Unification of fundamental interactions

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Standard Model

We have a standard model of elementary particle physics.
It is based on

- gauge symmetries $SU(3) \times SU(2) \times U(1)$
- three families of quarks and leptons
- a scalar Higgs boson

It is extremely successful,

- but there are many free parameters
- and some open questions.

Is there physics beyond the standard model?

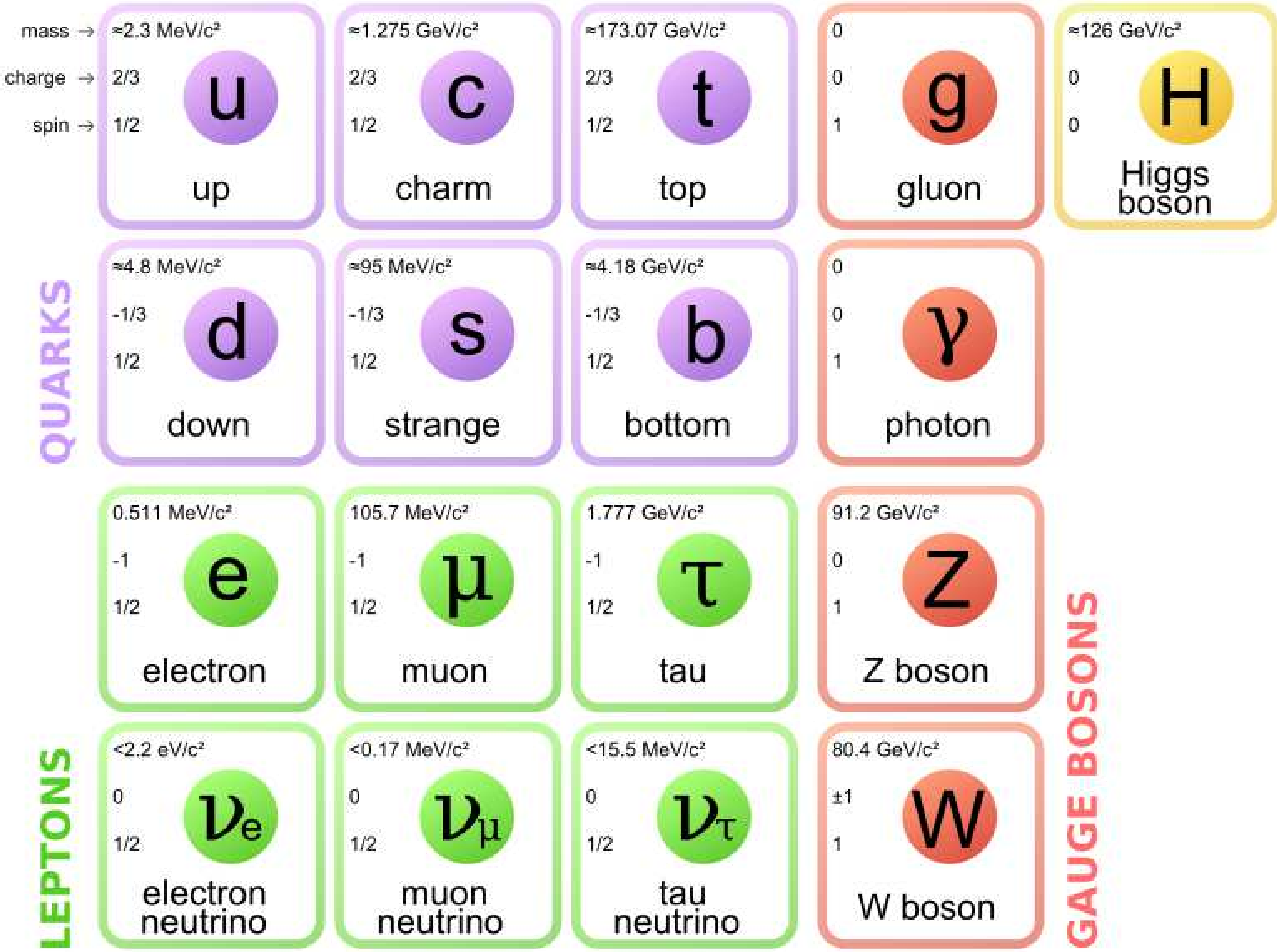
Outline

- The Standard Model (SM)
- Three basic questions
- Reasons to go beyond the SM
- Grand unification and supersymmetry
- Extra dimensions and "Local Grand Unification"
- Some group theory: The beauty of $SO(10)$
- Strong motivation for E_8
- Extra dimensions from String Theory
- How to test?

History

- Gravity 1915
- Quantum Electrodynamics (QED) ca. 1950
- Yang-Mills theory for weak interactions 1954
- "Higgs" mechanism 1964
- Electroweak standard model 1967
- Renormalizability of nonabelian gauge theories ca. 1972
- Quantum chromodynamics (QCD) ca. 1973
- Discovery of gauge bosons $W^{\pm}$ and Z^0 1983
- Discovery of Higgs boson 2012

Standard Model



A family of quarks and leptons

The gauge group is $SU(3) \times SU(2) \times U(1)_Y$

$$(u_\alpha, d_\alpha)_{Y=1/6} \qquad (\nu_e, e)_{Y=-1/2}$$

$$(\bar{u}_\alpha)_{Y=-2/3} \qquad (\bar{e})_{Y=1}$$

$$(\bar{d}_\alpha)_{Y=1/3}$$

with $\alpha = 1, 2, 3$ the $SU(3)$-index.
Observe that

$$\sum_i Y_i = 0 \qquad \text{and} \qquad \sum_i Y_i^3 = 0$$

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Three basic questions

Some fundamental questions remain unanswered

- The origin of the structure of a family?
- Why three copies ?
  Question of I. Rabi: who ordered the muon?
- Why gauge group $SU(3) \times SU(2) \times U(1)$?

and require physics beyond the Standard Model.

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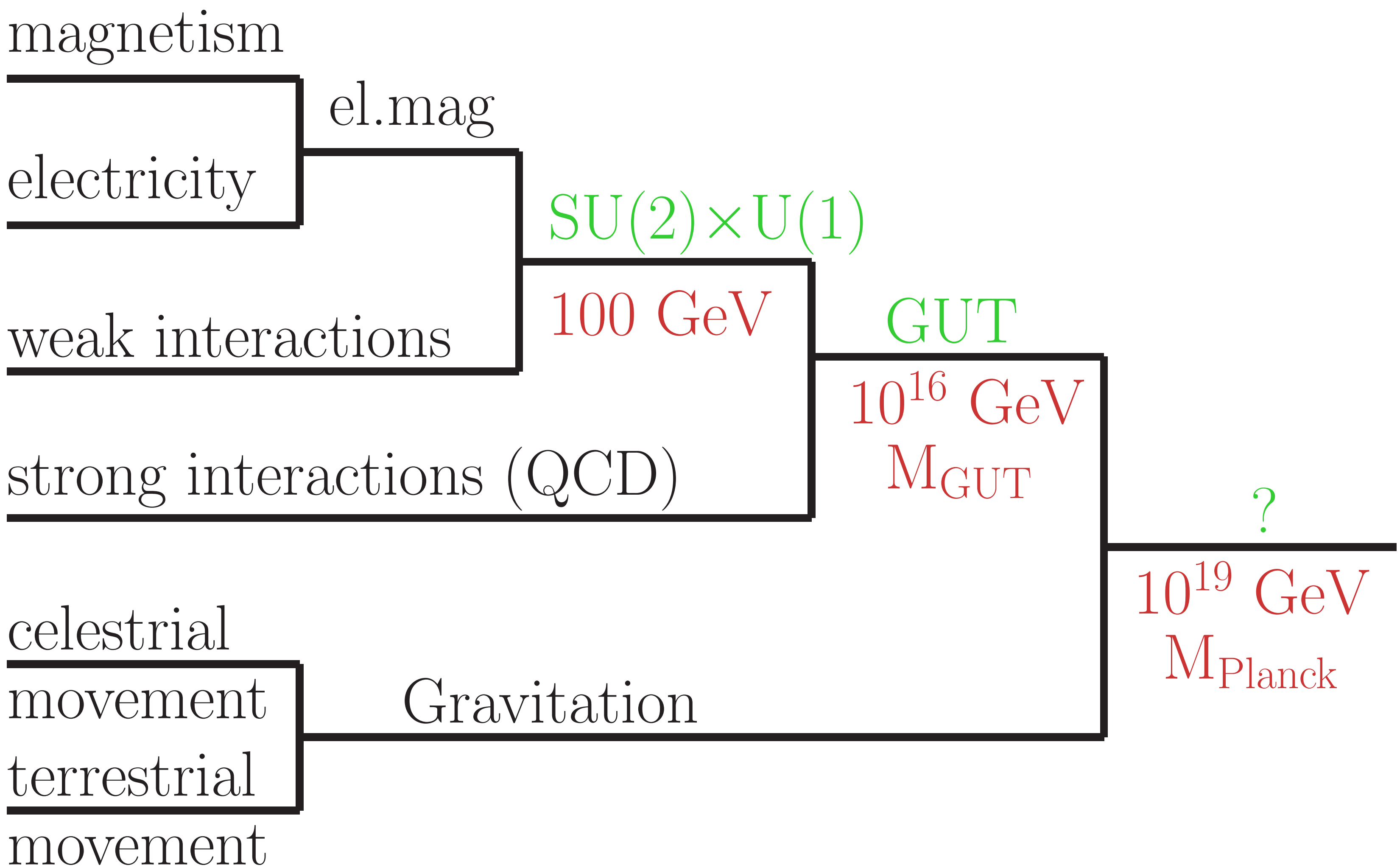
Some other reasons to go beyond the SM

