Flavor's Delight

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Outline

- The flavor structure of the Standard Model
- Bottom-up (BU) model building
- The need for top-down (TD) considerations
- Traditional, Modular and Eclectic Flavor Groups
- "Local Flavor Unification" as the origin of hierarchies for masses and mixing angles
- Strict rules from top-down model building
- The clash between BU- and TD-constructions
- Open questions

Importance of localized structures in extra dimensions

(Work with Baur, Knapp-Perez, Liu, Ratz, Ramos-Sanchez, Trautner, Vaudrevange, 2019-24)

The flavor structure of SM

Most of the parameters of the SM concern the flavor sector

- Quark sector: 6 masses, 3 angles and one phase
- Lepton sector: 6 masses, 3 angles, one phase and additional parameters from Majorana neutrino masses

The pattern of parameters

- Quarks: hierarchical masses und small mixing angles
- Leptons: two large and one small mixing angle, hierarchical mass pattern and extremely small neutrino masses

The Flavor structure of quarks and leptons is very different!

Bottom-up approach

Discrete symmetries have been used extensively in the discussion of flavor structures

- Many fits from bottom-up perspective with discrete symmetries ($S_3, A_4, T', S_4, A_5, \Delta(27), \Delta(54)$ etc.)
- Various choices of representations, with emphasis on irreducible 3-dim. representations
- Choices of flavon fields (for traditional flavor) and modular weights (for modular flavor)
- Bottom-up model building leads to many reasonable fits for various choices of groups and representations

But there are many arbitrary choices that seem to require a top-down explanation of flavor

Discrete Flavor Symmetries from TD

There are various types of flavor symmetries. One of them is called traditional flavor symmetry

- they are linearly realised
- need flavon fields for symmetry breakdown

A second type is modular symmetries

- motivated by string theory dualities (Lauer, Mas, Nilles, 1989)
- applied recently to lepton sector (Feruglio, 2017)
- nonlinearly realised (no flavon fields needed)
- Yukawa couplings are modular forms

Combine with traditional flavor symmetries to the so-called "eclectic flavor group" (Nilles, Ramos-Sanchez, Vaudrevange, 2020)

String Geometry of extra dimensions

Strings are extended objects and this reflects itself in special aspects of geometry (incl. winding modes).
We have:

- normal symmetries of extra dimensions as observed in quantum field theory – traditional flavor symmetries.
- String duality transformations lead to modular or symplectic flavor symmetries
- They combine to a unified picture within the concept of eclectic flavor symmetries

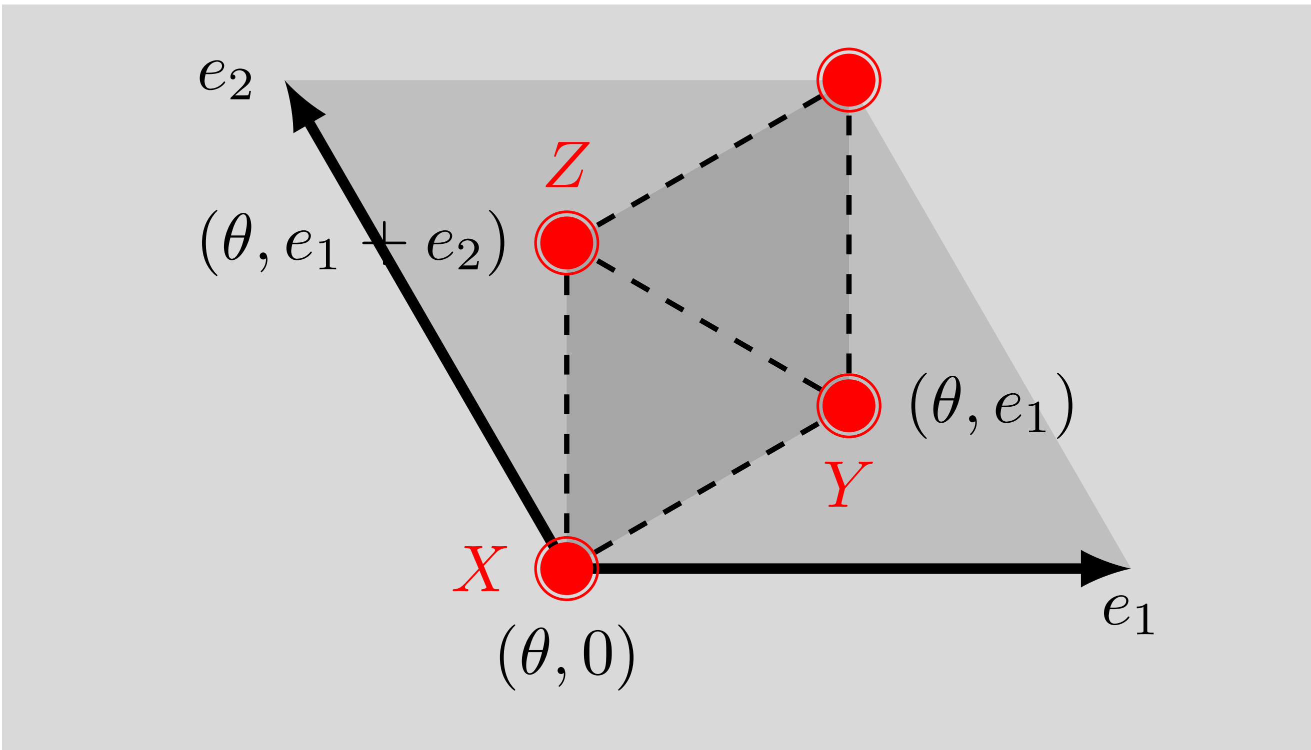
In the following we illustrate with a simple example

