The Modular Flavor of the Heterotic String

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Outline

- The flavor structure of the Standard Model
- Traditional flavor symmetries
- Modular flavor symmetries
- Eclectic Flavor Group
- Local Flavor Unification
- Origin of hierarchies for masses and mixing angles
- Lessons from top-down model building
- Global fit for lepton masses and mixing angles

Importance of localized structures in extra dimensions

(Work with Baur, Knapp-Perez, Liu, Ratz, Ramos-Sanchez, Trautner, Vaudrevange, 2019-23)

Heterotic String

Closed string with $N = 1$ supersymmetry in $d = 10$

- right movers live in 10 space-time dimensions
- left movers in 26 dimensions with 16 dimensions compactified on $E_8 \times E_8$ (or $SO(32)$) root lattice
- connected to Heterotic M-theory in $d = 11$

Compactification to $d = 4$ yields

- $(6, 22)$ dimensional Narain-Lattice
- breakdown of supersymmetry to $N = 1$ in $d = 4$ is connected to breakdown of E_8 gauge group
- yields observable and hidden sector in $E_8 \times E_8-$case

Heterotic Model Building

Makes natural contact to grand unified models

- natural appearance of $SO(10)$
- 16-dim spinor rep. for family of quarks and leptons
- the concept of "Local Grand Unification"

Construction of MSSM-like models

- compactification on CY, orbifolds or free fermionic construction yield many viable models
- rich set of discrete flavor symmetries as outer automorphism of the twisted Narain lattice
- appearance of duality symmeties and the concept of "Local Flavor Unification"

The flavor structure of SM

Most of the parameters of the SM concern the flavor sector

- Quark sector: 6 masses, 3 angles and one phase
- Lepton sector: 6 masses, 3 angles, one phase and additional parameters from Majorana neutrino masses

The pattern of parameters

- Quarks: hierarchical masses und small mixing angles
- Leptons: two large and one small mixing angle, hierarchical mass pattern and extremely small neutrino masses

The Flavor structure of quarks and leptons is very different!

Bottom-up approach

Discrete symmetries have been used extensively in the discussion of flavor structures

- Many fits from bottom-up perspective with discrete symmetries ($S_3, A_4, S_4, A_5, \Delta(27), \Delta(54)$ etc.)
- Flavor symmetries seem to require different models for quark and lepton sector (small mixing angles for quarks versus large mixing in lepton sector)
- Flavor symmetries are spontaneously broken. This requires the introduction of so-called flavon fields and additional parameters
- bottom-up model building leads to many reasonable fits for various choices of groups and representations

But we are still missing a top-down explanation of flavor

Traditional vs Modular Symmetries

So far the flavor symmetries had specific properties and we refer to them as traditional flavor symmetries

- they are linearly realised
- need flavon fields for symmetry breakdown

Another type of flavor symmetries are modular symmetries

- motivated by string theory dualities (Lauer, Mas, Nilles, 1989)
- applied recently to lepton sector (Feruglio, 2017)
- nonlinearly realised (no flavon fields needed)
- Yukawa couplings are modular forms

Combine with traditional flavor symmetries to the so-called "eclectic flavor group" (Nilles, Ramos-Sanchez, Vaudrevange, 2020)

String Geometry of extra dimensions

Strings are extended objects and this reflects itself in special aspects of geometry (incl. winding modes).
We have:

- normal symmetries of extra dimensions as observed in quantum field theory – traditional flavor symmetries.
- String duality transformations lead to modular or symplectic flavor symmetries
- They combine to a unified picture within the concept of eclectic flavor symmetries

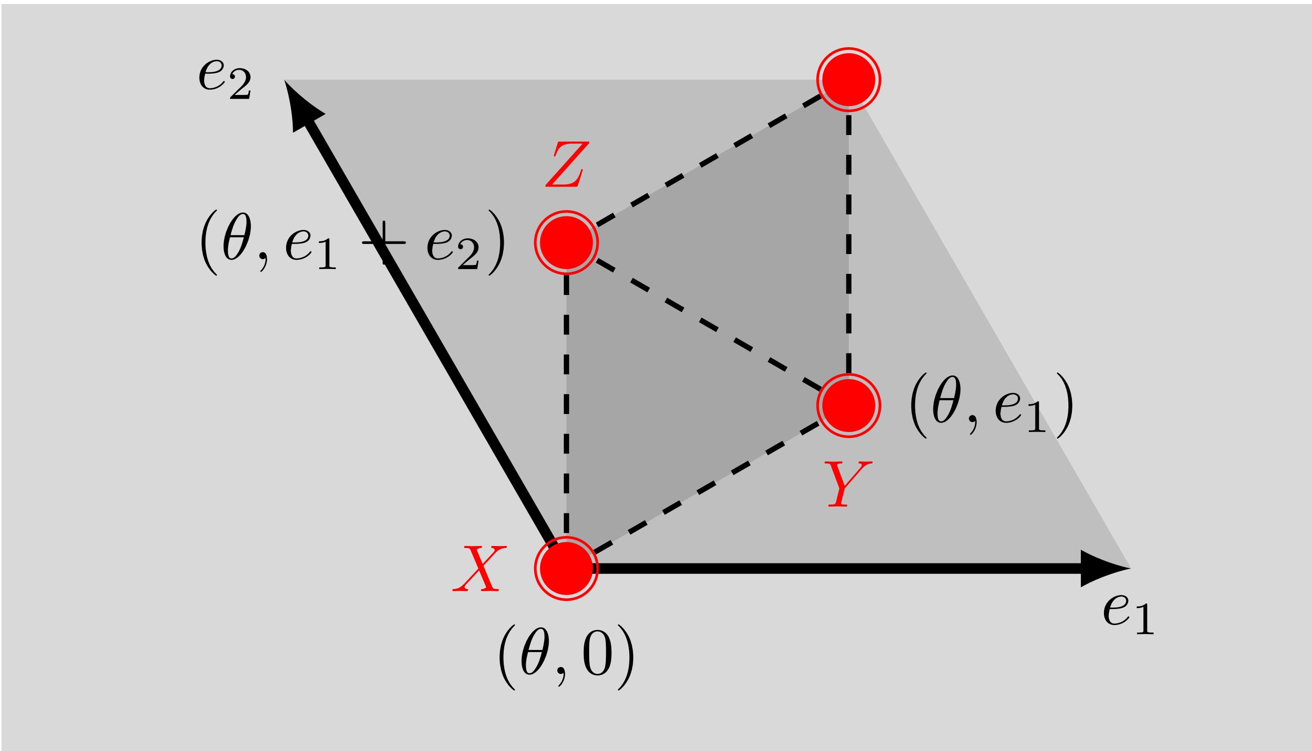
In the following we illustrate with a simple example

