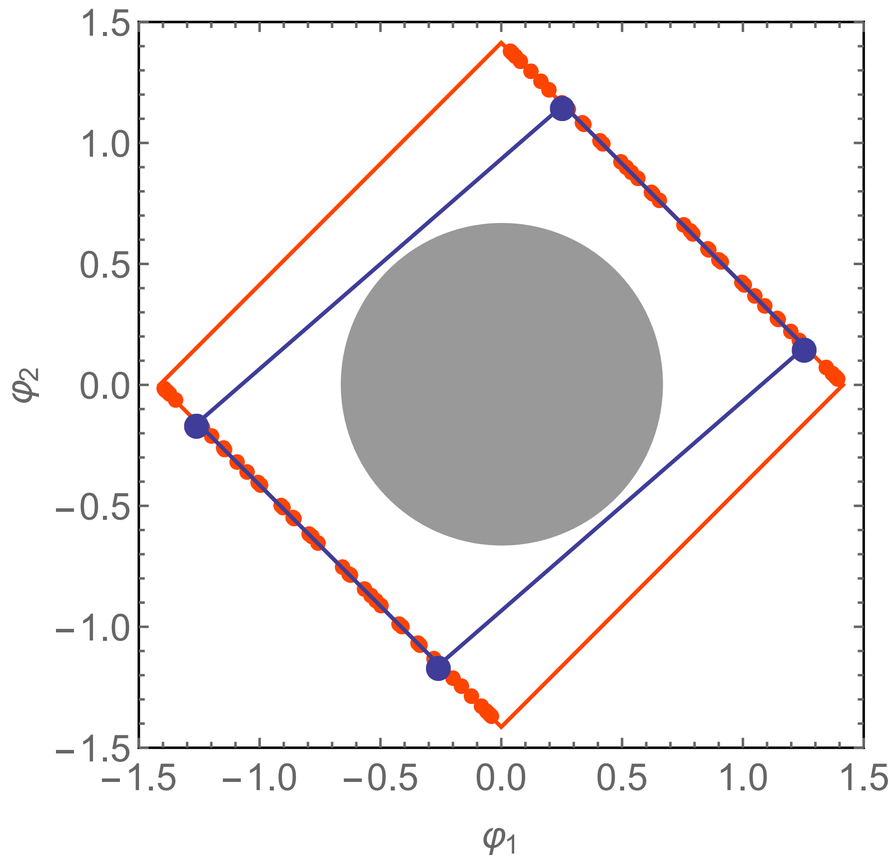
The wiggles in the case of small effective f

Wiggles in the aligned potential



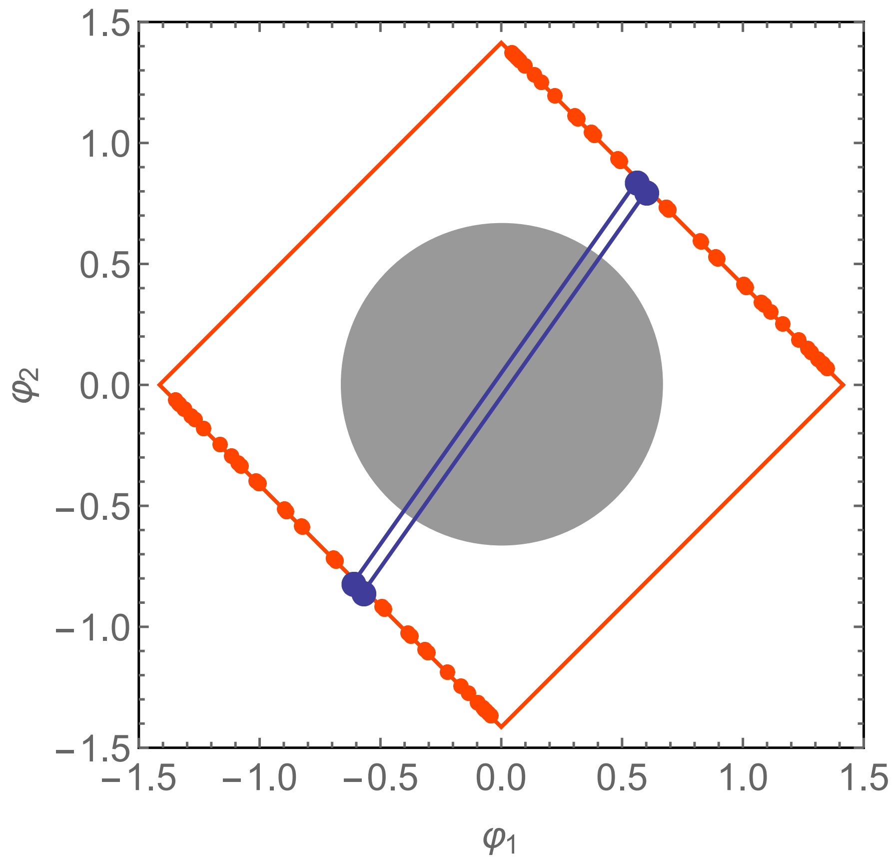
axion alignment with effective superPlanckian f

Non-aligned Axions



The convex hull restrictions are trivially satisfied

Aligned Axions



Subleading terms (red) satisfy the restrictions

Modulated Natural Inflation

It seems plausible that under some circumstances the wiggles become important and

- spoil the flat direction,
- provide un upper limit on decay constant $f_{\rm eff}$.