- twisted 2D-torus with localized matter fields
- relevant for compactifications with "elliptic fibrations"

Traditional Flavor Symmetries

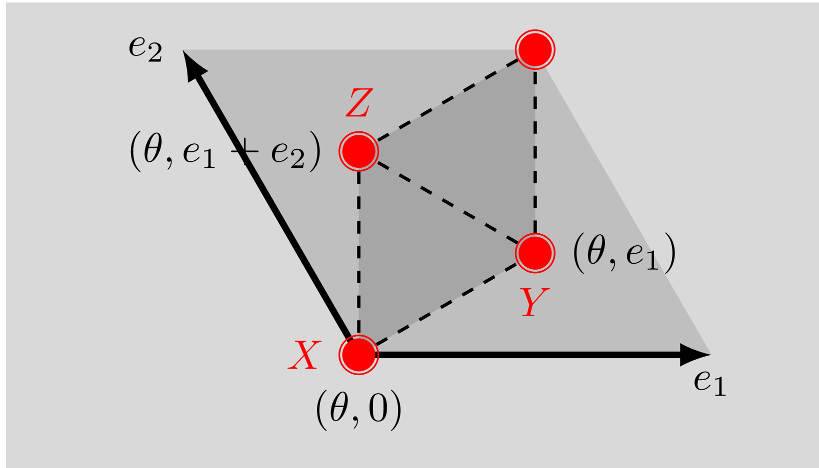
In string theory discrete symmetries can arise form geometry and string selection rules.

As an example we consider the orbifold T_2/Z_3



Discrete symmetry $\Delta(54)$

- untwisted and twisted fields
- S_3 symmetry from interchange of fixed points
- $Z_3 \times Z_3$ symmetry from string theory selection rules



- $\Delta(54)$ as multiplicative closure of S_3 and $Z_3 \times Z_3$
- $\Delta(54)$ – a non-abelian subgroup of $SU(3)$
- e.g. flavor symmetry for three families of quarks (as triplets of $\Delta(54)$) (Kobayashi, Nilles, Ploger, Raby, Ratz, 2006)

String dualities

Consider a particle on a circle with radius R

- discrete spectrum of momentum modes (KK-modes)
- density of spectrum is governed by m/R (m integer)
- heavy modes decouple for $R \to 0$

Now consider a string

- KK modes as before m/R
- Strings can wind around circle
- spectrum of winding modes governed by nR
- massless modes for $R \to 0$

T-duality

This interplay of momentum and winding modes is the origin of T-duality where one simultaneously interchanges

- momentum $\rightarrow$ winding
- $R \rightarrow 1/R$

This transformation maps a theory to its T-dual theory.

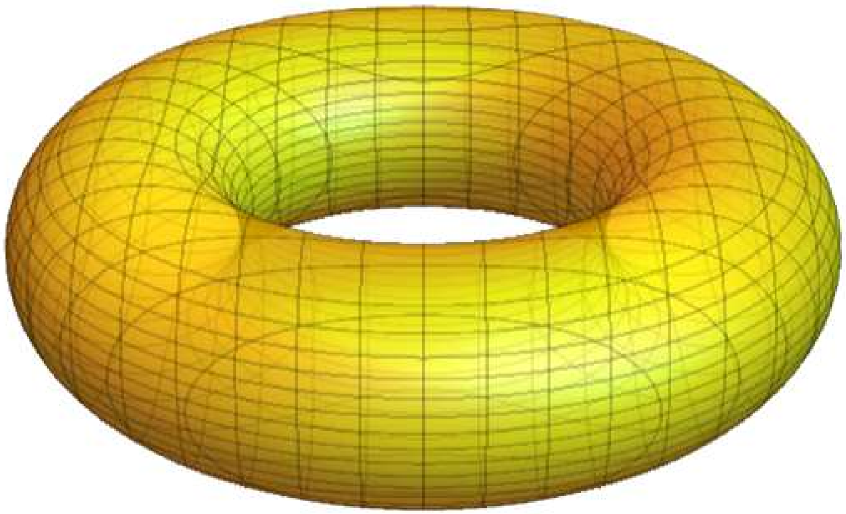
- self-dual point is $R^2 = \alpha' = 1/M_{\rm string}^2$