- twisted 2D-torus with localized matter fields
- relevant for compactifications with elliptic fibrations

Traditional Flavor Symmetries

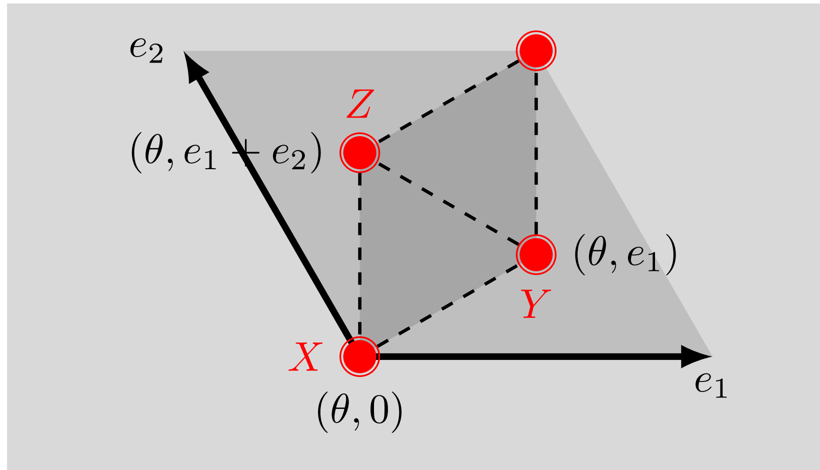
In string theory discrete symmetries can arise form geometry and string selection rules.

As an example we consider the orbifold T_2/Z_3



Discrete symmetry $\Delta(54)$

- untwisted and twisted fields
- S_3 symmetry from interchange of fixed points
- $Z_3 \times Z_3$ symmetry from string theory selection rules



- $\Delta(54)$ as multiplicative closure of S_3 and $Z_3 \times Z_3$
- $\Delta(54)$ – a non-abelian subgroup of $SU(3)_{\text{flavor}}$
- e.g. flavor symmetry for three families of quarks (as triplets of $\Delta(54)$)

String dualities

Consider a particle on a circle with radius R

- discrete spectrum of momentum modes (KK-modes)
- density of spectrum is governed by m/R (m integer)
- heavy modes decouple for $R \to 0$

Now consider a string

- KK modes as before m/R
- Strings can wind around circle
- spectrum of winding modes governed by nR
- massless modes for $R \to 0$

T-duality

This interplay of momentum and winding modes is the origin of T-duality where one simultaneously interchanges

- momentum $\rightarrow$ winding
- $R \rightarrow 1/R$

This transformation maps a theory to its T-dual theory.

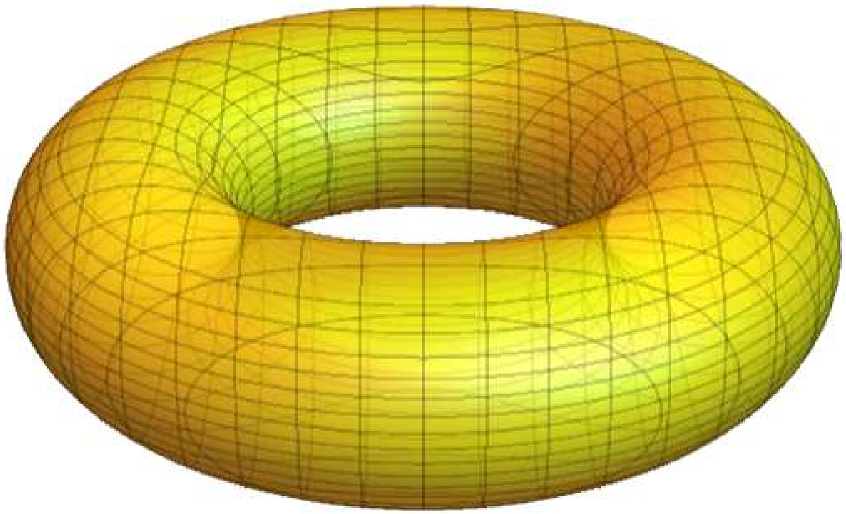
- self-dual point is $R^2 = \alpha' = 1/M^2_{\text{string}}$

If the string scale M_{string} is large, the low energy effective theory describes the momentum states and the winding states are heavy.