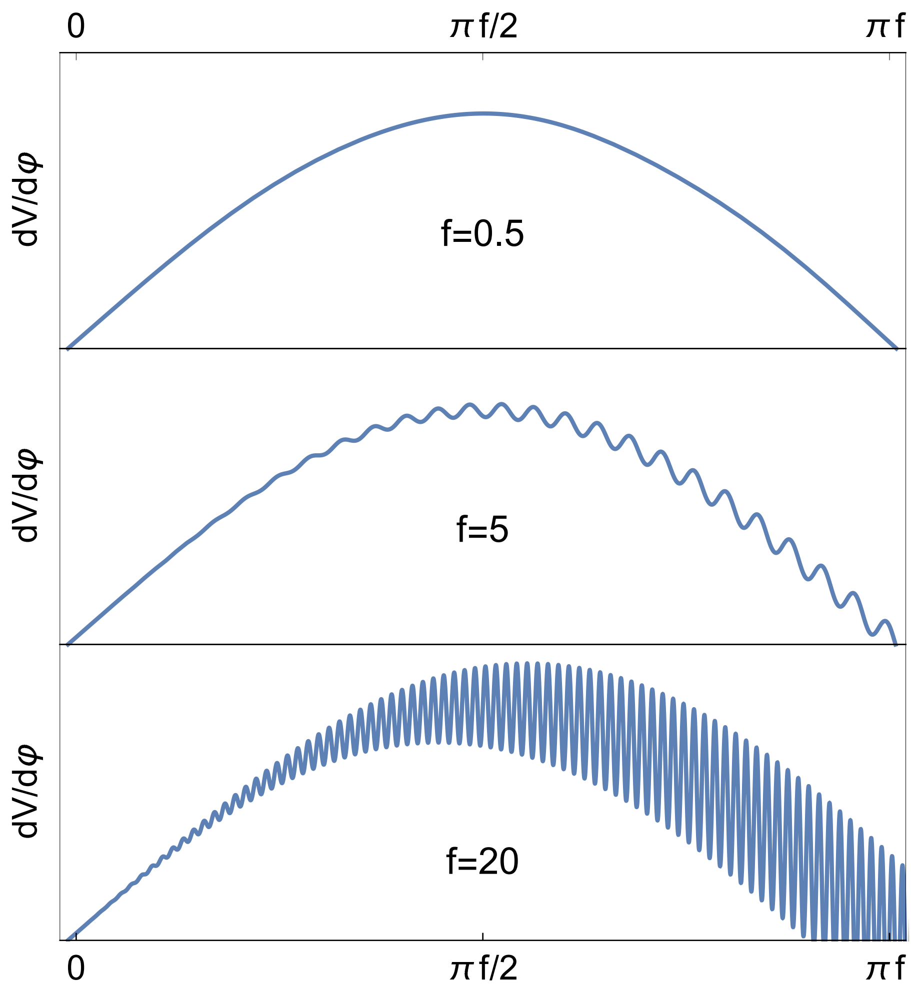
Explicit calculations are necessary to clarify the situation,

- but might be beyond our present capabilities;
- observational confirmation is extremely important.

Restrictions from WGC are satisfied here both in the aligned and non-aligned case.

(Kappl, Nilles, Winkler, 2015; Choi, Kim, 2015)