- dark matter of the universe
- baryon asymmetry, neutrino oscillations
- “Landau Pole” of electromagnetic $U(1)$

The Quest for Unification



Grand Unification

Embed the SM gauge group

- into a single grand unified group!
- examples are $SU(5)$ and $SO(10)$.

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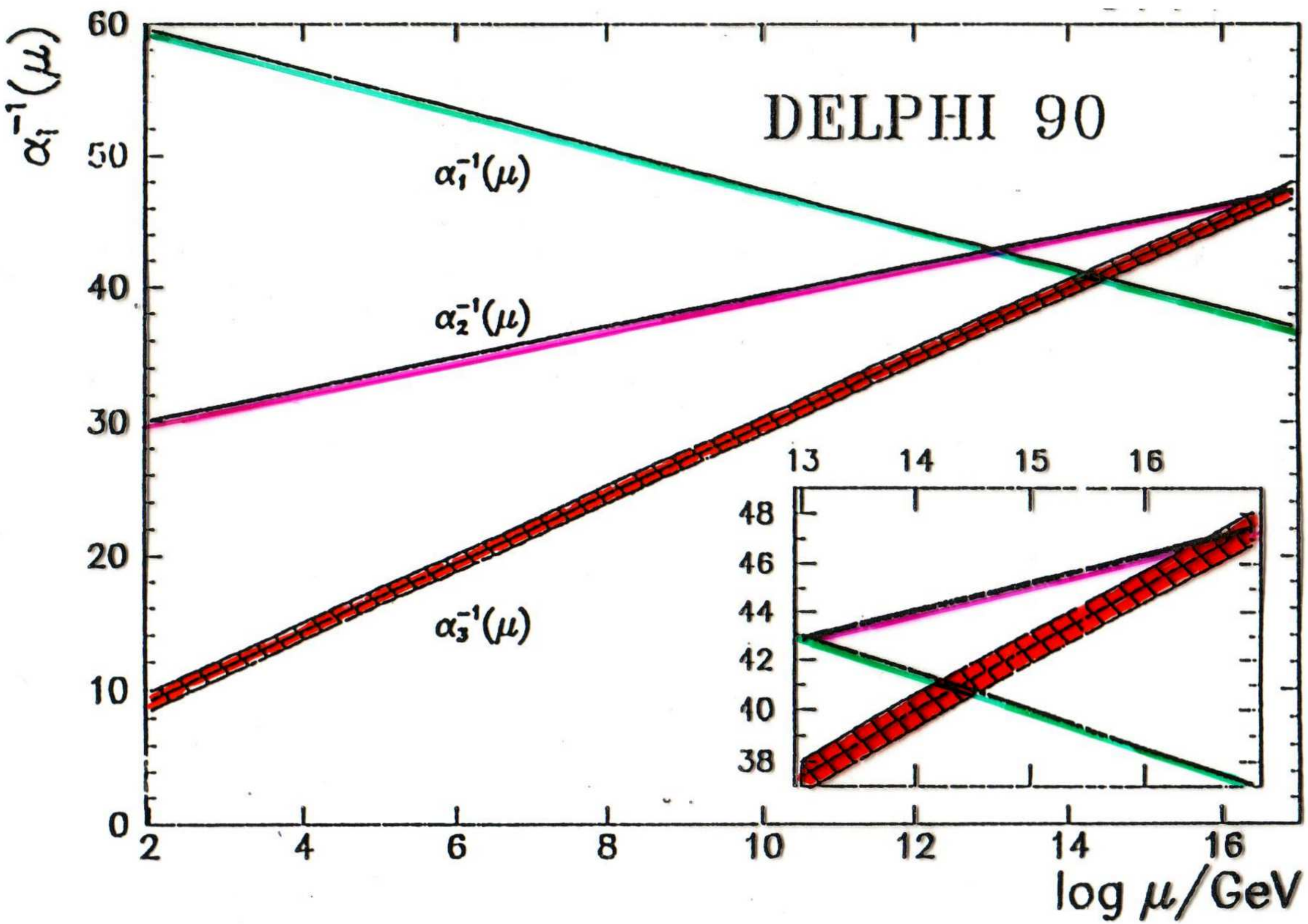
- into a single grand unified group!
- examples are $SU(5)$ and $SO(10)$.

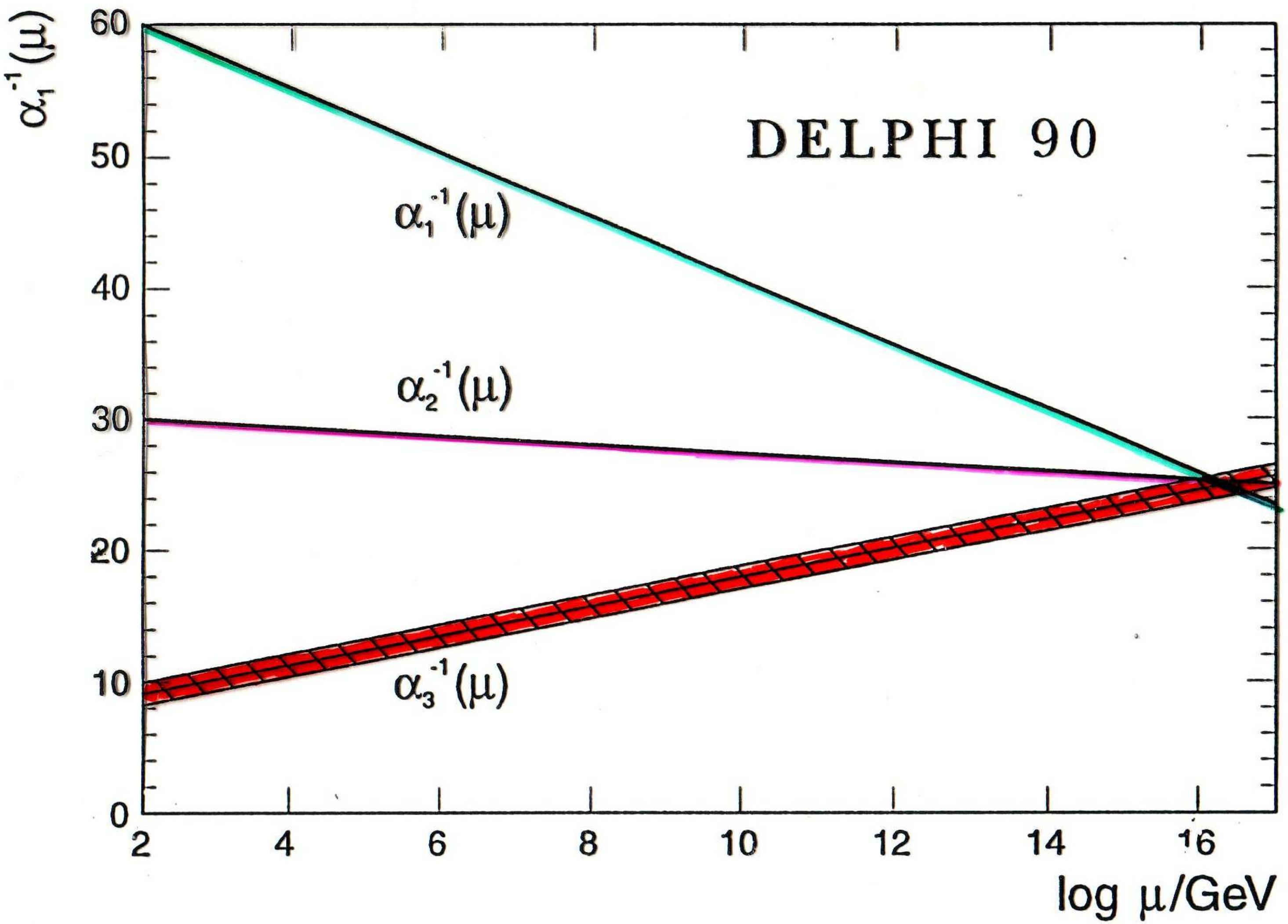
But there are a few obstacles:

- "equality" of gauge coupling constants
- the "doublet-triplet" splitting problem
- the breakdown of the grand unified gauge group.

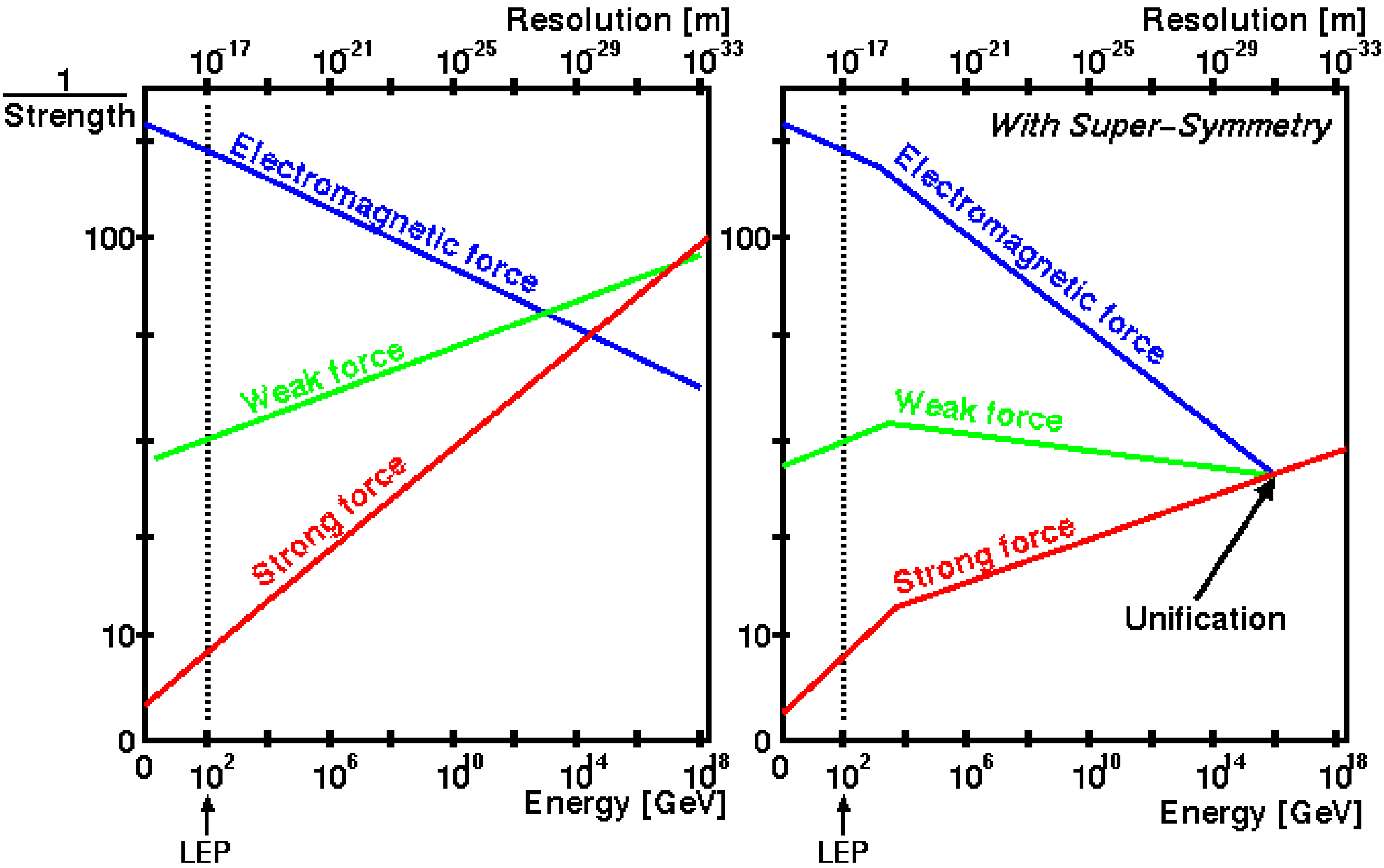
Standard Model



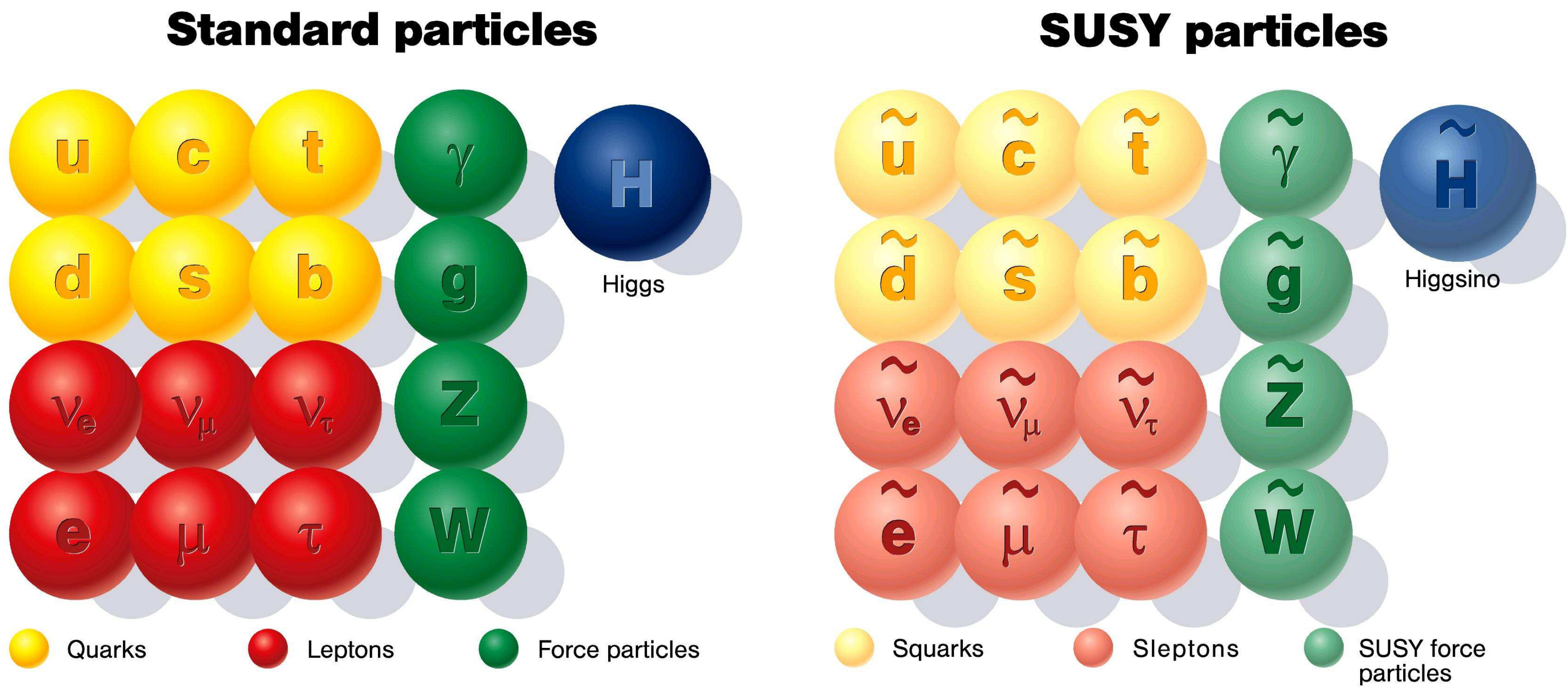
Supersymmetric SM



Susy thresholds



New particles



Supersymmetry

Unification of matter and radiation

- consistent with grand unification
- stabilizes the weak scale
- provides candidates for dark matter
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Preferred grand unified gauge groups