Can T-duality play a relevant role for flavor symmetry?

Torus compactification

Strings can wind around several cycles



Complex modulus M (in complex upper half plane)

Modular Transformations

Modular transformations (dualities) exchange windings and momenta and act nontrivially on the moduli of the torus.

In $D = 2$ these transformations are connected to the group $SL(2, Z)$ acting on Kähler and complex structure moduli.

The group $SL(2, Z)$ is generated by two elements

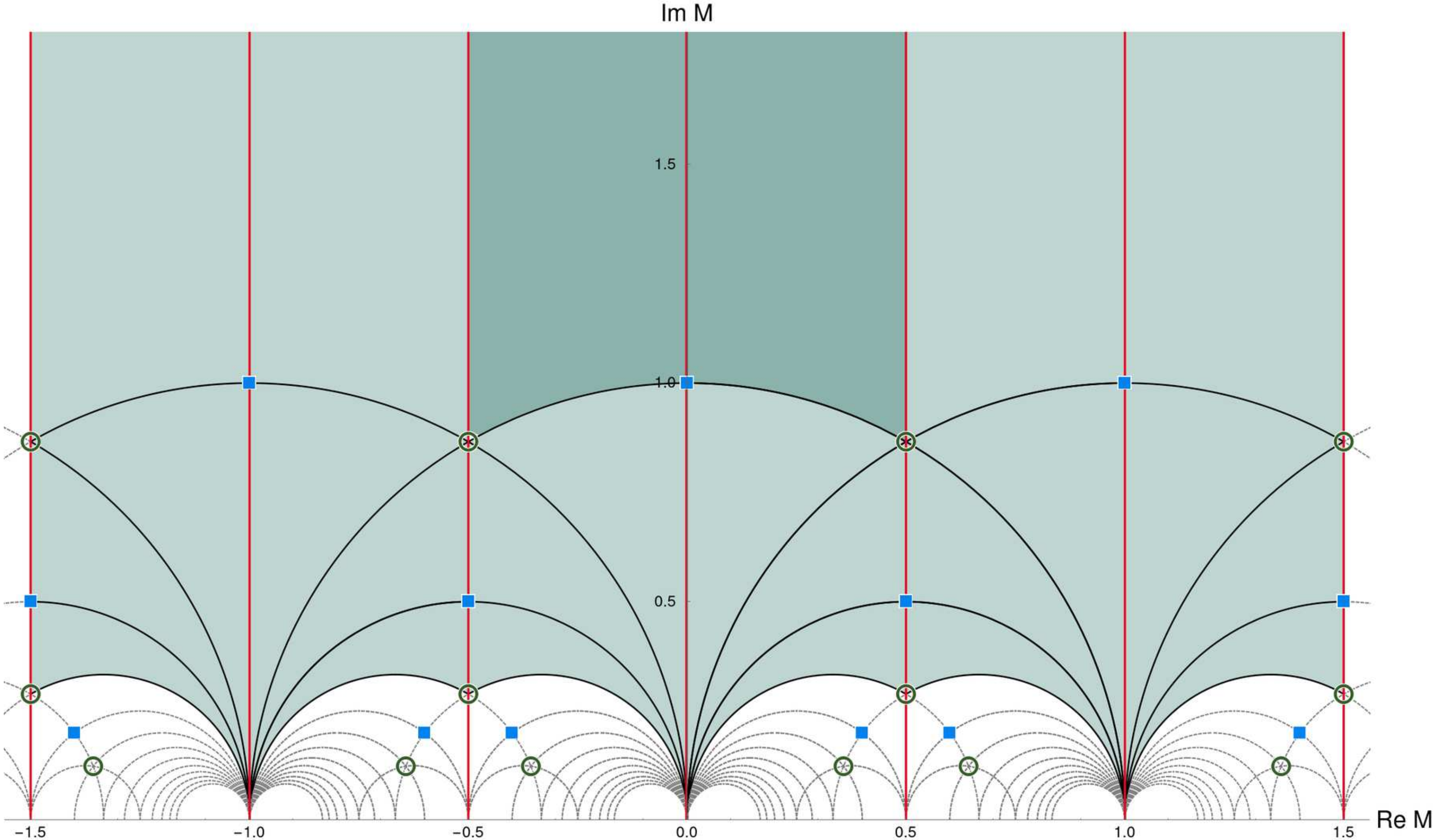
$$\mathrm{S}\ ,\ \mathrm{T}: \quad \text{with } \ \mathrm{S}^4 = 1 \ \text{ and } \ \mathrm{S}^2 = (\mathrm{ST})^3$$

A modulus M transforms as

$$\mathrm{S}: \quad M \to -\frac{1}{M} \quad \text{and} \quad \mathrm{T}: M \to M + 1$$

Further transformations might include $M \to -\overline{M}$ and mirror symmetry between Kähler and complex structure moduli.

