

HIGHER DIMENSIONAL SPACETIME

AND THE

UNIFICATION OF FUNDAMENTAL INTERACTIONS

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PARTICLE PHYSICS HAS A

STANDARD MODEL

FOR STRONG, WEAK AND

ELECTROMAGNETIC INTERACTIONS

IT IS BASED ON THE

GAUGE SYMMETRY

$SU(3) \times SU(2) \times U(1)$

OUTLINE

- * THE STANDARD MODEL (SM)
- * REASONS TO GO **BEYOND** THE SM
- * BOTTOM-UP APPROACH (**SUSY** + **GUTs**)
- * SYMMETRIES (SOME **GROUP THEORY**)
- * TOP-DOWN APPROACH (STRING THEORY)
- * CONCLUSIONS

* GRAVITY ~ 1915

* QED ~ 1950

* YANG MILLS THEORY ~ 1954

* STANDARD MODEL ~ 1970

● RADIATION (GAUGE BOSONS)

$SU(3) \times SU(2) \times U(1)$

↖
GLUONS (OCTET)

↘
 W^{\pm}, Z, γ

● MATTER:

3 FAMILIES OF QUARKS AND LEPTONS

$$\begin{pmatrix} u & u & u \\ d & d & d \end{pmatrix} \gamma = \frac{1}{6}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \gamma = -\frac{1}{2}$$

$$\begin{pmatrix} \bar{u} & \bar{u} & \bar{u} \end{pmatrix} \gamma = -\frac{2}{3}$$

$$\bar{e} \gamma = 1$$

$$\begin{pmatrix} \bar{d} & \bar{d} & \bar{d} \end{pmatrix} \gamma = \frac{1}{3}$$

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$$\bar{e} \gamma = 1$$

$$\begin{pmatrix} \bar{d} & \bar{d} & \bar{d} \end{pmatrix} \gamma = \frac{1}{3}$$

$$\bar{\nu} \gamma = 0$$

THREE FUNDAMENTAL QUESTIONS:

* ORIGIN OF STRUCTURE
OF A GIVEN FAMILY ?

* WHY 3 COPIES OF THE
SAME STRUCTURE ?

* WHY $8U(3) \times 8U(2) \times U(1)$

TO OBTAIN AN ANSWER WE NEED TO GO
BEYOND THE STANDARD MODEL

OTHER REASONS FOR PHYSICS BEYOND SM:

* NEUTRINO PROPERTIES

* MECHANISM FOR BARYOGENESIS

* EXISTENCE OF DARK MATTER

BOTTOM-UP APPROACH

* EVOLUTION OF GAUGE COUPLINGS

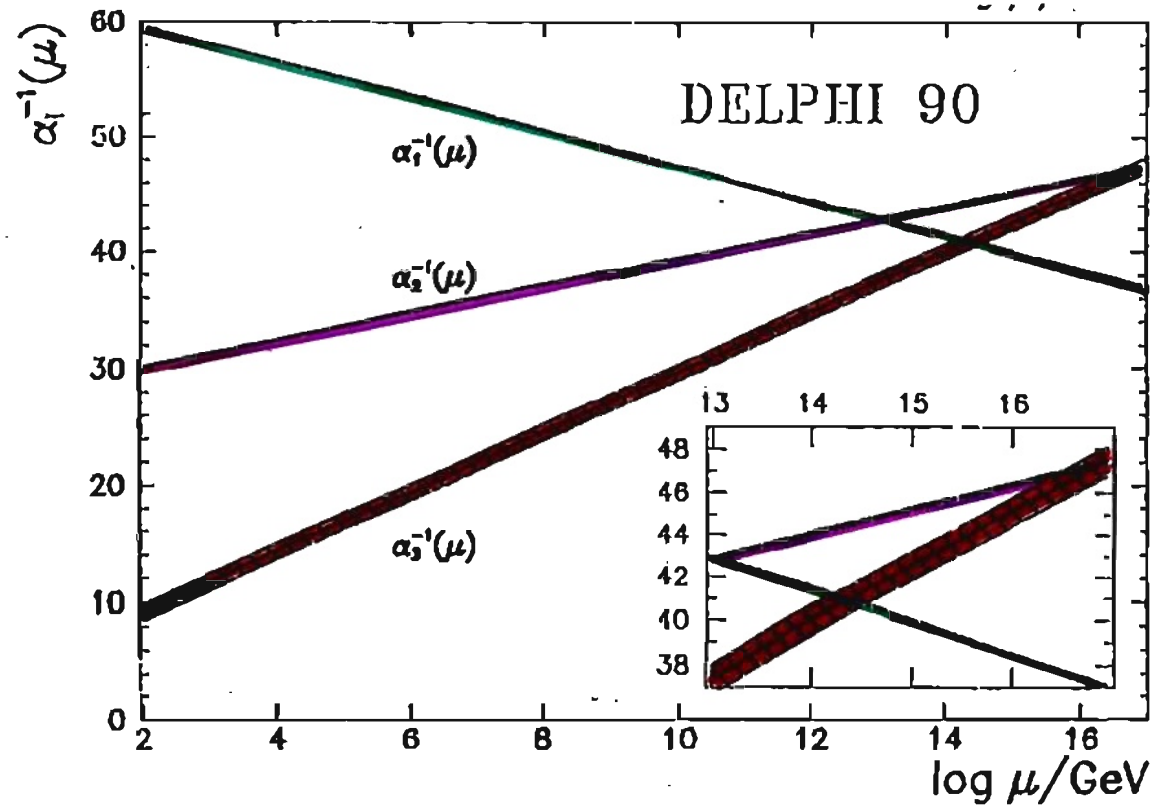
LEADS TO GUTS + SUSY

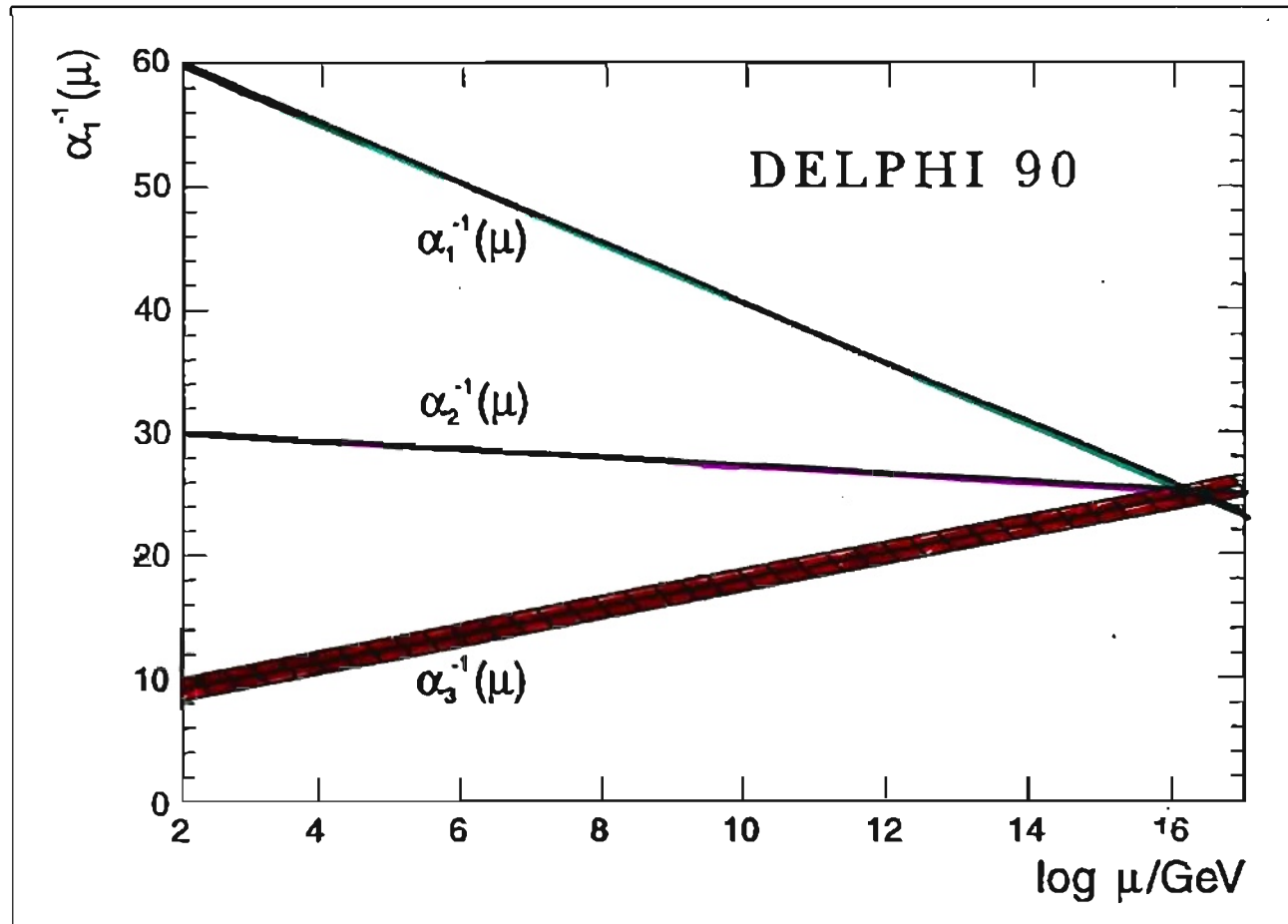
$$M_{\text{GUT}} \sim 10^{16} \text{ GeV}$$