- $SO(10)$ and $SU(5)$ include $SU(3) \times SU(2) \times U(1)$
- explain the structure of families of quarks and leptons
  - $\bar{5}$ + 10 representations of $SU(5)$
  - 16-dimensional spinor representation of $SO(10)$

Binary code for quarks and leptons

$$(n_1, n_2, n_3, n_4, n_5) \quad \text{with} \quad n_i = 0, 1 \quad \text{and} \quad \sum_i n_i = \text{even}$$

- $(1,1,1,1,0)$ 5 combinations
  - $(1,1,0;1,1)$ $\bar{d}$
  - $(1,1,1;0,1)$ (ν_e, e)
- $(1,1,0,0,0)$ 10 combinations
  - $(1,1,0;0,0)$ $\bar{u}$
  - $(1,0,0;1,0)$ (u, d)
  - $(0,0,0;1,1)$ $\bar{e}$
- $(0,0,0,0,0)$ 1 combination $\bar{\nu}_e$

Basic questions: where are we?

We have made some pogress.

- The origin of the structure of a family:
  answer is 16-dim. spinor representation of $SO(10)$
- Why three copies: not known yet,
  but group theory is proven to be unsuccessful.
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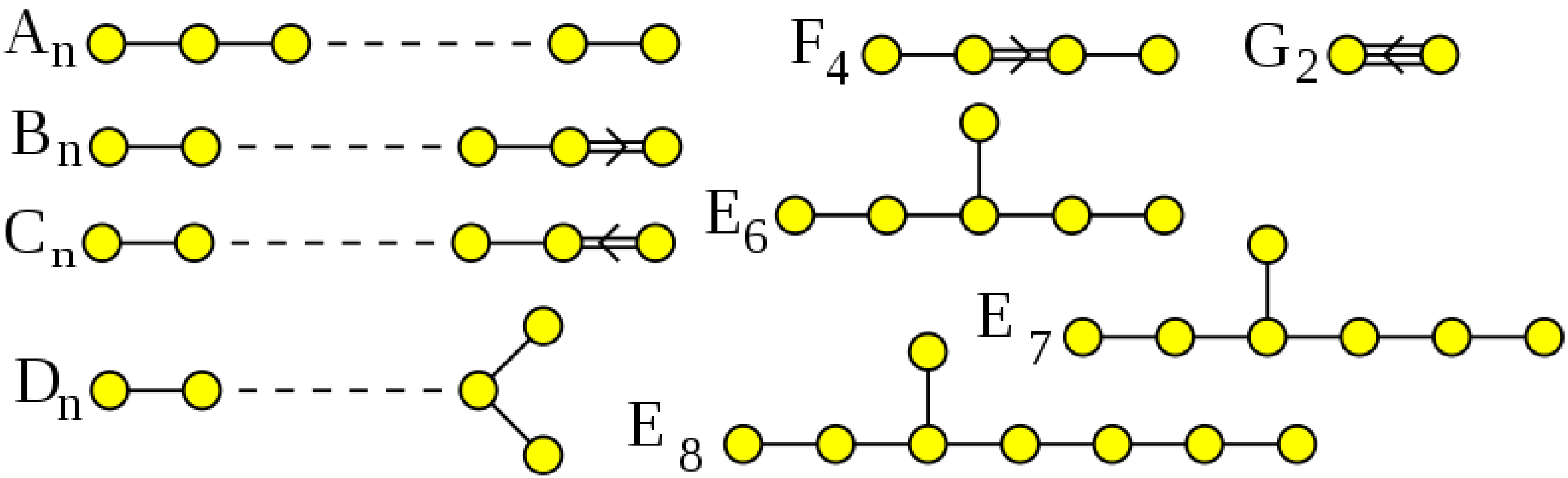
Reminder: still some problems with grand unification:

- evolution of couplings requires supersymmetry
- “doublet-triplet” splitting
- breakdown of grand unified group

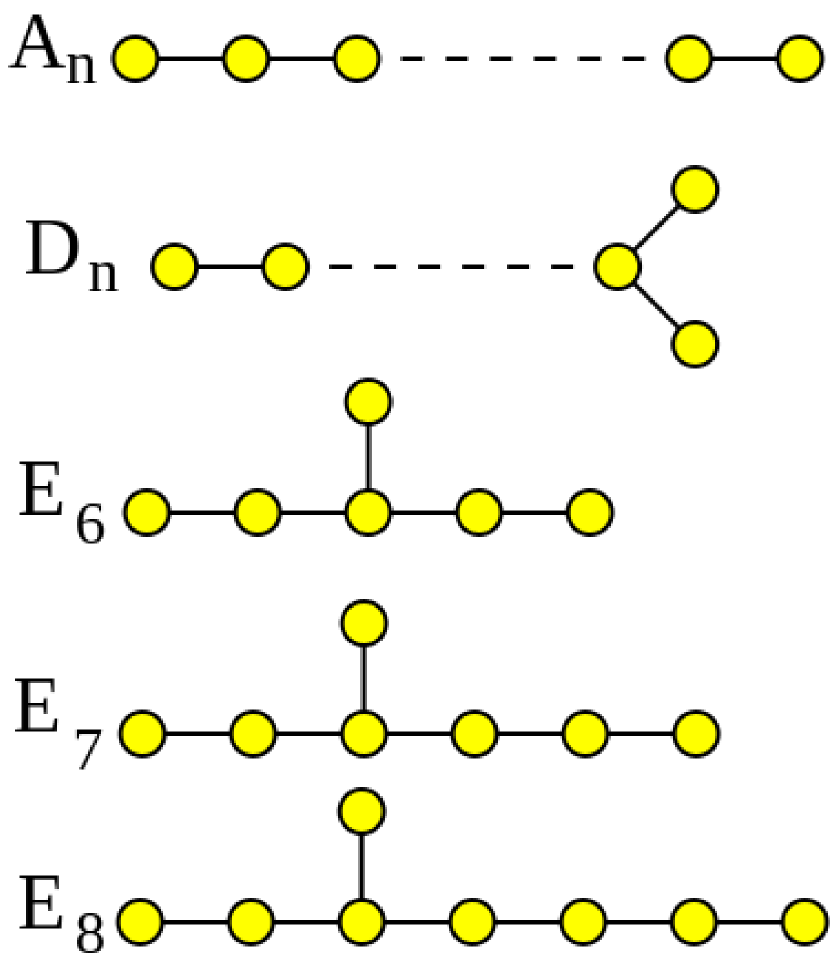
Why SO(10): Dynkin diagrams



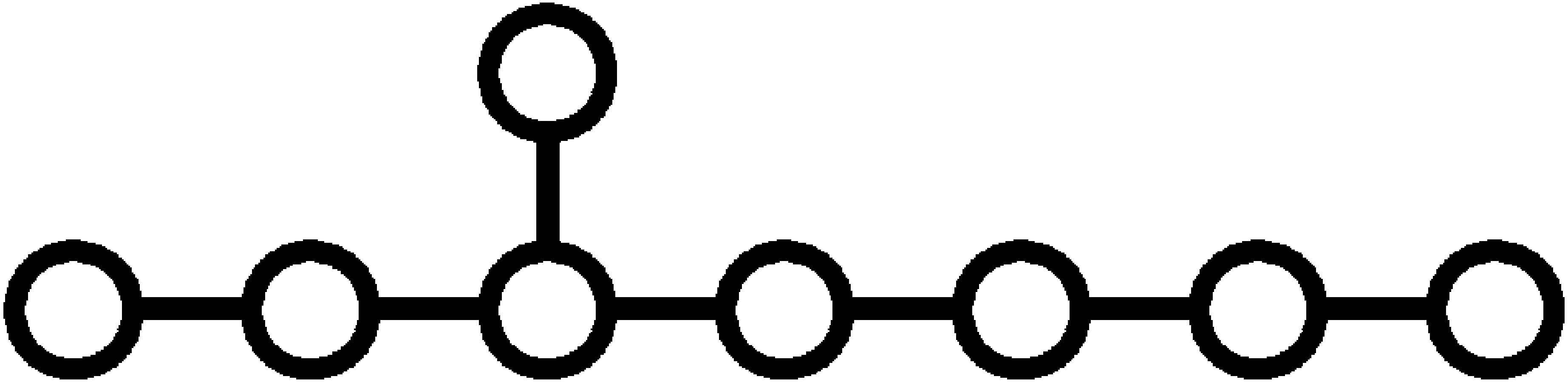
Lie groups come in 4 infinite series $SU(N)$, $SP(2N)$, $SO(2N+1)$, $SO(2N)$ and 5 exceptional groups.