If the string scale $M_{\rm string}$ is large, the low energy effective theory describes the momentum states and the winding states are heavy.

Can T-duality play a relevant role for flavor symmetry?

Torus compactification

Strings can wind around several cycles



Complex modulus M (in complex upper half plane)

Modular Transformations

Modular transformations (dualities) exchange windings and momenta and act nontrivially on the moduli of the torus.

In $D = 2$ these transformations are connected to the group $SL(2, Z)$ acting on Kähler and complex structure moduli.

The group $SL(2, Z)$ is generated by two elements

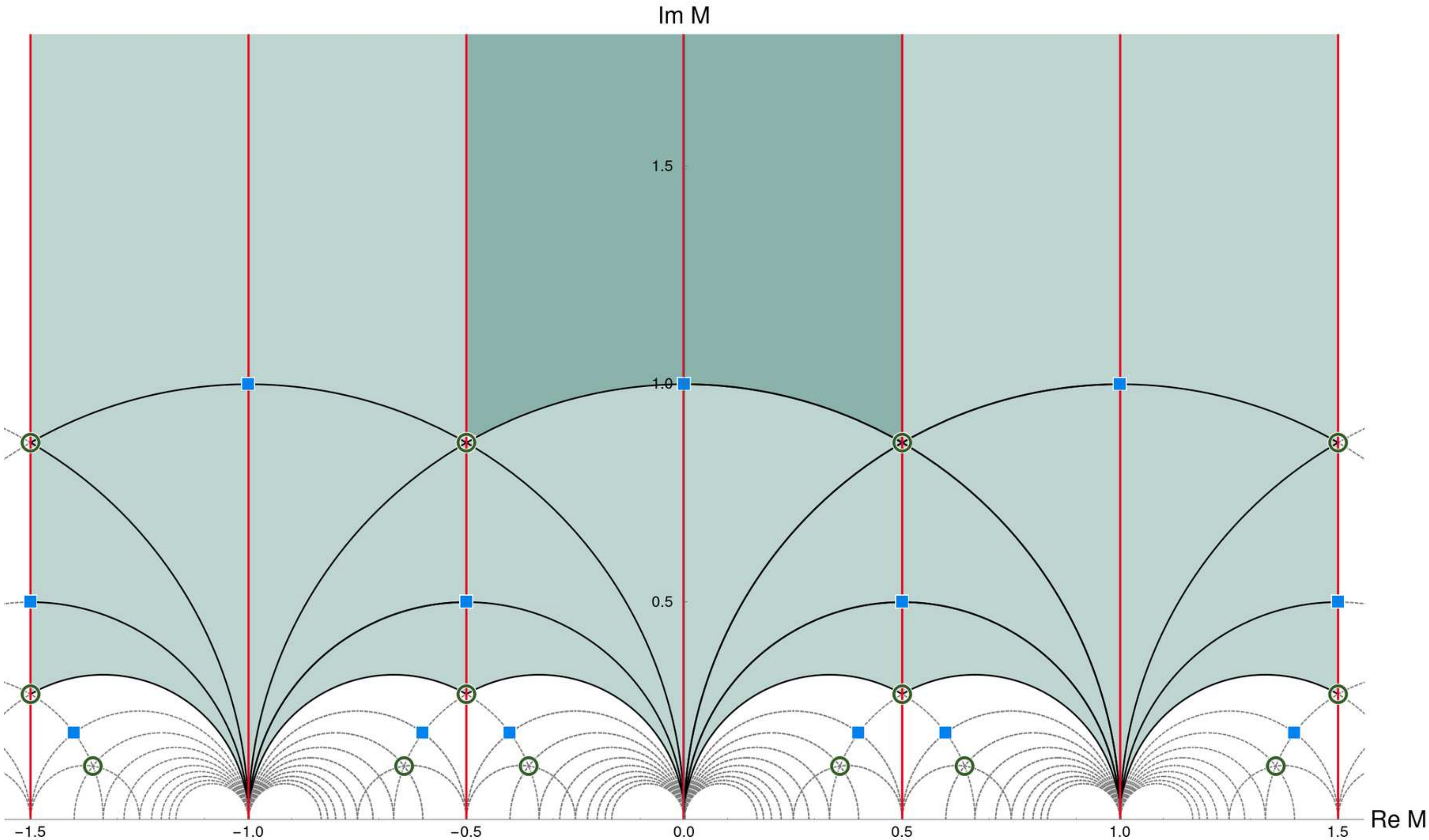
$$\mathrm{S}\ ,\ \mathrm{T}: \quad \text{with}\ \ \mathrm{S}^4 = 1\ \ \text{and}\ \ \mathrm{S}^2 = (\mathrm{ST})^3$$