$$M_{\text{SUSY}} \sim 10^3 \text{ GeV}$$

SUSY UNIFIES FERMIONS AND BOSONS,

THUS MATTER AND RADIATION





* NEUTRINO "MASSES" $\sim 10^{-3} \text{eV}$

MIGHT FIND EXPLANATION IN

SEE-SAW MECHANISM

(INVOLVING A RIGHT-HANDED NEUTRINO)

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

$$m_1 \sim M$$

($m \ll M$)

$$m_2 \sim \frac{m^2}{M}$$

$$M \sim 10^{16} \text{GeV} \quad m \sim 10^2 \text{GeV}$$

$$\rightarrow m_1 \sim 10^{16} \text{GeV}$$

$$m_2 \sim 10^{-3} \text{eV}$$

* GRAND UNIFIED GROUPS

→ $SU(5)$ FAMILY $\bar{5} + 10$

→ $SO(10)$ FAMILY 16

* SYMMETRY BREAKDOWN

* PROTON DECAY

* REPETITION OF FAMILIES

* SOME GROUP THEORY



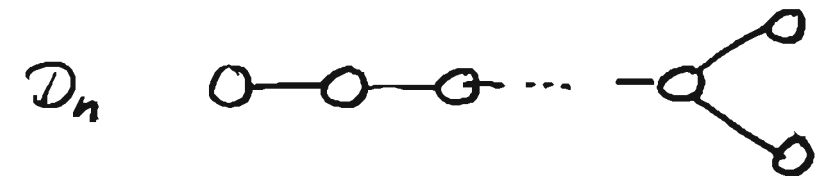
$SU(n+1)$



$SO(2n+1)$



$SP(2n)$



$SO(2n)$

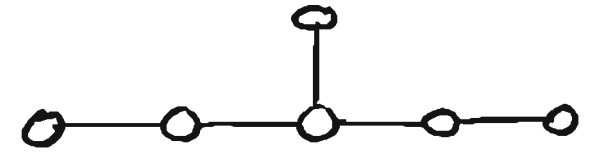
FOUR INFINITE SERIES



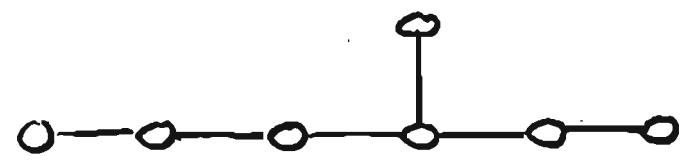
G_2



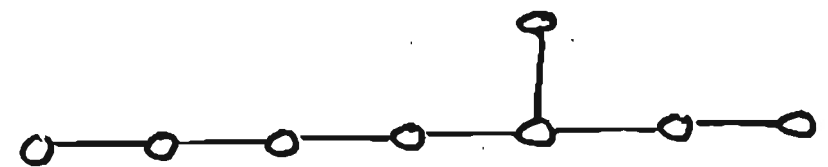
F_4



E_6



E_7



E_8

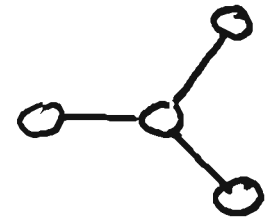
FIVE EXCEPTIONAL GROUPS

ATTRACTIVE POSSIBILITIES

E_8 : AS LARGEST EXCEPTIONAL GROUP

G_2 : AS SMALLEST " "

$D_4 = SO(8)$ BECAUSE OF SYMMETRY



COULD THESE MATHEMATICAL STRUCTURES
BE REALIZED IN NATURE?