Not all of them are useful for grand unification as they do not provide chiral representations to explain parity violation of weak interactions.

Simply Laced Lie Groups



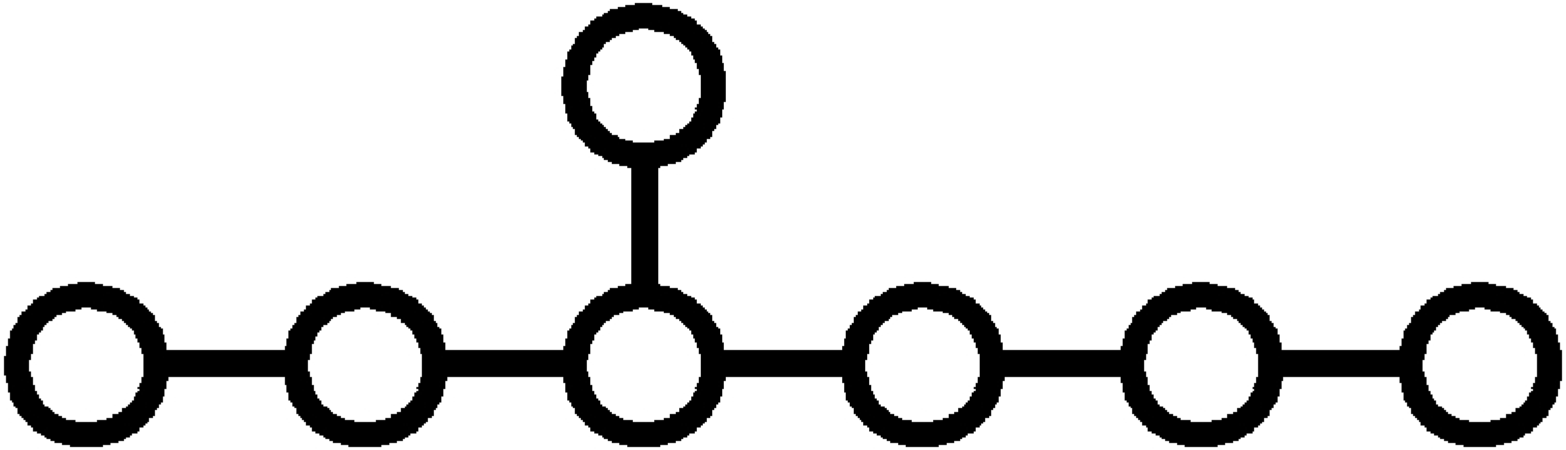
Maximal Group E_8



E_8 is the maximal group.

There are, however, no chiral representations in $d = 4$.

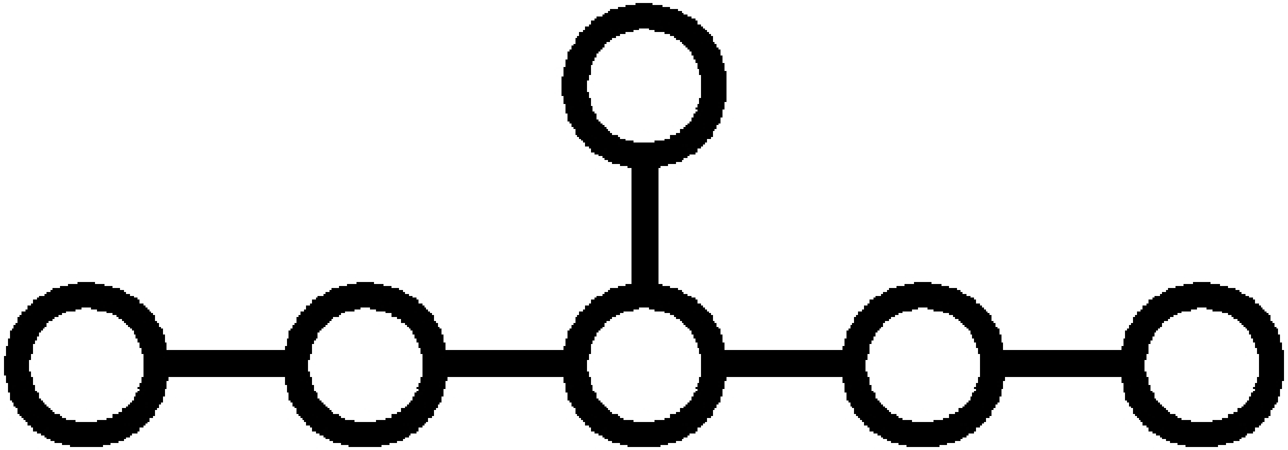
E_7



Next smaller is E_7.

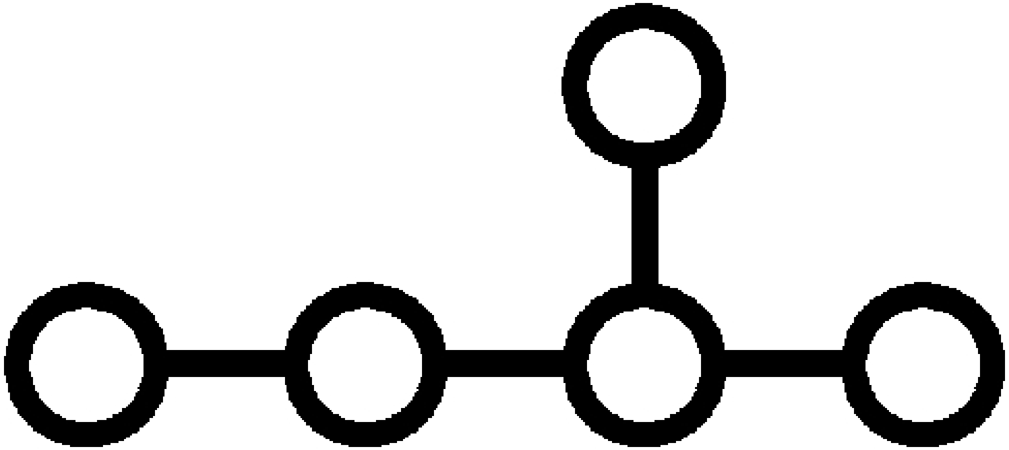
No chiral representations in $d = 4$ either.

E_6



E_6 allows for chiral representations even in $d = 4$.

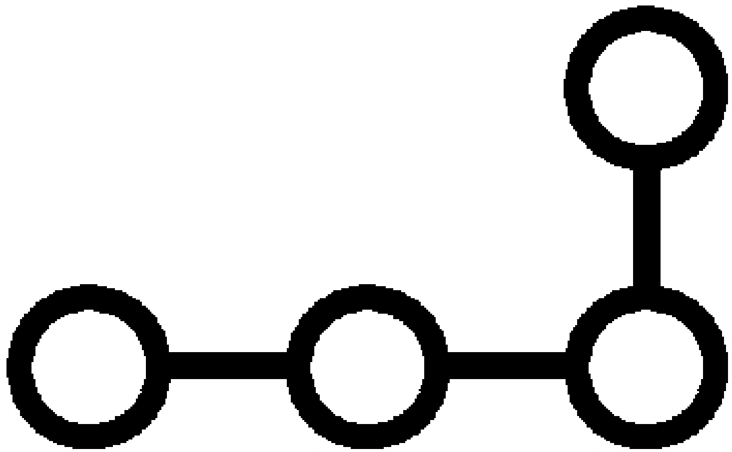
$E_5 = D_5$



E_5 is usually not called exceptional.