Fundamental Domain



Three fixed points at M$= i$, $\omega = \exp(2\pi i/3)$ and $i\infty$

Modular Forms

String dualities give important constraints on the action of the theory via the modular group $SL(2,Z)$:

$$\gamma: \quad M \to \frac{aM+b}{cM+d}$$

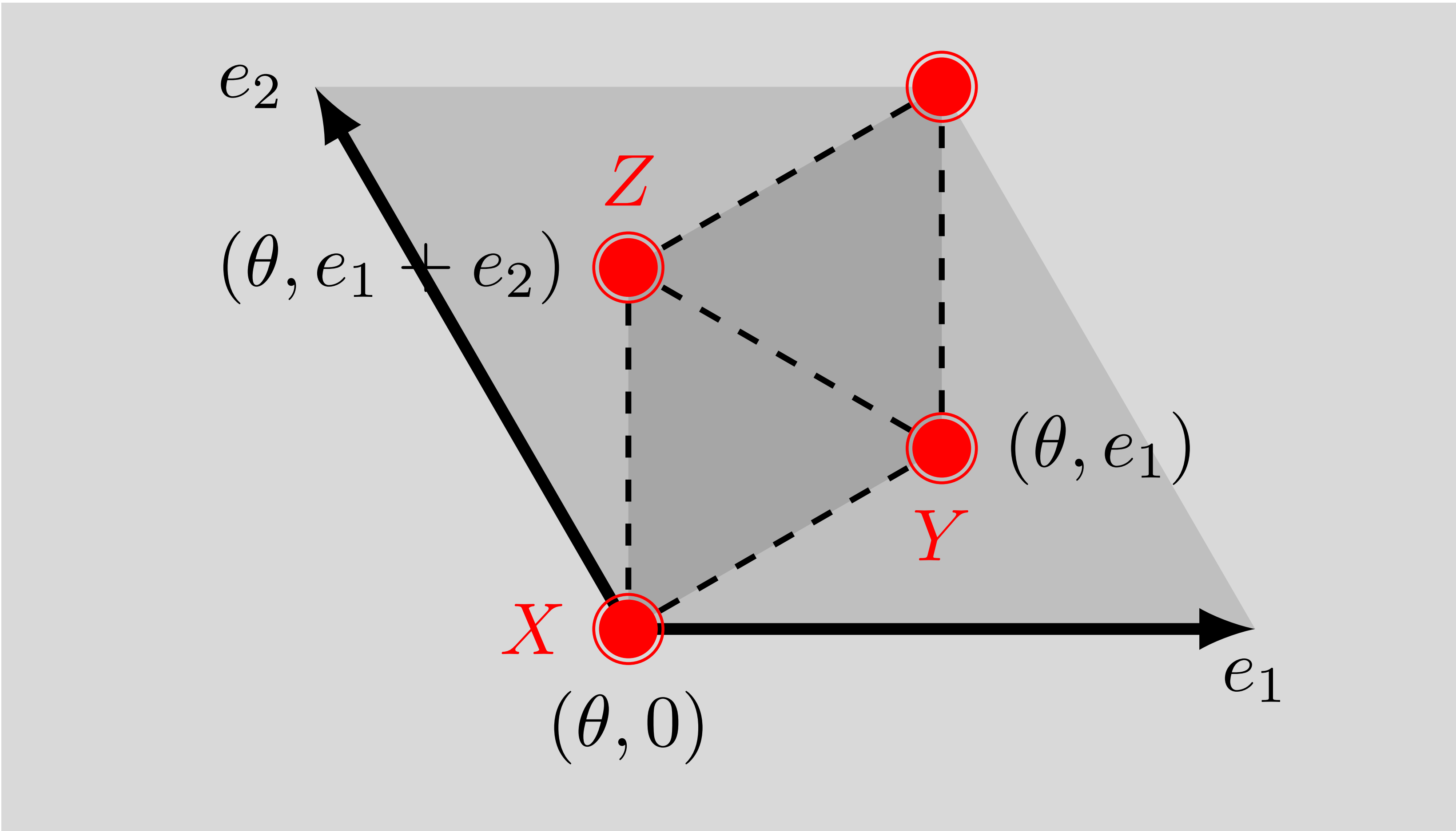
with $ad-bc=1$ and integer a,b,c,d.

Matter fields transform as representations $\rho(\gamma)$ and modular functions of weight k

$$\gamma: \quad \phi \to (cM+d)^k \rho(\gamma)\phi \ .$$

Yukawa-couplings transform as modular functions as well. $G = K + \log|W|^2$ must be invariant under T-duality

Orbifold T_2/Z_3



Yukawa Couplings

Yukawa couplings are modular forms that depend nontrivially on the modulus M.

Consider, for example,

- the twisted fields of the T_2/Z_3 orbifold.
- located at the fixed points X, Y and Z.