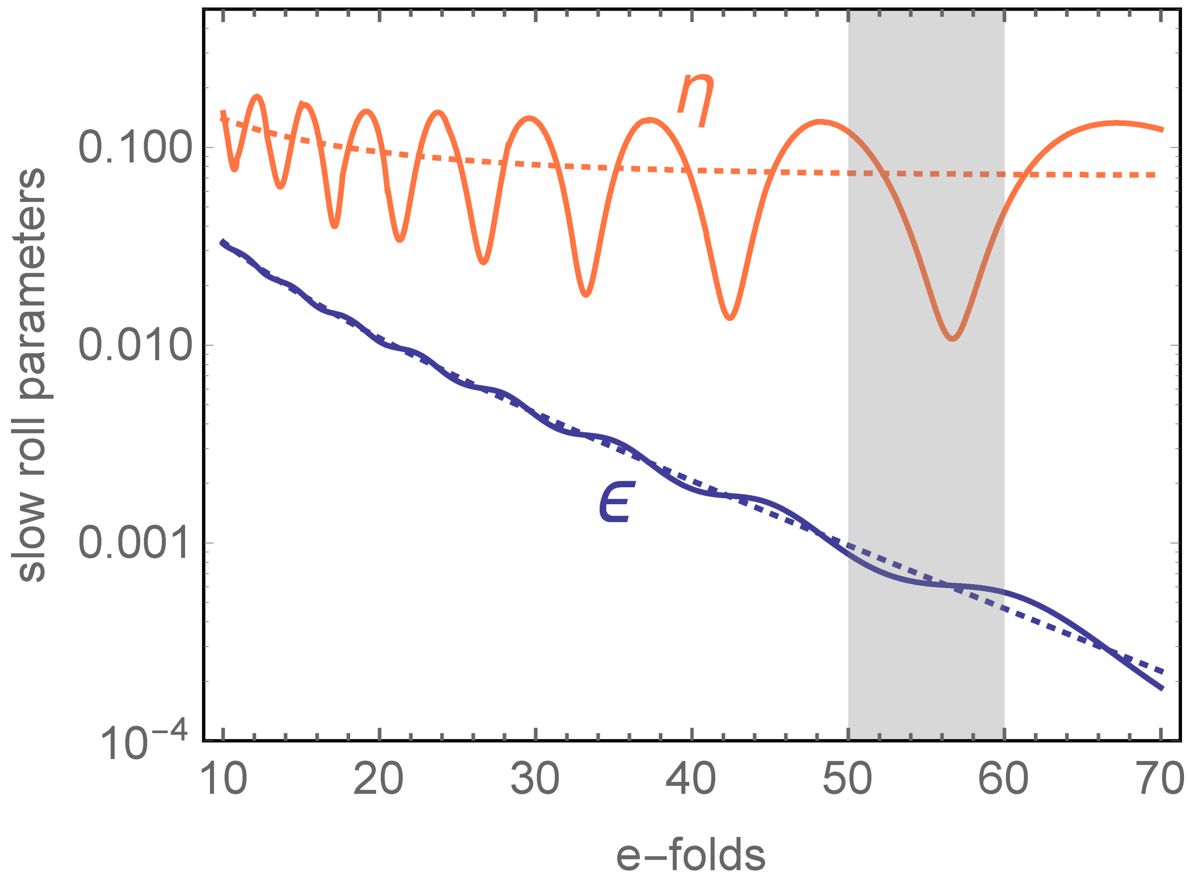
Slope of Potential



Wiggly structure becomes even more important

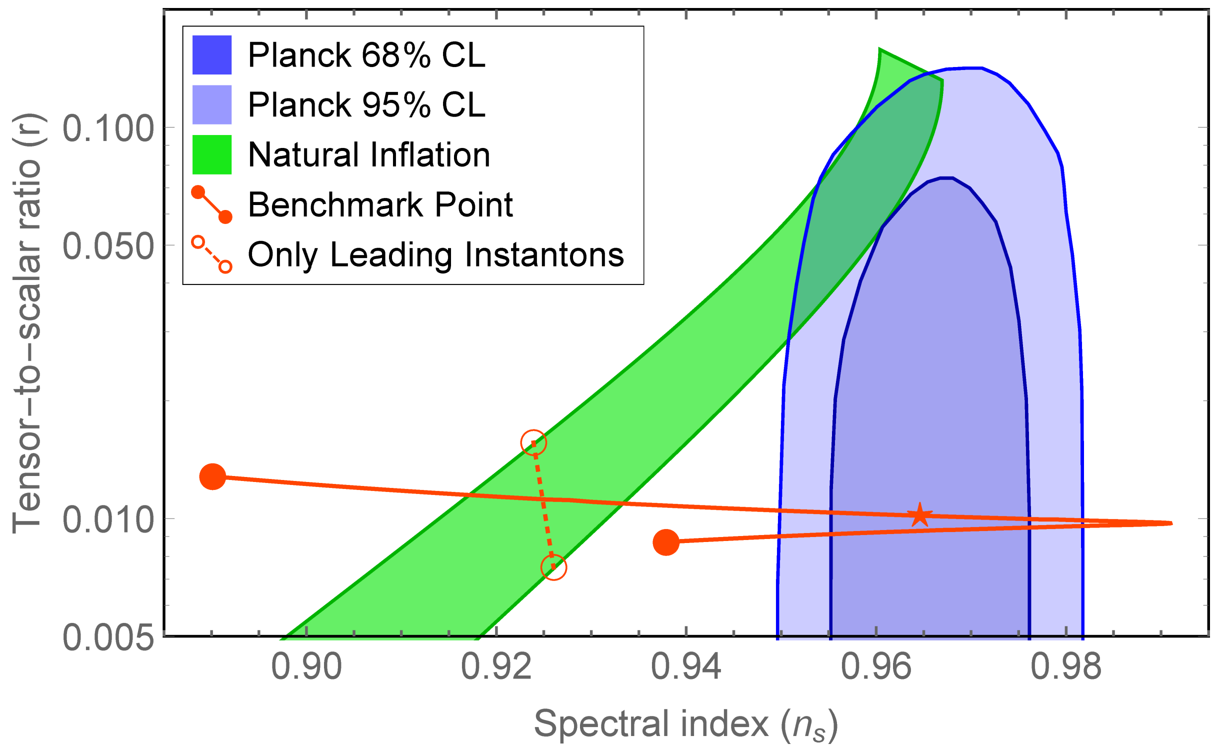
(Kappl, Nilles, Winkler, 2015)