A modulus M transforms as

$$\mathrm{S}: \quad M \to -\frac{1}{M} \quad \text{and} \quad \mathrm{T}: M \to M + 1$$

Further transformations might include $M \to -\overline{M}$ and mirror symmetry between Kähler and complex structure moduli.

Fundamental Domain



Three fixed points at M$= i$, $\omega = \exp(2\pi i/3)$ and $i\infty$

Modular Forms

String dualities give important constraints on the action of the theory via the modular group $SL(2,Z)$:

$$\gamma : \quad M \rightarrow \frac{aM+b}{cM+d}$$

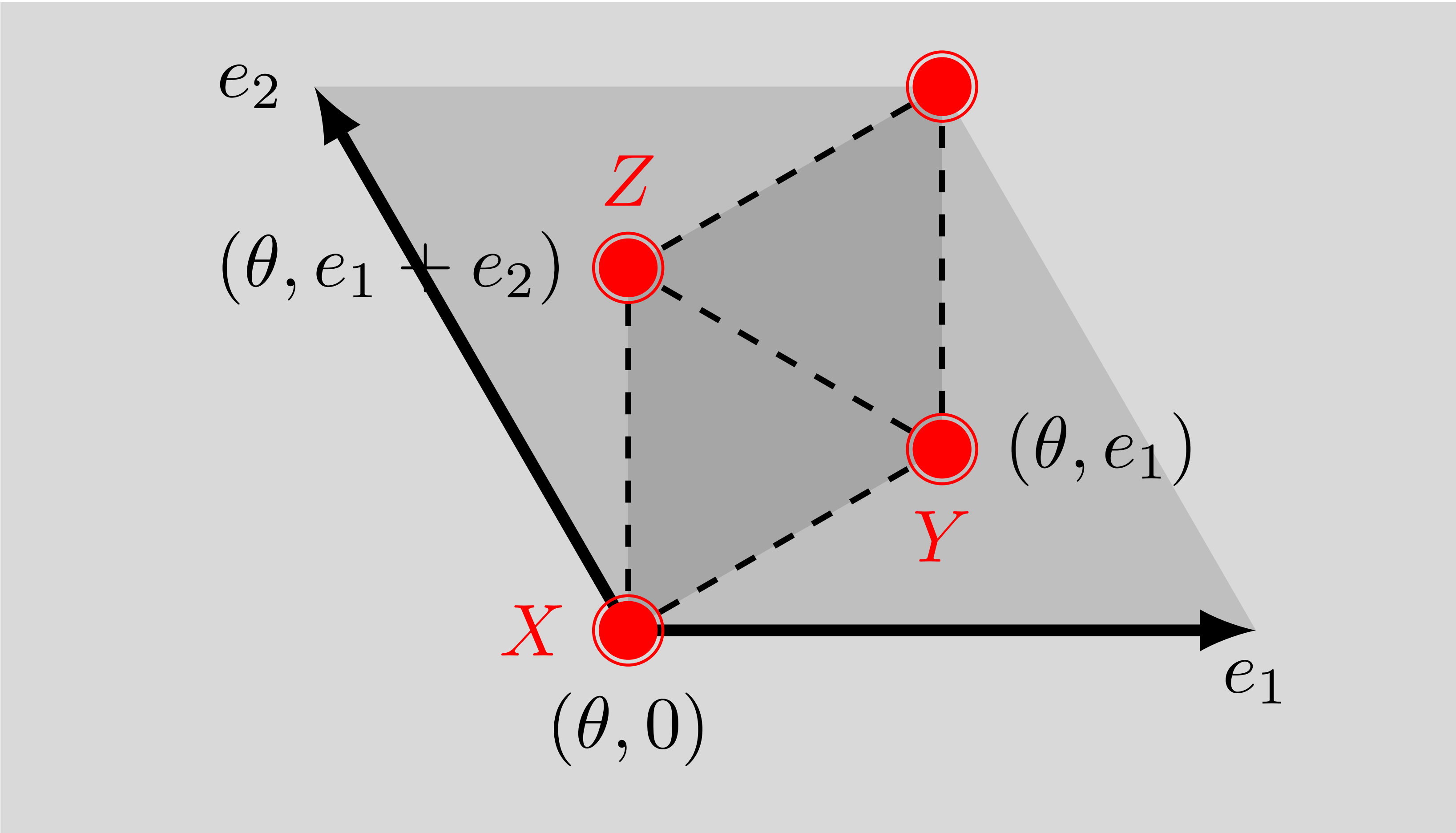
with $ad - bc = 1$ and integer a, b, c, d.