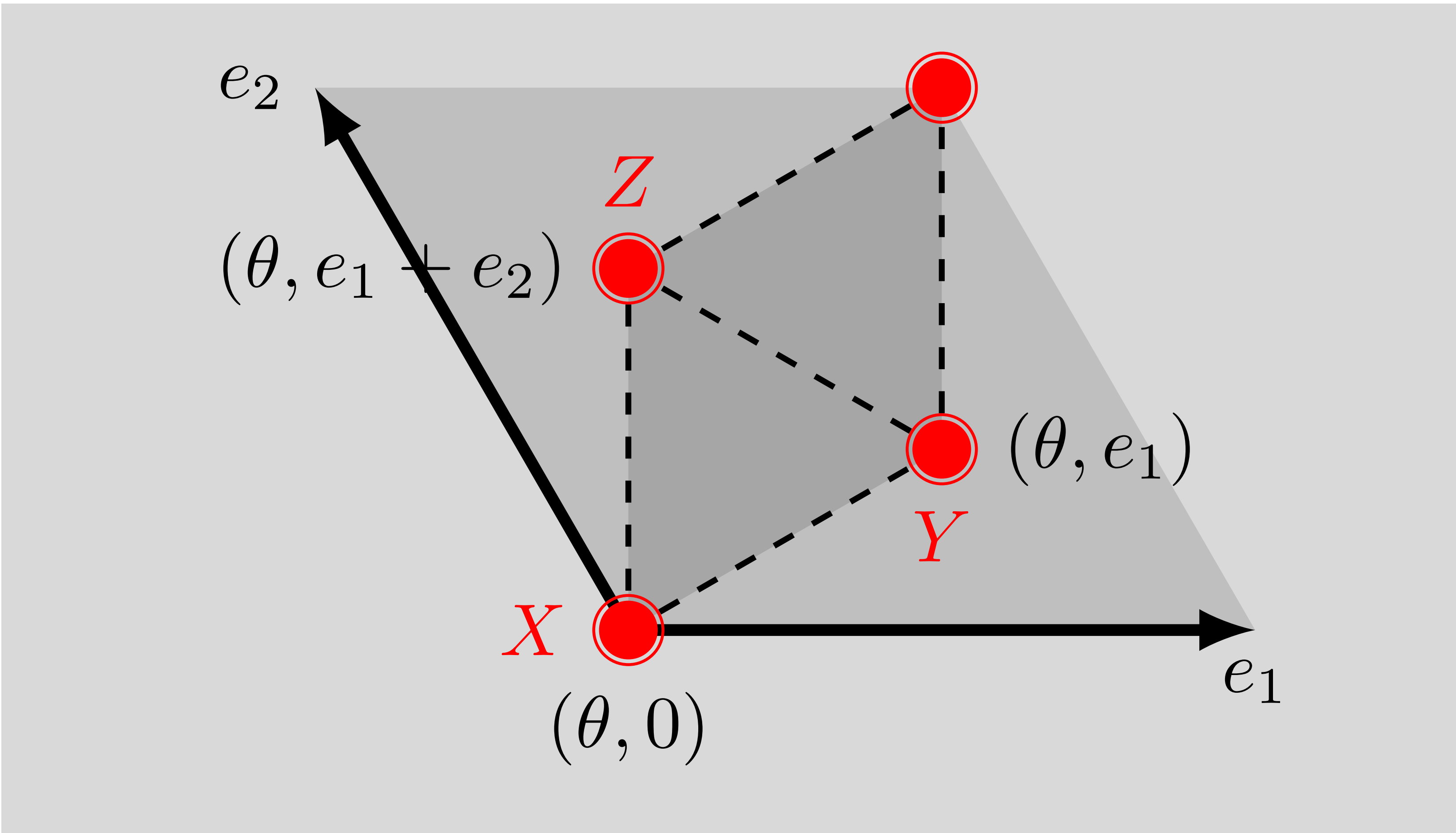
Allowed couplings are:

$$f(M)(X^3 + Y^3 + Z^3) + g(M)XYZ$$

$f(M)$ and $g(M)$ are modular functions of weight k

For large M the coupling $g(M)$ is exponentially suppressed, while $f(M)$ remains finite.

Towards Modular Flavor Symmetry



Modular flavor symmetry

On the T_2/Z_3 orbifold some of the moduli are frozen,

- lattice vectors e_1 and e_2 have the same length
- angle is 120 degrees

Modular transformations form a subgroup of $SL(2, Z)$

- $\Gamma(3) = SL(2, 3Z)$ as a mod(3) subgroup of $SL(2, Z)$
- discrete modular flavor group $\Gamma'_3 = SL(2, Z)/\Gamma(3)$
- the discrete modular group is $\Gamma'_3 = T' \sim SL(2, 3)$ (which acts nontrivially on twisted fields); the double cover of $\Gamma_3 \sim A_4$ (which acts only on the modulus).
- the CP transformation $M \to -\overline{M}$ completes the picture.

Full discrete modular group is $GL(2, 3)$.