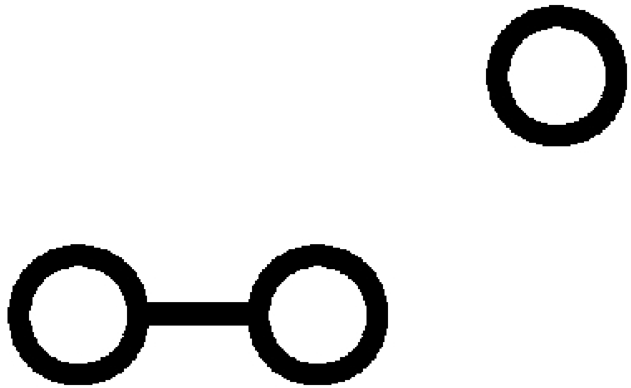
It coincides with $D_5 = SO(10)$.

$E_4 = A_4$



E_4 coincides with $A_4 = SU(5)$.

E_3



E_3 coincides with $A_2 \times A_1$ which is $SU(3) \times SU(2)$.

Strong motivation for E_8

E_8 would require higher dimensions

- E_8 is strongly motivated from string theory ($E_8 \times E_8$ heterotic string and M/F theory)
- E_8 has chiral representations in $d = 8n + 2$
- String theory requires $d = 10$
- E_8 broken in process of compactification (e.g. to E_5)

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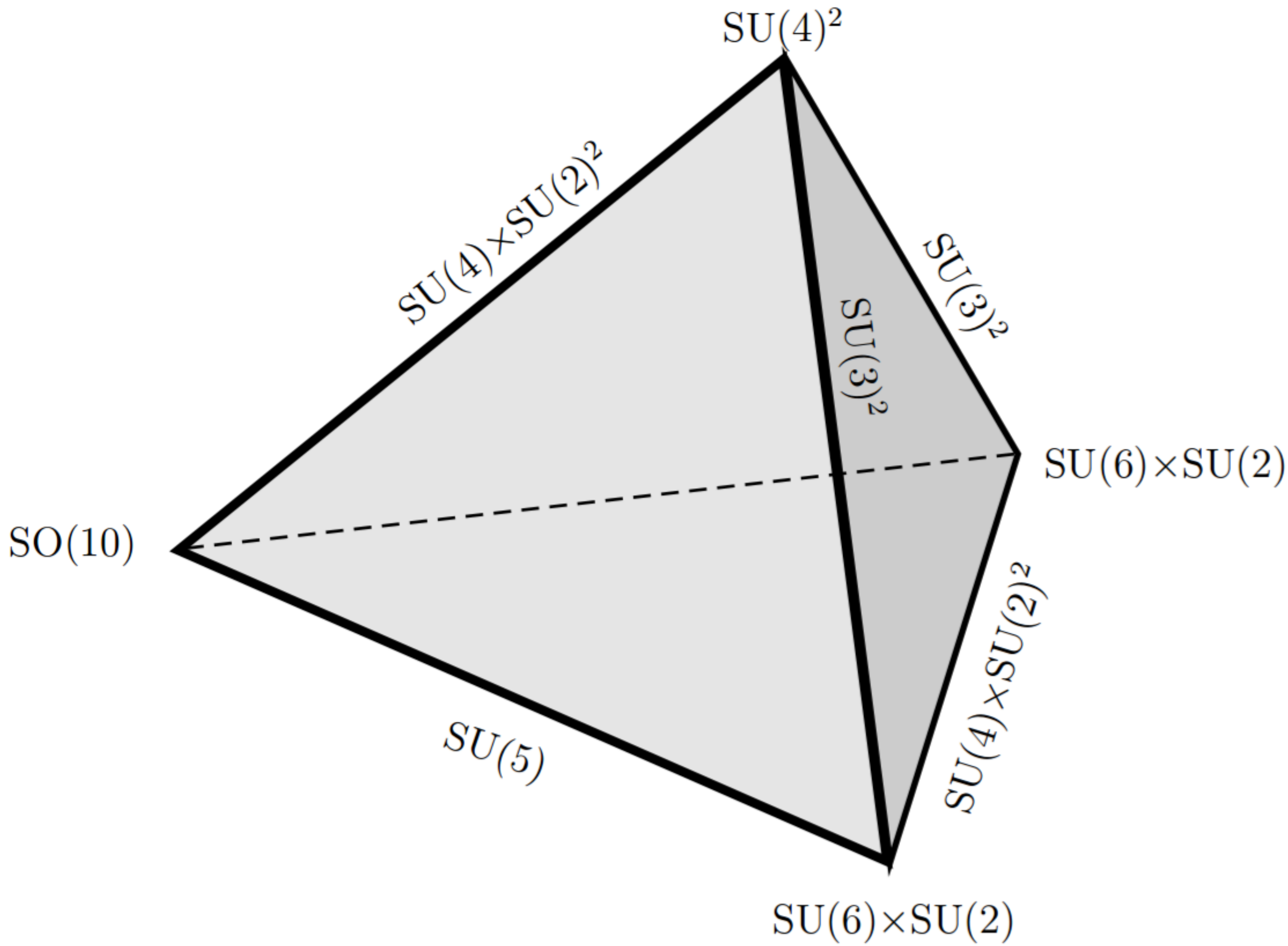
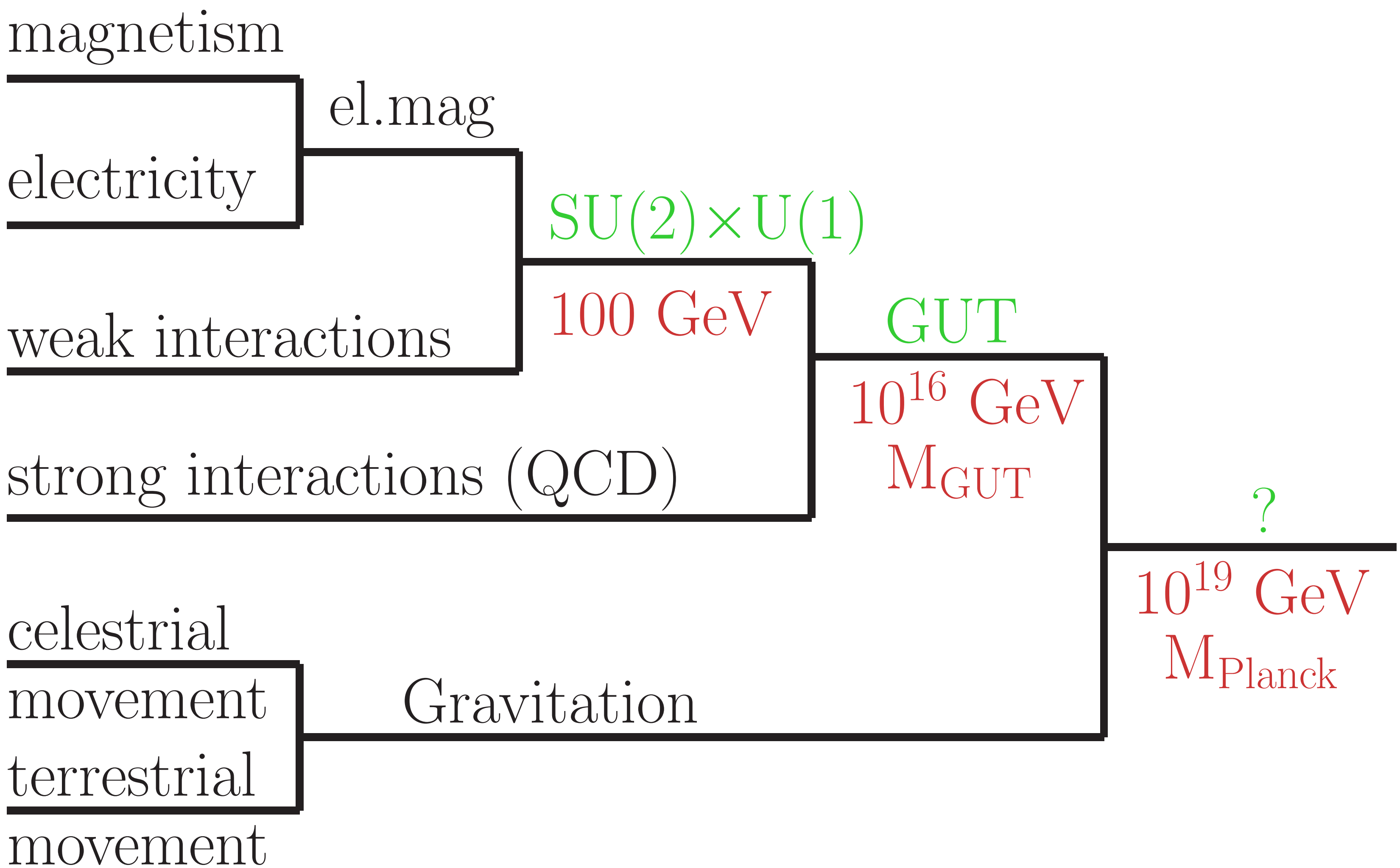
Extra dimensions allow for the concept of
“Local Grand Unification”:

- this solves the doublet-triplet splitting problem
- and provides the breakdown of the GUT group.

Local Grand Unification



The Quest for Unification



Three basic questions, again

Some basic questions are answered.

- The origin of the structure of a family:
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- Why three copies:
  topological properties of compactified extra dimensions
- Why $SO(10)$?
  It is the grand-grand daughter E_5 of E_8.
- Local Grand unification:
  allows for “incomplete multiplets”

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Answers require physics beyond the SM!

- We need new experimental input.

Physics beyond the SM

Standard model is incomplete

- problems with unification
- dark matter
- baryogenesis
- inclusion of gravity

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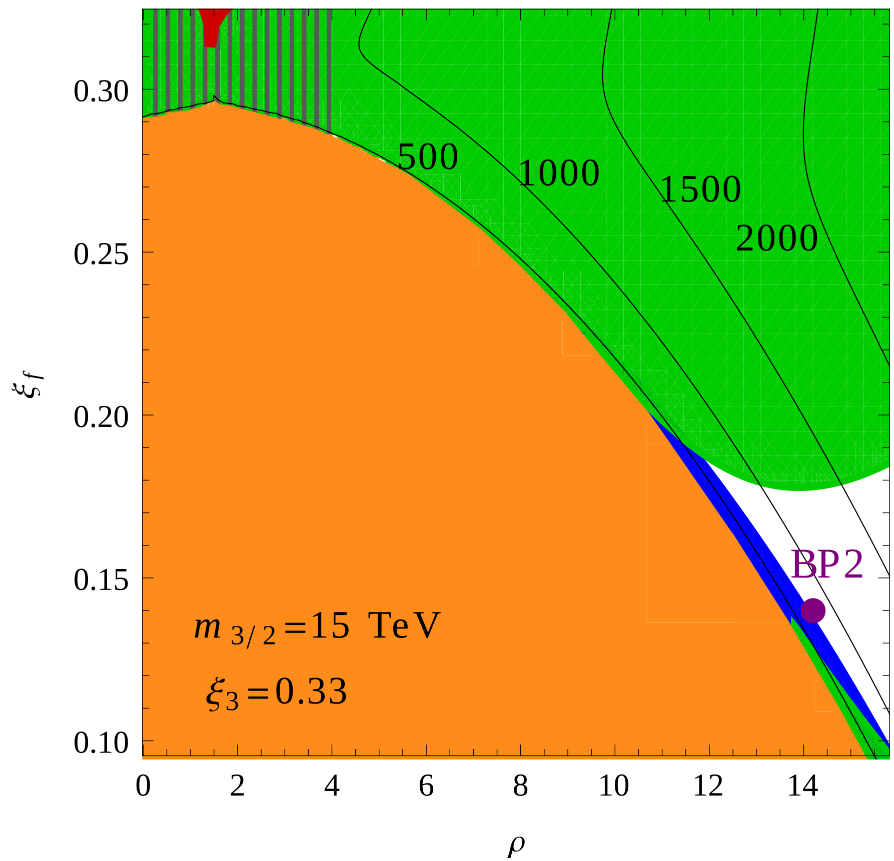
There must be new physics somewhere.

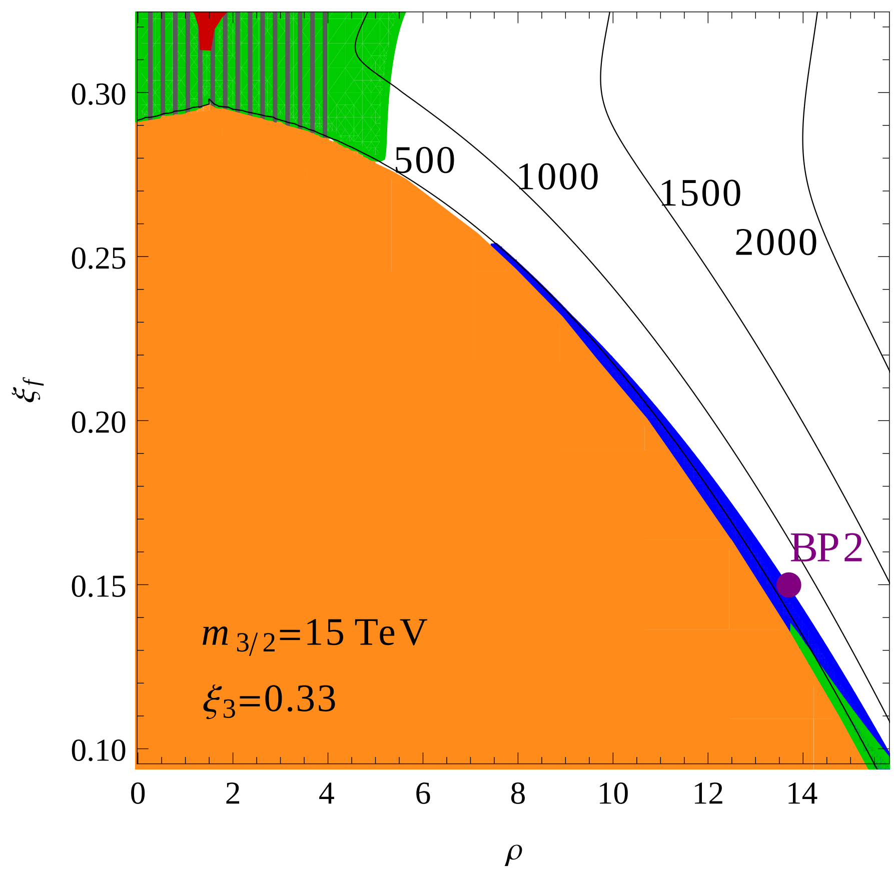
- Where is it?
- Is it at the TeV scale?
- Why is there no signal yet at the LHC?

LHC and physics beyond SM



Strong constraints on MSSM from 126 GeV Higgs mass.
The coloured regions are excluded while the hatched region indicates the current reach of the LHC.