Slow roll parameters



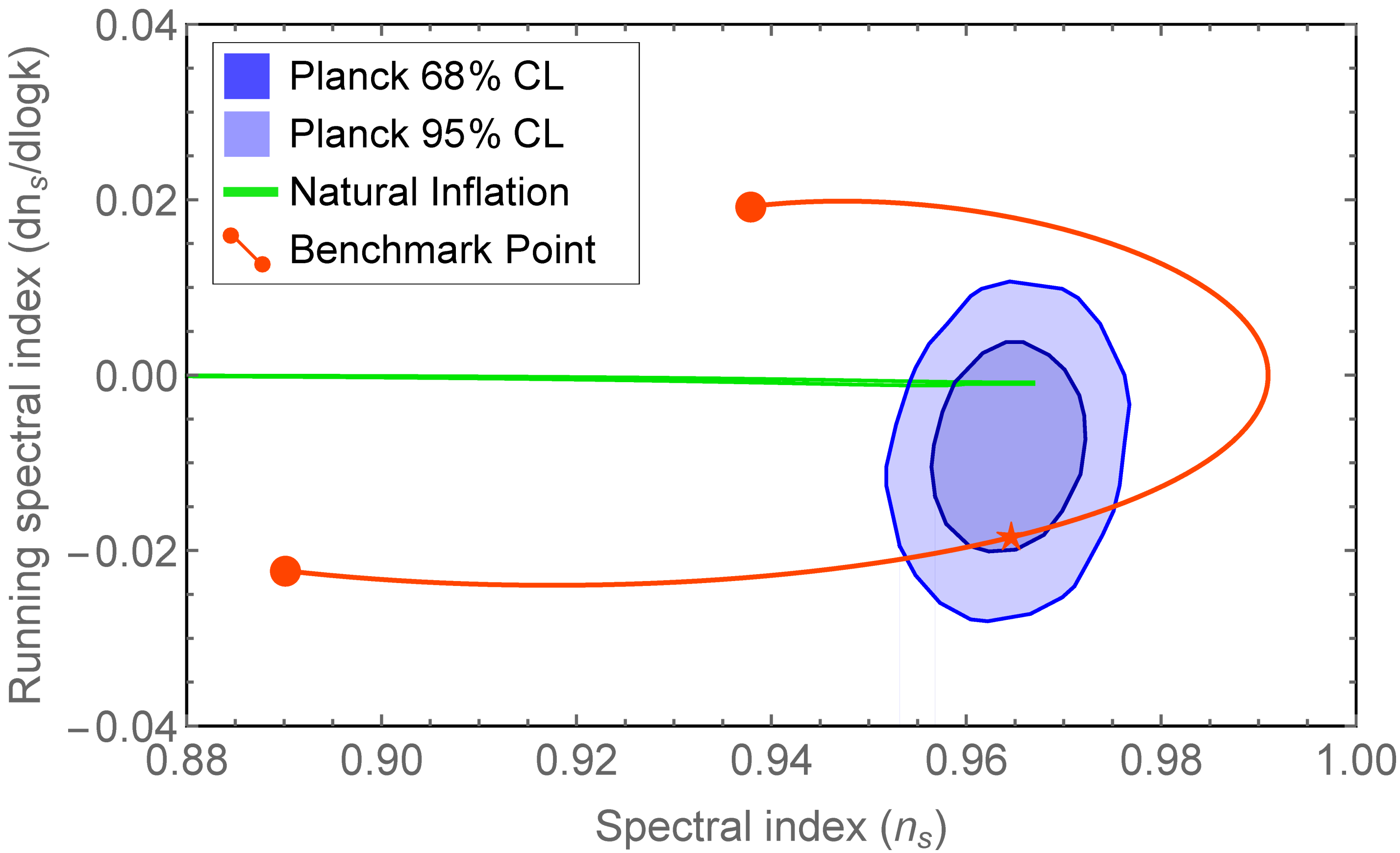
ϵ (η) depend on first (second) derivative

$n_s - r$ plane



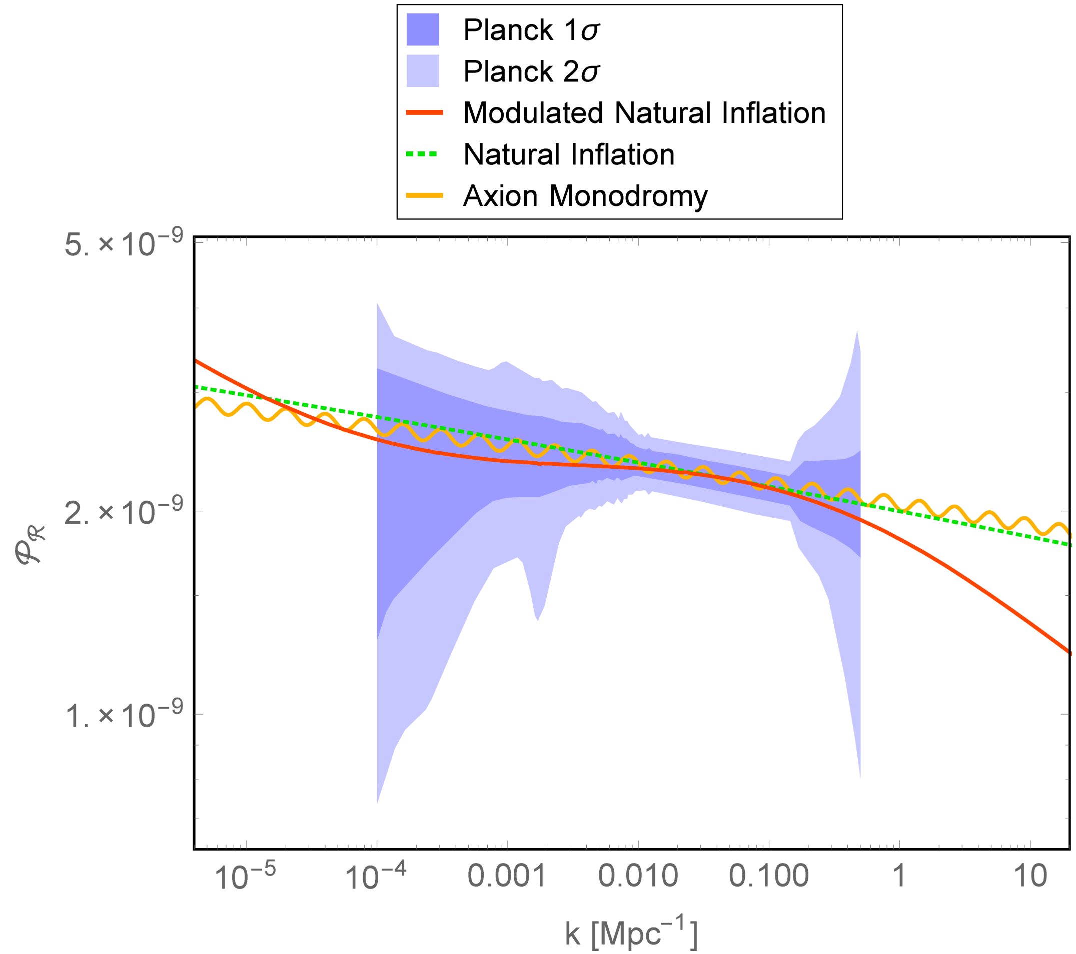
Strong variations of n_s with the number of e-folds