Matter fields transform as representations $\rho(\gamma)$ and modular functions of weight k

$$\gamma : \quad \phi \rightarrow (cM+d)^k \rho(\gamma) \phi \ .$$

Yukawa-couplings transform as modular functions as well.
$G = K + \log |W|^2$ must be invariant under T-duality

Towards Modular Flavor Symmetry



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Modular flavor symmetry

On the T_2/Z_3 orbifold some of the moduli are frozen,

- e_1 and e_2 have the same length, angle is 120 degrees

Modular transformations form a subgroup of $SL(2,Z)$

- $\Gamma(3) = SL(2,3Z)$ as a mod(3) subgroup of $SL(2,Z)$
- discrete modular flavor group $\Gamma'_3 = SL(2,Z)/\Gamma(3)$
- the discrete modular group is $\Gamma'_3 = T' \sim SL(2,3)$
  the double cover of $\Gamma_3 \sim A_4$
  (with representations $1, 1', 1'', 2, 2', 2'', 3$).
- twisted fields are in $1 + 2'$ rep. of T', (3 of $\Delta(54)$)

(Lauer, Mas, Nilles, 1989)

- the CP transformation $M \to -\overline{M}$ completes the picture.

Eclectic Flavor Groups

We have thus two types of flavor groups

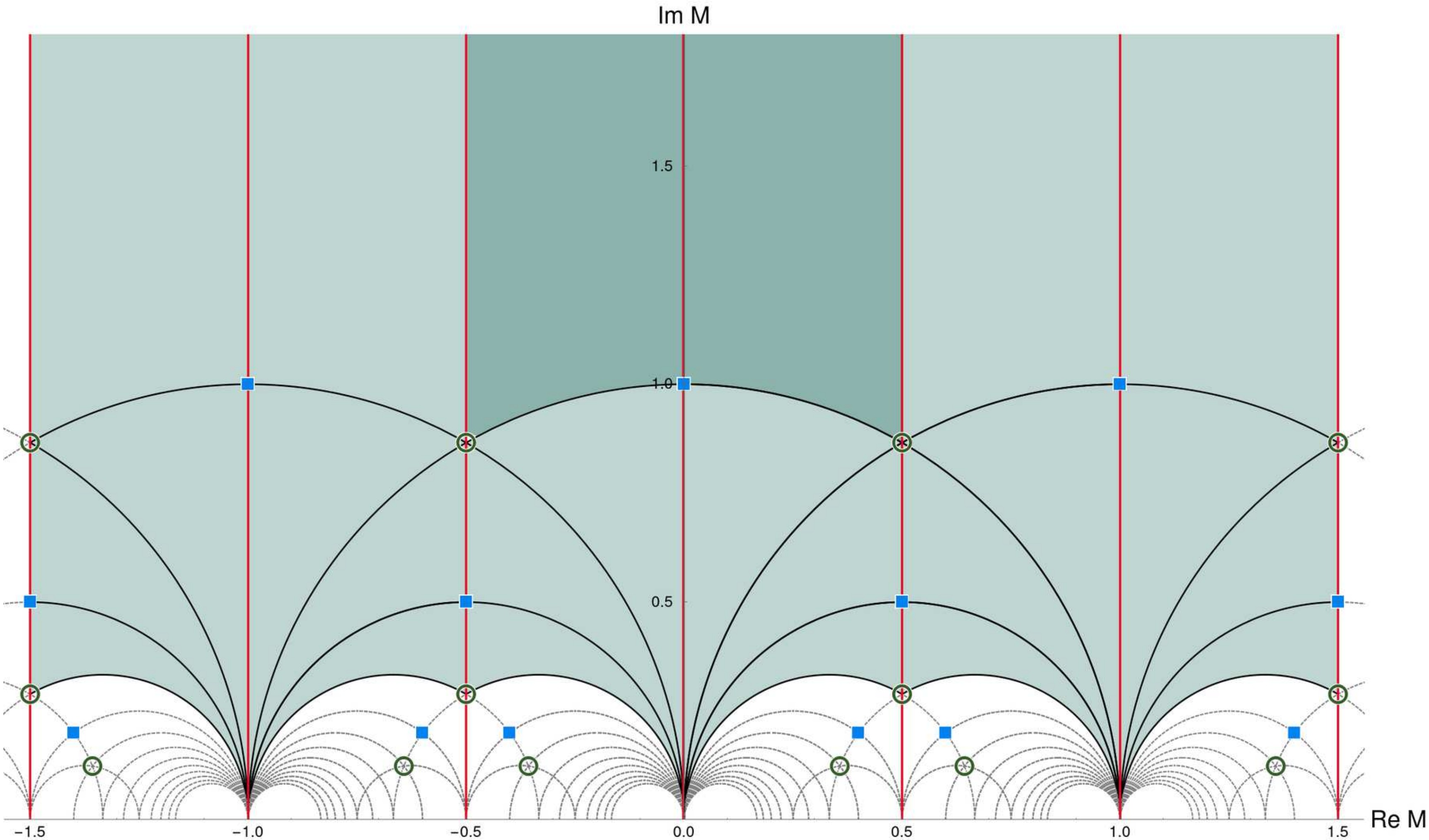
- the traditional flavor group that is universal in moduli space (here $\Delta(54)$)
- the modular flavor group that transforms the moduli nontrivially (here T')