PARITY VIOLATION IN WEAK INTERACTIONS

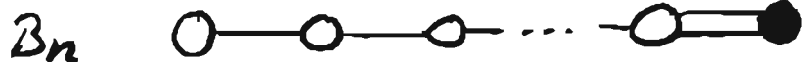
* SOME GROUP THEORY



$SU(n+1)$



G_2



$SO(2n+1)$



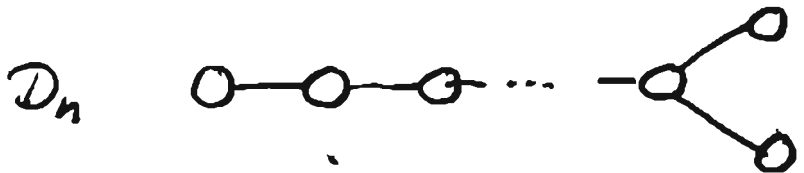
F_4



$SP(2n)$



E_6

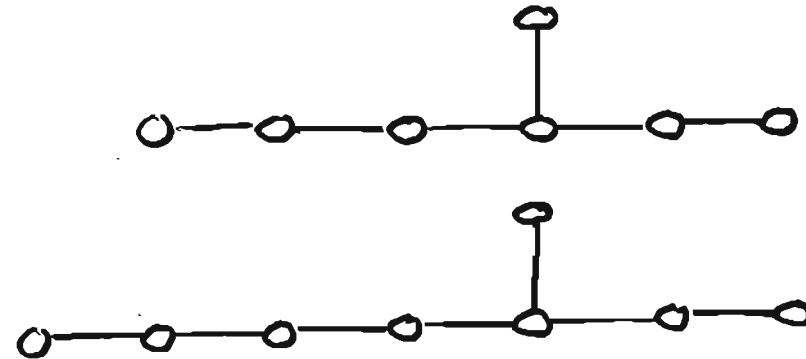


$SO(2n)$



E_7

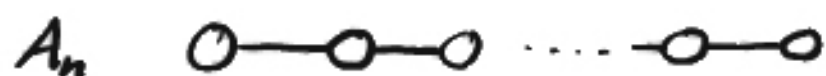
FOUR INFINITE SERIES



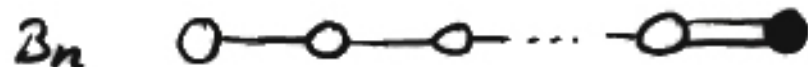
E_8

FIVE EXCEPTIONAL GROUPS

* SOME GROUP THEORY



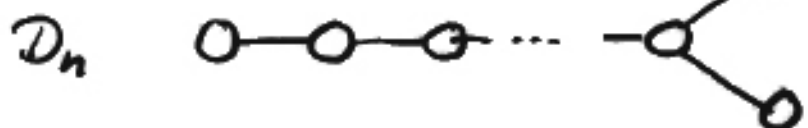
$SU(n+1)$



$SO(2n+1)$



$SP(2n)$



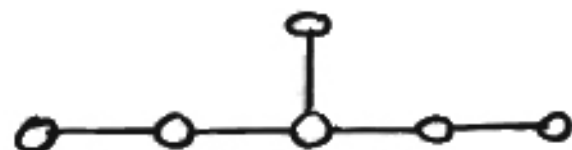
$SO(2n)$



G_2



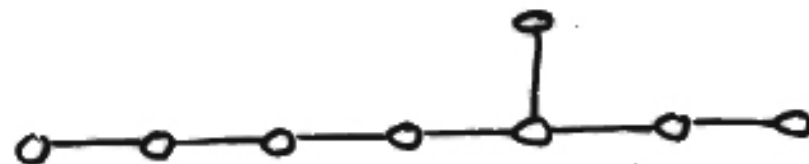
F_4



E_6



E_7



E_8

FOUR INFINITE SERIES

V-A STRUCTURE
OF WEAK INTERACTIONS
SELECTS

FIVE EXCEPTIONAL GROUPS

* SO E_8, ζ_2 (and $SO(8)$) ARE SOME

* $SU(5)$ AND $SO(10)$ ARE INCLUDED

* WHY NOT $SO(256)$?

* WHAT ABOUT THE NUMBER OF FAMILIES

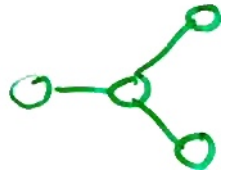
TOP-DOWN APPROACH

SUPERSTRINGS THEORY MIGHT PROVIDE
A QUANTUM THEORY OF GRAVITY

* CRITICAL DIMENSION $D=10$ (NOT $D=4$)

* SUSY REQUIRED FOR CONSISTENCY

(CLOSELY RELATED TO PROPERTIES OF



* $E_8 \times E_8$ (HETEROTIC) SUPERSTRINGS TH.

HIGHER DIMENSIONS (IN PARTICULAR $D=8N+2$)

CAN ACCOMMODATE PARITY VIOLATION

THE ROLE OF COMPACTIFICATION

$$D=10 \longrightarrow \boxed{M_4} \text{ (MINKOWSKI)} \times \boxed{C} \text{ (COMPACT)}$$



SIZE OF EXTRA
DIMENSION "SMALL"

PROPERTIES OF $D=4$ THEORY
DEPEND ON DETAILS OF
COMPACTIFICATION
(GEOMETRY, TOPOLOGY...)