Eclectic Flavor Groups

We have thus two types of flavor groups

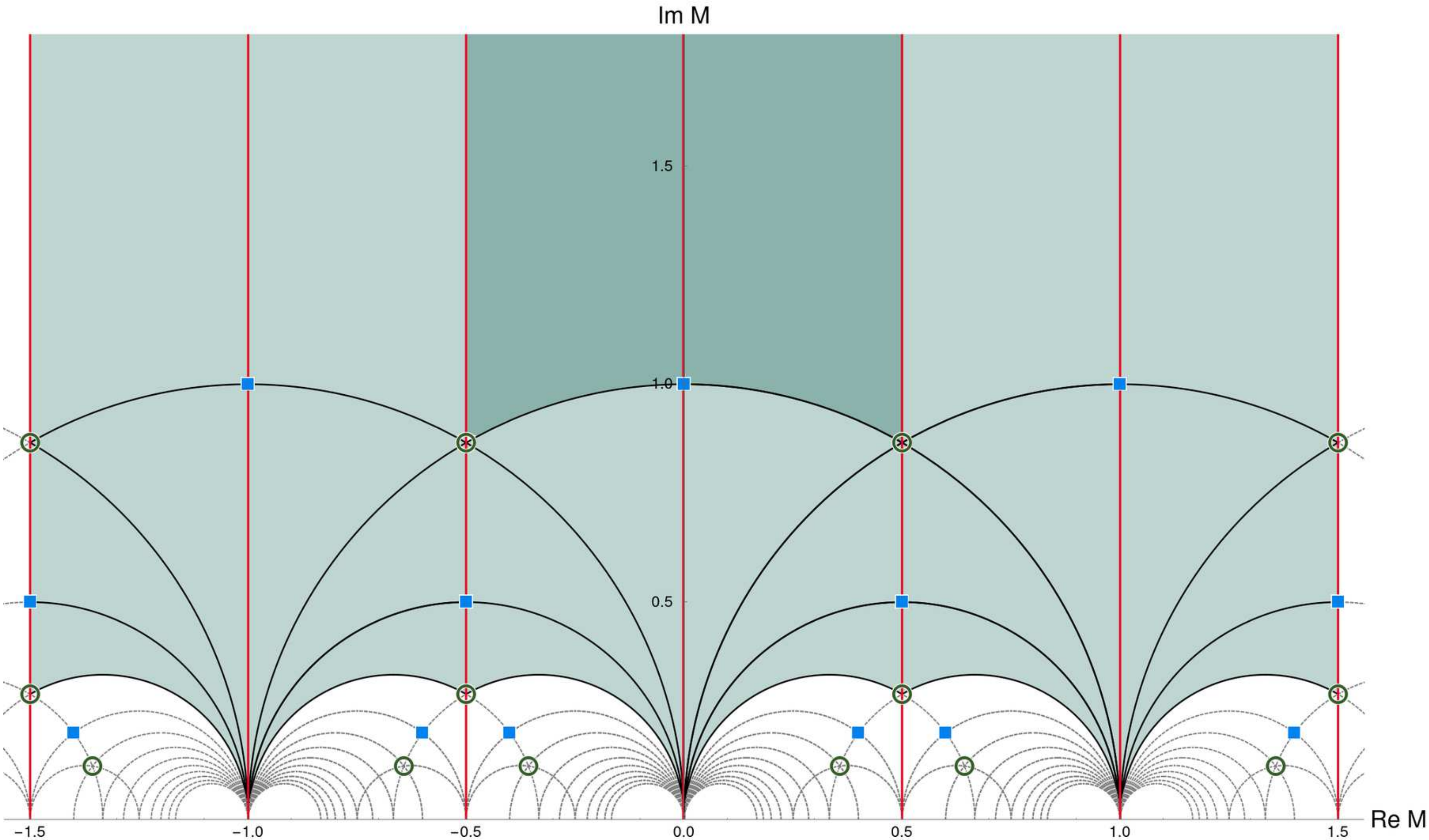
- the traditional flavor group that is universal in moduli space (here $\Delta(54)$)
- the modular flavor group that transforms the moduli nontrivially (here T')

The eclectic flavor group is defined as the multiplicative closure of these groups. Here we obtain for T_2/Z_3

- $\Omega(1) = SG[648, 533]$ from $\Delta(54)$ and $T' = SL(2,3)$
- $SG[1296, 2891]$ from $\Delta(54)$ and $GL(2,3)$ including CP

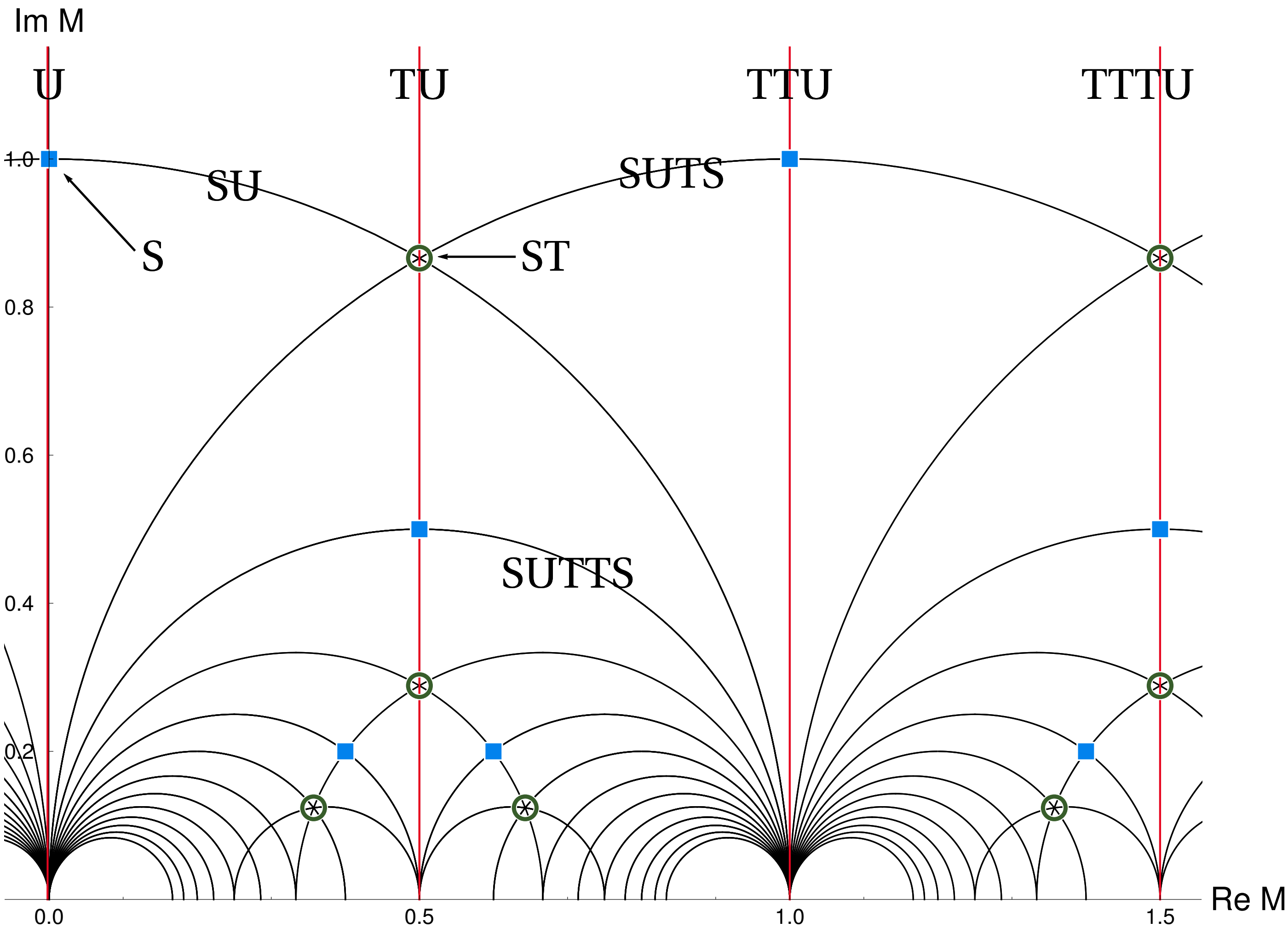
The eclectic group is the largest possible flavor group for the given system, but it is not necessarily linearly realized.