The eclectic flavor group is defined as the multiplicative closure of these groups. Here we obtain for T_2/Z_3

- $\Omega(1) = SG[648, 533]$ from $\Delta(54)$ and $T' = SL(2,3)$
- $SG[1296, 2891]$ from $\Delta(54)$ and $GL(2,3)$ including CP

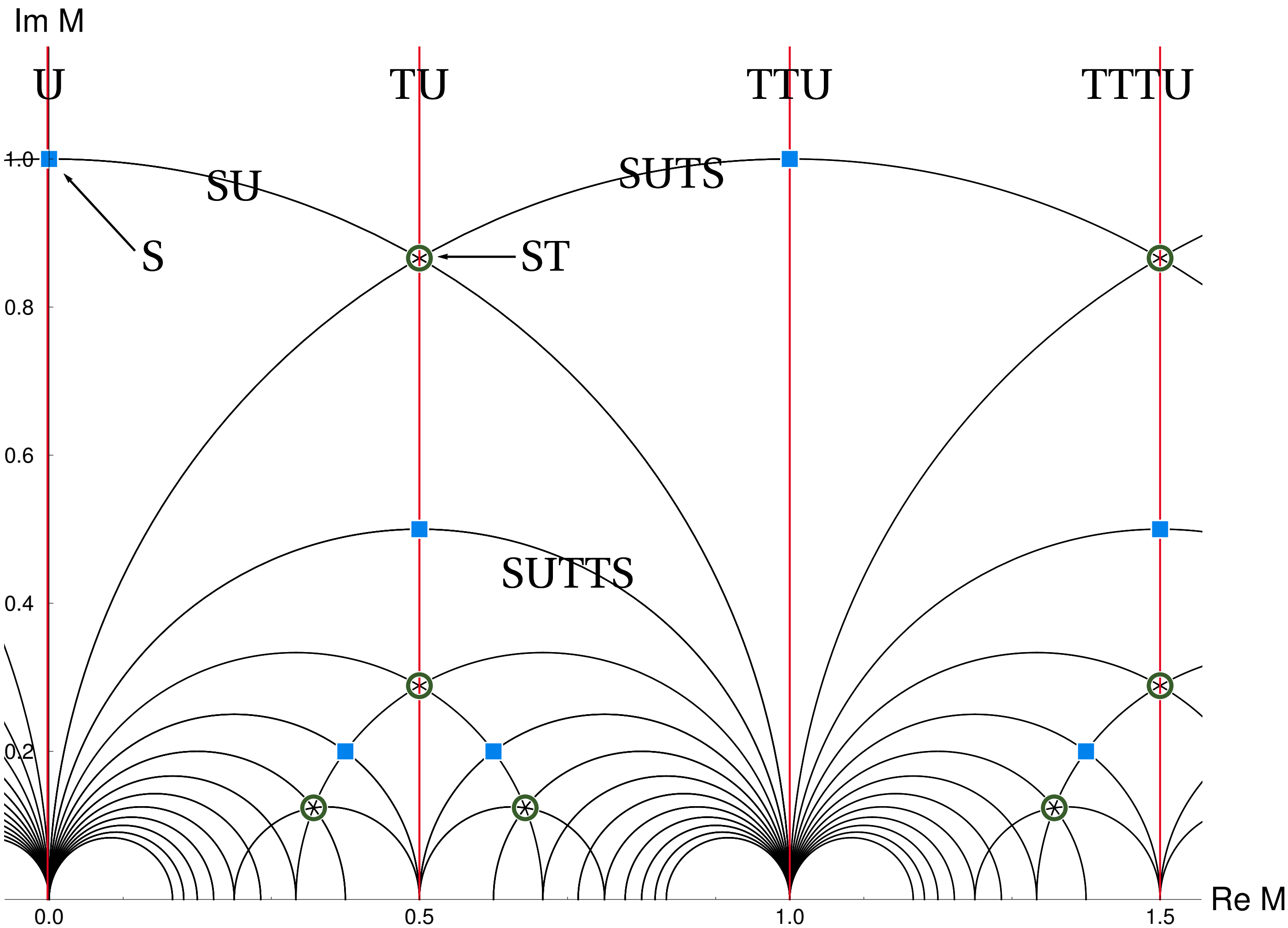
The eclectic group is the largest possible flavor group for the given system, but it is not necessarily linearly realized.