THE ROLE OF COMPACTIFICATION

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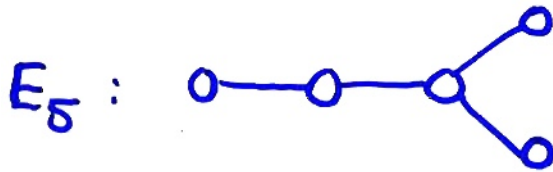
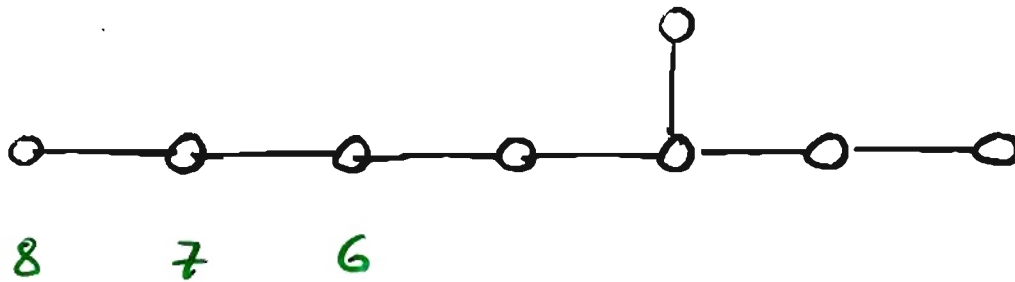
NUMBER OF STATES
DEPEND ON TOPOLOGY
OF COMPACT SPACE

→ EXPLANATION OF REPETITION
OF FAMILIES OF QUARKS + LEPTONS

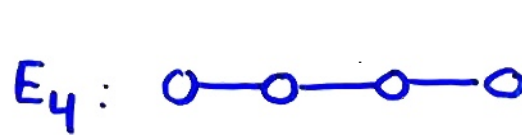
TOP-DOWN MEETS BOTTOM-UP : E_n SERIES

$$E_8 \longrightarrow E_7 \longrightarrow E_6 \longrightarrow E_5 \longrightarrow E_4 \longrightarrow \dots$$

$$F_4 \longrightarrow G_2$$



$SO(10)$



$SU(5)$



$SU(3) \times SU(2) \times U(1)$

A BINARY CODE FOR A FAMILY OF QUARKS + LEPTONS

16 DIMENSIONAL SPINOR REPR. OF $SO(10)$

$$(n_1, \dots, n_5) \quad n_i = 0, 1 \quad \sum_{i=1}^5 n_i = \text{even}$$

$(0, 0, 0, 0, 0)$ 1 $SU(3) \times SU(2) \times U(1)$ SINGLET $(\bar{\nu})$

$(1, 1, 0, 0, 0)$ 10 $\begin{bmatrix} (1, 0; \underline{100}) \\ (0, 1; \underline{100}) \end{bmatrix} \begin{pmatrix} u \\ d \end{pmatrix} \quad [(1, 1; 0, 0, 0)] \bar{e}$
 $(0, 0; \underline{110}) \bar{u}$

$(1, 1, 1, 1, 0)$ 5 $\begin{bmatrix} (1, 0; 111) \\ (0, 1; 111) \end{bmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad [(1, 1; 1, 1, 0)] \bar{d}$

OUR QUESTIONS:

* ORIGIN OF STRUCTURE
OF FAMILY :

GROUP THEORY
(6 OF SO(10))

* WHY 3 COPIES OF
SAME STRUCTURE :

PROPERTIES OF
COMPACT SPACE

* WHY $SU(3) \times SU(2) \times U(1)$:

E_8 BROKEN TO E_3
VIA $E_5 = SO(10)$

THINGS TO CLARIFY:

- * BREAKDOWN OF SYMMETRIES
- * INCOMPLETE MULTIPLETS
(e.g. IS $SU(10)$ REALIZED IN $D=4$?)
- * CONNECTION TO STRINGS THEORY
MODEL BUILDING (HETEROTIC BRANE WORLD)
- * GEOMETRIC OR TOPOLOGICAL ORIGIN
OF 3 FAMILIES
- * INTERPLAY OF GAUGE SYMMETRY AND SUSY

WE ARE EAGERLY WAITING FOR
NEW EXPERIMENTAL RESULTS:

- * COLLIDER EXPERIMENTS (LHC...)
- * DARK MATTER SEARCH
- * PROTON STABILITY
- * BARYOGENESIS (OR LEPTOGENESIS)
→ NEUTRINO PROPERTIES

(MORE DETAILS: HPN et al, arXiv:0806.3905)