Local Flavor Unification



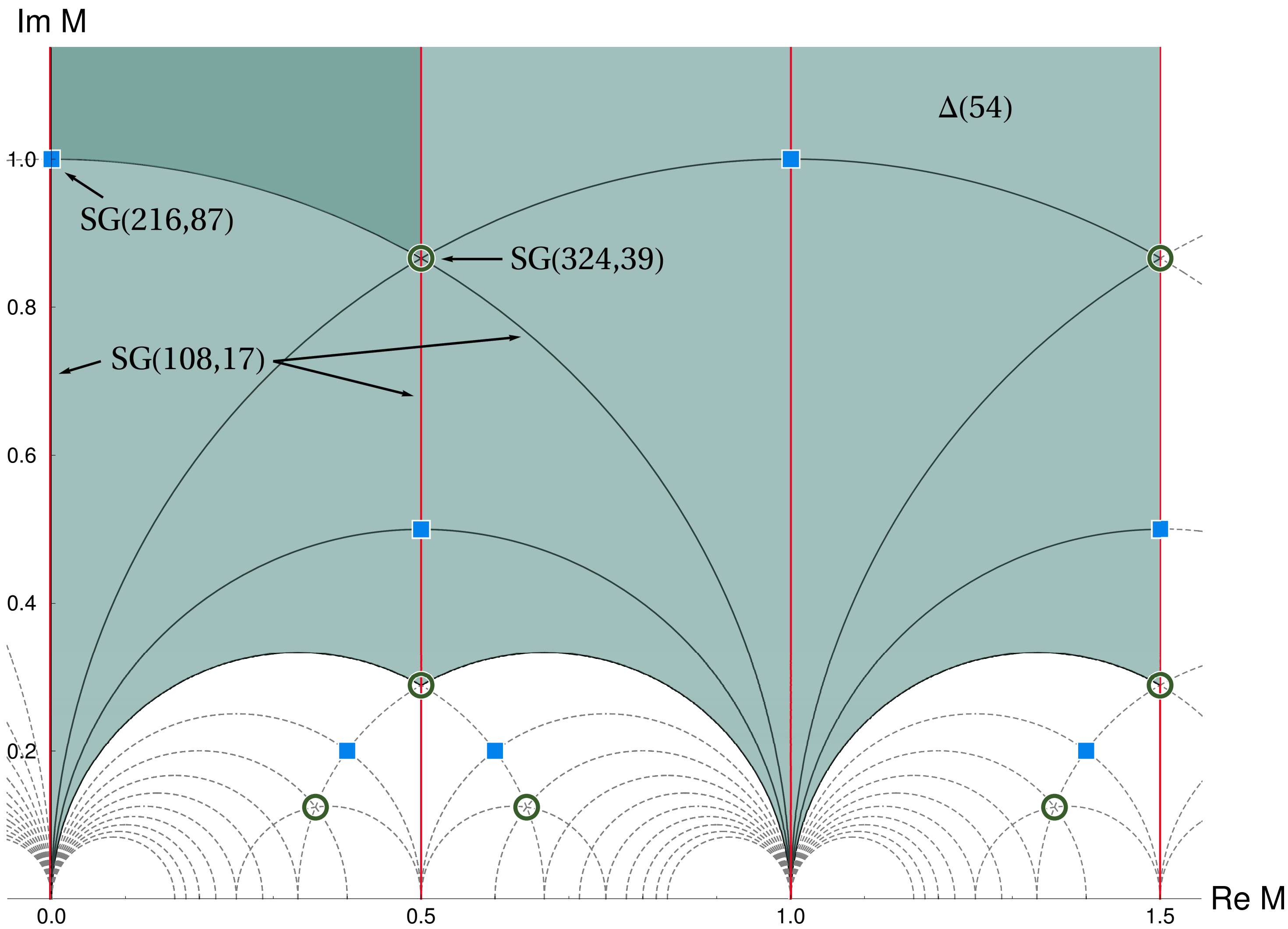
Moduli space of $\Gamma(3)$

Fixed lines and points



$$\mathrm{S}: M \to -\frac{1}{M}, \quad \mathrm{T}: M \to M+1 \quad \text{and} \quad \mathrm{U}: M \to -\overline{M}$$

Moduli space of flavour groups



"Local Flavor Unification"

Unification of Flavor and CP

Summary of predictions of the string picture:

- traditional flavor symmetries (universal in moduli space)
- modular flavor symmetries and CP are non-universal in moduli space

They unify in the eclectic picture of flavor symmetry.
You cannot just have one without the other.