Local Flavor Unification



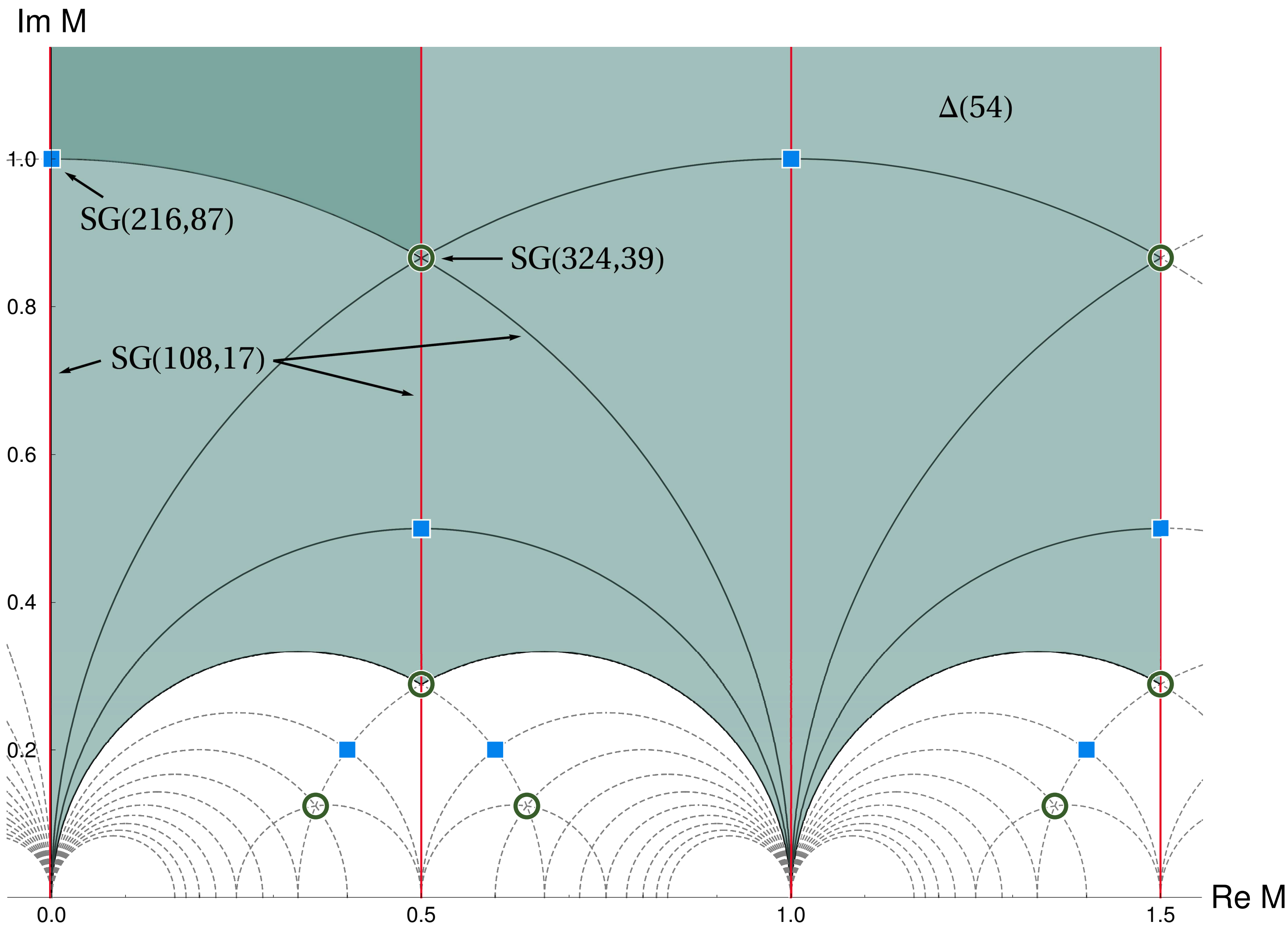
Moduli space of $\Gamma(3)$

Fixed lines and points



$$\mathrm{S} : M \to -\frac{1}{M}, \quad \mathrm{T} : M \to M+1 \quad \text{and} \quad \mathrm{U} : M \to -\overline{M}$$