Running of spectral index



Comparison of spectral index with Planck data

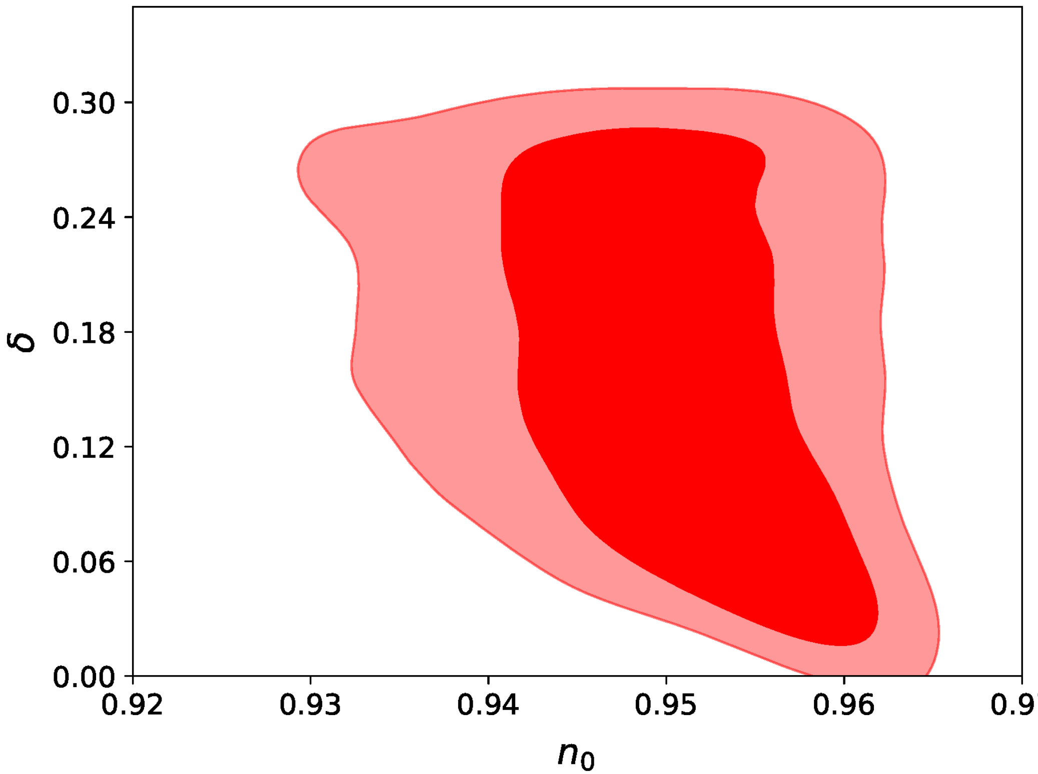
Scalar power spectrum



Comparison to Planck reconstructed power spectrum

Fit to CMB data

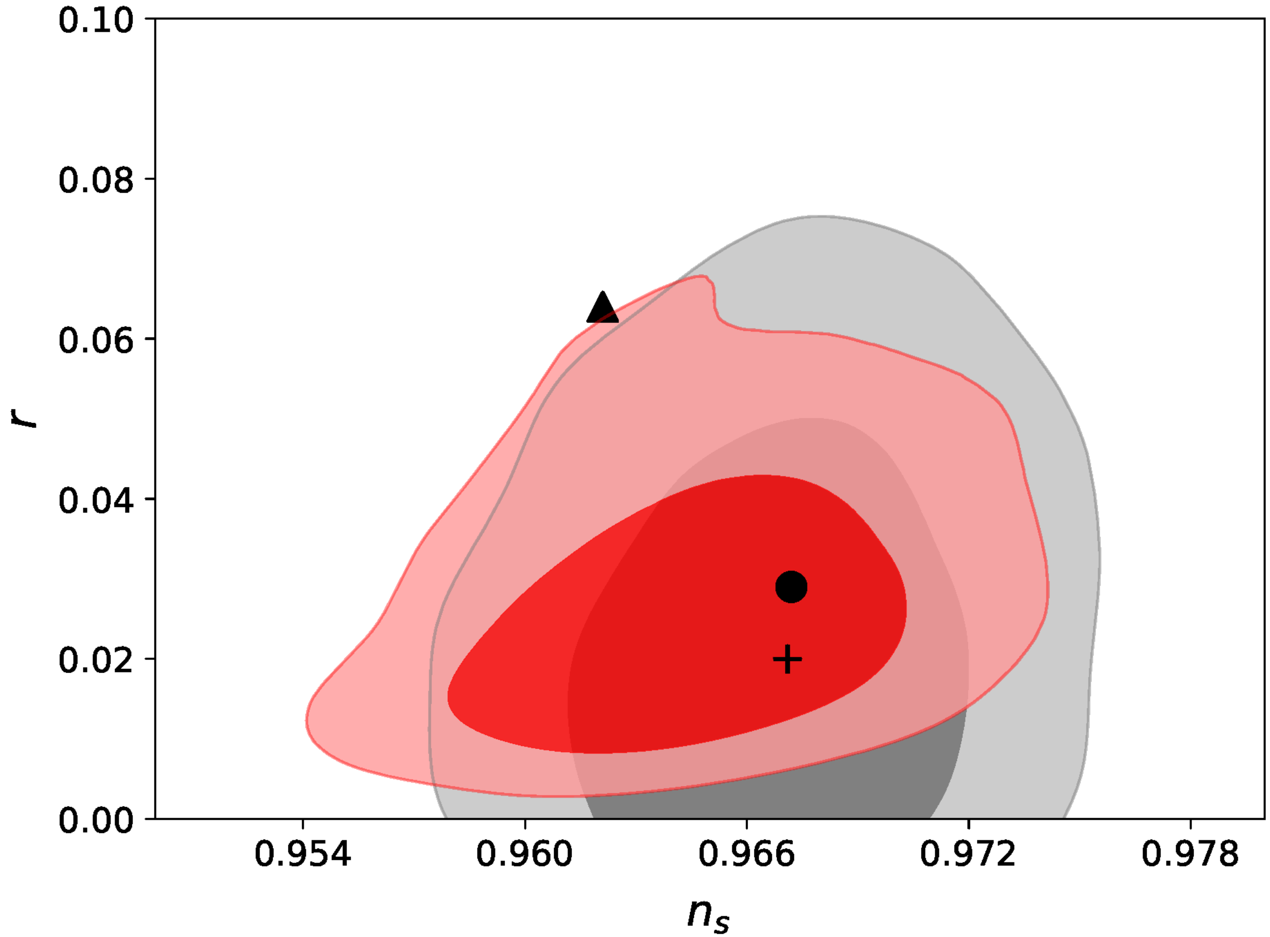