The non-universality in moduli space leads to

- local flavor unification at specific points in moduli space
- hierarchical structures of masses, mixing angles and phases in vicinity of fixed points or lines
- potentially different pictures for quarks and leptons

Top-Down versus Bottom-Up

This opens up a new arena for flavor model building:

- so far $\Delta(54) \times T'$ is the favourite "top-down" model
- need more explicit string constructions
- but it is not only the groups but also the representations and modular weights of matter fields that are relevant (top-down models very restrictive)
- there is still a huge gap between "top-down" and "bottom-up" constructions
- modular flavour group from outer automorphisms of traditional flavor group (Nilles, Ramos-Sanchez, Vaudrevange, 2020)
- recently applied in "bottom-up" constructions for $\Delta(27)$ as traditional flavor symmetry (Ding, King, Li, Liu, Lu, 2023)

UV-IR connection

String dualities connect winding to momentum modes. Winding modes are heavy. Could there be an effect at low energies?

- "Stringy Miracles" and naturalness in string theory – need introduction of "Rule 4" (Font, Ibanez, Nilles, Quevedo, 1988)
- selection rules of CFT lead to vanishing of certain couplings that could not be understood through symmetries of the low energy theory
- extended later including "Rule 5" and "Rule 6"

  (Kobayashi, Parameswaran, Ramos-Sanchez, Zavala, 2011)
- these "Stringy Miracles" remained a puzzle till recently

Restrictions from eclectic flavor symmetry explain these "rules" (Nilles, Ramos-Sanchez, Vaudrevange, 2020)

Modular Flavor

How could that be?

- We need to analyse modular flavor symmetry more carefully.
- modular group $SL(2, Z)$ with $S^4 = 1$ and $S^2 \neq 1$
- $PSL(2, Z)$ with $S^2 = 1$ acts on moduli
- additional Z_2 corresponds to the double cover of finite modular group (originates from CFT selection rules)
- it is also part of the traditional flavor group. It looks "traditional" but it is intrinsically "modular"
- in the string models this Z_2 acts on "twisted" oscillator modes of the underlying string theory

(Work in progress)

Example T_2/Z_3

Superpotential is restricted by the eclectic flavor group

- $SG[648, 533] = \Omega(1)$ from $\Delta(54)$ and T'
- a Z_2 symmetry is common to $\Delta(54)$ and T'
- responsible for double cover T' of A_4
- extends $\Delta(27)$ to $\Delta(54)$
- $\Delta(54)$ contains nontrivial singlet $1'$ as well as two 3-dimensional representations 3_1 and 3_2
- vev of $1'$ breaks $\Delta(54)$ to $\Delta(27)$ with one triplet rep.
- twisted oscillator modes transform as $1'$ rep. of $\Delta(54)$

The Z_2 as part of $\Delta(54)$ and T' acts nontrivially on the oscillator modes and explains these "Stringy Miracles"

Messages

The top-down approach to flavor symmetries leads to a

- unification of traditional (discrete) flavor, CP and modular symmetries within an eclectic flavor scheme
- modular flavor symmetry is a prediction of string theory
- traditional flavor symmetry is universal in moduli space
- there are non-universal enhancements (including CP at some places (broken in generic moduli space))
- CP is a natural consequence of the symmetries of the underlying string theory
- the potential flavor groups are large and non-universal
- spontaneous breakdown as motion in moduli space

Outlook

This opens up a new arena for flavor model building and connections to bottom-up constructions:

- need more explicit string constructions
- role of traditional versus modular symmetries
- the concept of local flavor symmetries allows different flavor groups for different sectors of the theory, as for example quarks and leptons
- but it is not only the groups but also the representations of matter fields that are relevant. Not all of the possible representations appear in the massless sector.

There is still a huge gap between "top-down and bottom-up" constructions

Open Questions

So far $\Delta(54) \times T'$ seems to be the favourite model

- numerous bottom-up models with these groups
- successful realistic string model from Z_3 orbifold

(Baur, Nilles, Ramos-Sanchez, Trautner, Vaudrevange, 2022)

It has been observed that many of the successful fits are in the vicinity of fixed points and lines

(Feruglio, 2022-23; Petcov, Tanimoto, 2022; Abe, Higaki, Kawamura, Kobayashi, 2023)

- moduli stabilization favours boundary of moduli space, but leads to AdS-minima
- uplift moves them slightly away from the boundary and leads to flavor hierarchies

(Knapp-Perez, Liu, Nilles, Ramos-Sanchez, Ratz, 2023)

Summary

String theory provides the necessary ingredients for flavor:

- traditional flavor group
- discrete modular flavor group
- a natural candidate for CP
- the concept of local flavour unification

The eclectic flavor group provides the basis:

- this includes a non-universality of flavor symmetry in moduli space
- allows a different flavor structure for quarks and leptons