Moduli space of flavour groups



"Local Flavor Unification"

Unification of Flavor and CP

Summary of predictions of the string picture:

- traditional flavor symmetries (universal in moduli space)
- modular flavor symmetries and CP are non-universal in moduli space

They unify in the eclectic picture of flavor symmetry.
You cannot just have one without the other.

The non-universality in moduli space leads to

- local flavor unification at specific points in moduli space
- hierarchical structures of masses, mixing angles and phases in vicinity of fixed points or lines
- potentially different pictures for quarks and leptons

Rules from the top-down approach

Top-down constructions are very restrictive with high predictive power.

- so far we have only few choices of discrete modular flavor groups $S_3, T', 2D_3$ (no A_4, A_5, S_4 etc.)
- the representations are fixed (e.g. no triplet of T')
- the modular weights are fixed (not free prameters) and correlated with the representations of the modular group
- you cannot discuss traditional and modular flavor symmetry in isolation, they are intrinsically linked and restrict Kähler- and superpotential

The top-down construction reflects the symmetries of the underlying string and provides a valid UV-completion.

Bottom-up constructions

These rules are violated in most BU-constructions.

- They mostly ignore the existence of either traditional or modular flavor symmetry
- Consider various modular groups not present in TD approach (like A_4, S_4, A_5 etc.)
- Choose representations not found in TD-constructions (e.g. triplet of T')
- Most importantly they choose modular weights at will and this provides many new parameters
- it is like adding a Frogatt-Nielsen mechanism by hand

As a result we still have to wait for a connection between TD- and BU-approaches

The "Representation Dilemma"

In the bottom-up approach for the modular group $\Gamma_3 \sim A_4$

- one assigns e.g.
  triplet 3 and (non-)trivial singlet representations $1, 1', 1''$
  for left-handed and right-handed leptons respectively
- one assumes freedom in the choice of modular weights

Compared to the top-down approach based on the group T' (the double cover of Γ_3)

- twisted fields transform as $2' + 1$ (not as triplet)
- in addition only trivial singlets in the low energy sector
- modular weights of light fields restricted to $-2/3, -5/3$

It might be that many of the bottom-up models fail to have a consistent UV-completion.

Messages

The top-down approach to flavor symmetries leads to a

- unification of traditional (discrete) flavor, CP and modular symmetries within an eclectic flavor scheme
- modular flavor symmetry is a prediction of string theory
- traditional flavor symmetry is universal in moduli space
- there are non-universal enhancements (including CP at some places (broken in generic moduli space))
- CP is a natural consequence of the symmetries of the underlying string theory
- the potential flavor groups are large and non-universal
- spontaneous breakdown as motion in moduli space

Outlook

This opens up a new arena for flavor model building and connections to bottom-up constructions:

- need more explicit string constructions
- more BU-constructions respecting the "TD-rules"
- the concept of local flavor symmetries allows different flavor groups for different sectors of the theory, as for example quarks and leptons
- but it is not only the groups but also the representations of matter fields that are relevant. Not all of the possible representations appear in the massless sector.

There is still a huge gap between "top-down and bottom-up" constructions

Open Questions

So far: the winner is $\Delta(54) \times T'$

- numerous bottom-up models with these groups
- successful realistic string models from Z_3 orbifolds
- Z_2, Z_4 and Z_6 as alternatives at this level (work in progress)

"Local Flavor Unification" predicts that many of the successful fits are in the vicinity of fixed points and lines

(Feruglio, 2022-23; Petcov, Tanimoto, 2022; Abe, Higaki, Kawamura, Kobayashi, 2023)

- moduli stabilization favours boundary of moduli space, but AdS-minima (Cvetic, Font, Ibanez, Lüst, Quevedo, 1991)
- uplift moves them slightly away from the boundary

(Knapp-Perez, Liu, Nilles, Ramos-Sanchez, Ratz, 2023)

Summary

String theory provides the necessary ingredients for flavor:

- traditional flavor group
- discrete modular flavor group
- a natural candidate for CP
- the concept of local flavour unification

The eclectic flavor group provides the basis:

- this includes a non-universality of flavor symmetry in moduli space
- allows a different flavor structure for quarks and leptons

Moduli fixing