$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right] \left[1 + \delta \cos\left(\frac{\phi}{f_{\rm mod}}\right)\right]$$



n_0 is scalar index of natural inflation without modulations

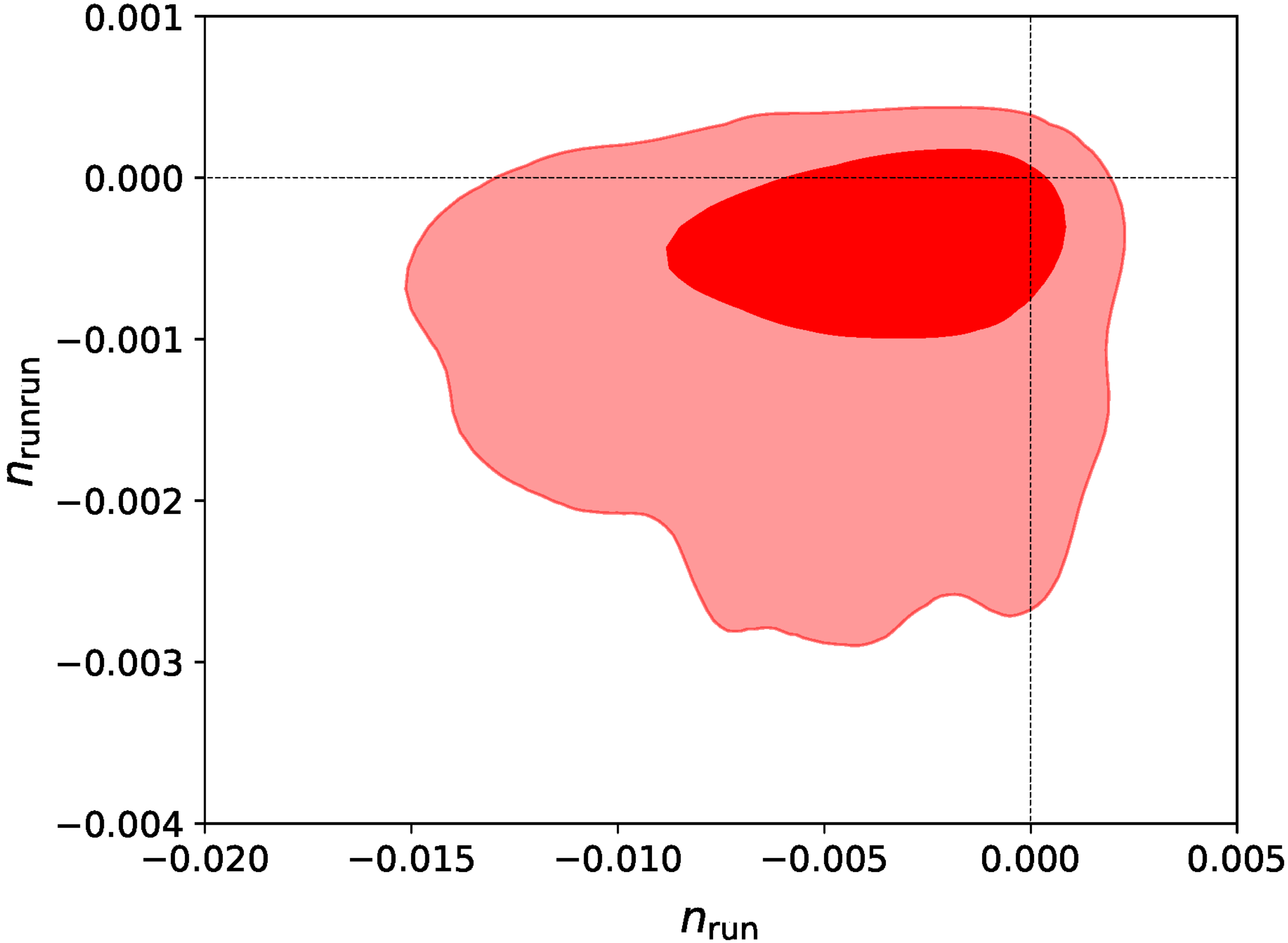
(Winkler, Gerbino, Benetti, 2019)

Comparison to ΛCDM + r

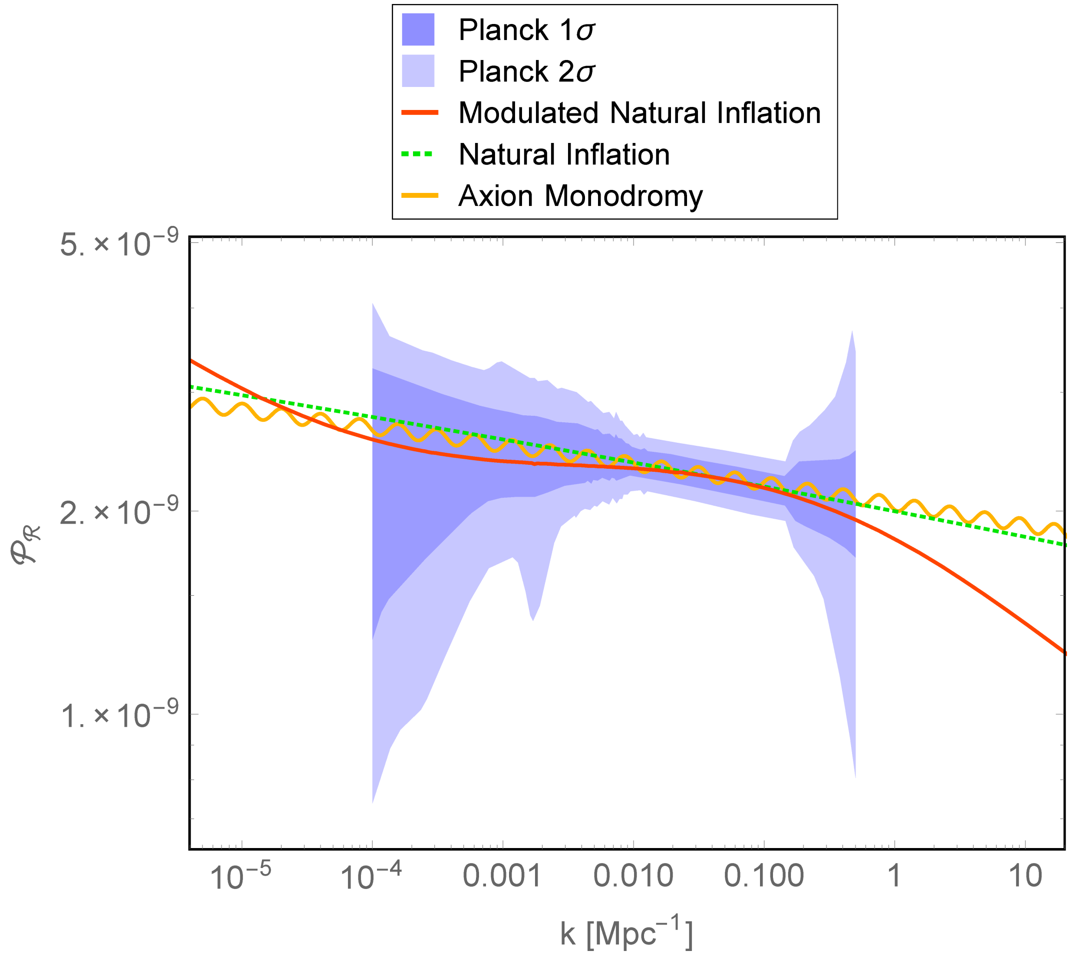


Modulated Natural Inflation in red. Lower limit: $r > 0.002$

Running of scalar index

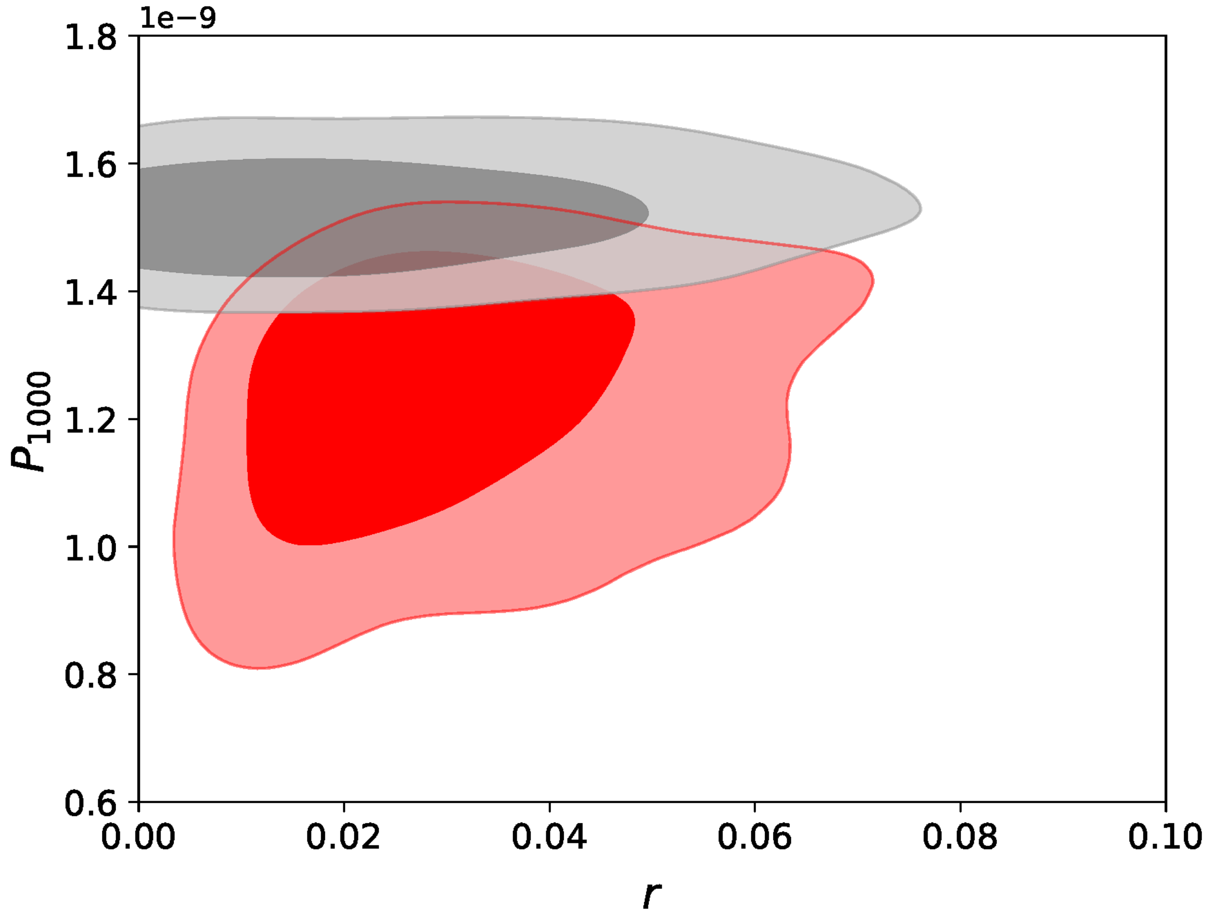


Scalar power spectrum



Strongest contstraints for $0.01 < k < 0.01$

Power at $k = 1000[\mathrm{Mpc}^{-1}]$



Less power at small scales (large k, high multipoles)

Results

- Natural Inflation (NI) is essentially ruled out
  - too large tensor modes
  - incompatible with the weak gravity conjecture (WGC)
- Modulated Natural Inflation (MNI) satisfies WGC
  - but there is an upper limit on f
  - best fit for MNI $4.1 < f < 5.4$ at 1σ-level
- MNI predicts a lower limit: $r > 0.002$ at 99% CL
- $n_s \sim 0.96 - 0.97$ is slightly smaller than in ΛCDM (requires $n_0 > 0.93$ of Natural Inflation)
- MNI has scale dependent modulations in scalar spectrum with reduced power at small scales (large k)

Summary and Outlook

Modulated Natural inflation

- gives excellent fit to current CMB-data
  (comparable in quality to that of ΛCDM + r)
- is consistent with theoretical conjectures
- predicts sizeable tensor modes $r > 0.002$
- predicts lower power at small scales
  (large k, high multipoles)

MNI vs. ΛCDM can be tested in future CMB observations:

- Simons Observatory: $\sigma(r) = 0.003$ (soon)
- CMB-S4 and LiteBIRD: $\sigma(r) = 0.001$ (2028-2029)
- PICO: $r > 5 \times 10^{-4}$ (proposed satellite mission)