Pre-LHC expectations



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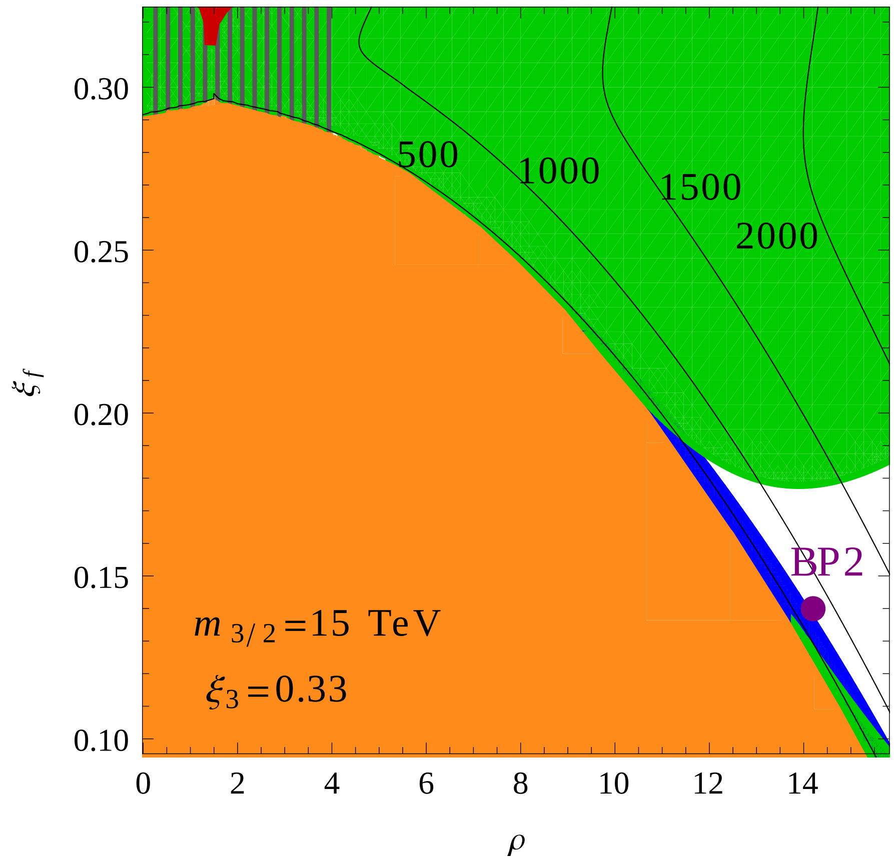
LHC and physics beyond SM



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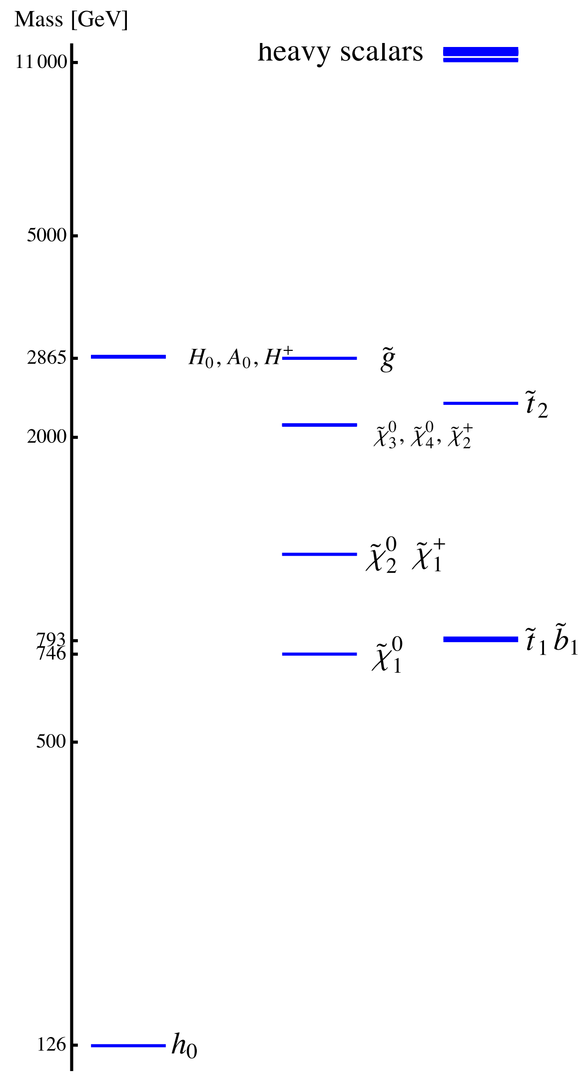
Benchmark model



The quest for “Precision Susy”

Two important arguments for supersymmetry

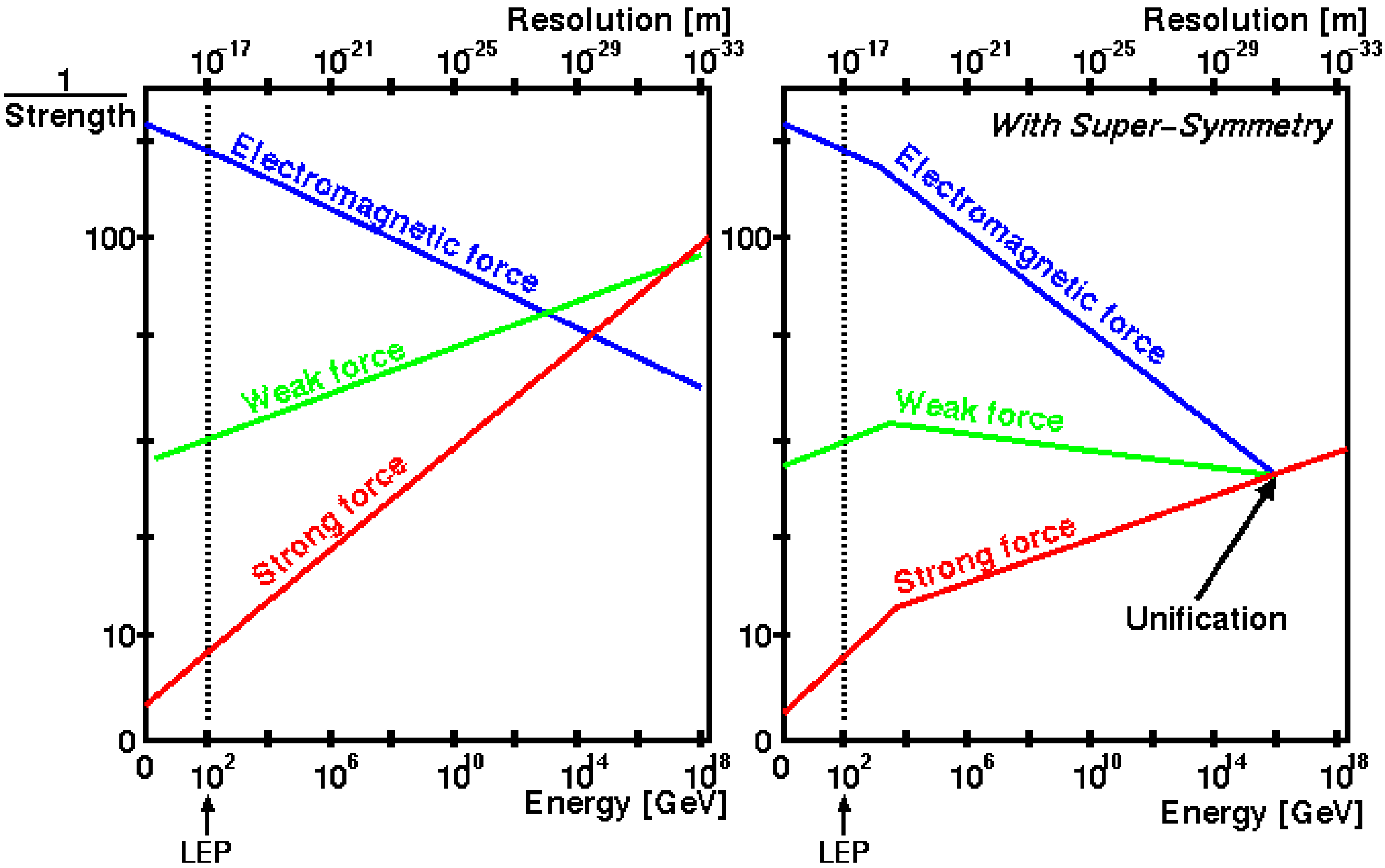
- solution to the hierarchy problem
- gauge coupling unification

We want to take these two arguments as serious as possible and reanalyze the MSSM within this scheme. We make two assumptions:

- demand precision gauge unification
- require smallest supersymmetric masses possible

What are the consequences for the search at LHC?

Susy thresholds



Precision gauge unification

$$\frac{1}{g_i^2(M_{\mathsf{GUT}})} = \frac{1}{g_i^2(M_Z)} - \frac{b_i^{\mathsf{MSSM}}}{8\pi^2} \ln\left(\frac{M_{\mathsf{GUT}}}{M_Z}\right) + \frac{1}{g_{i,\mathsf{Thr}}^2}$$

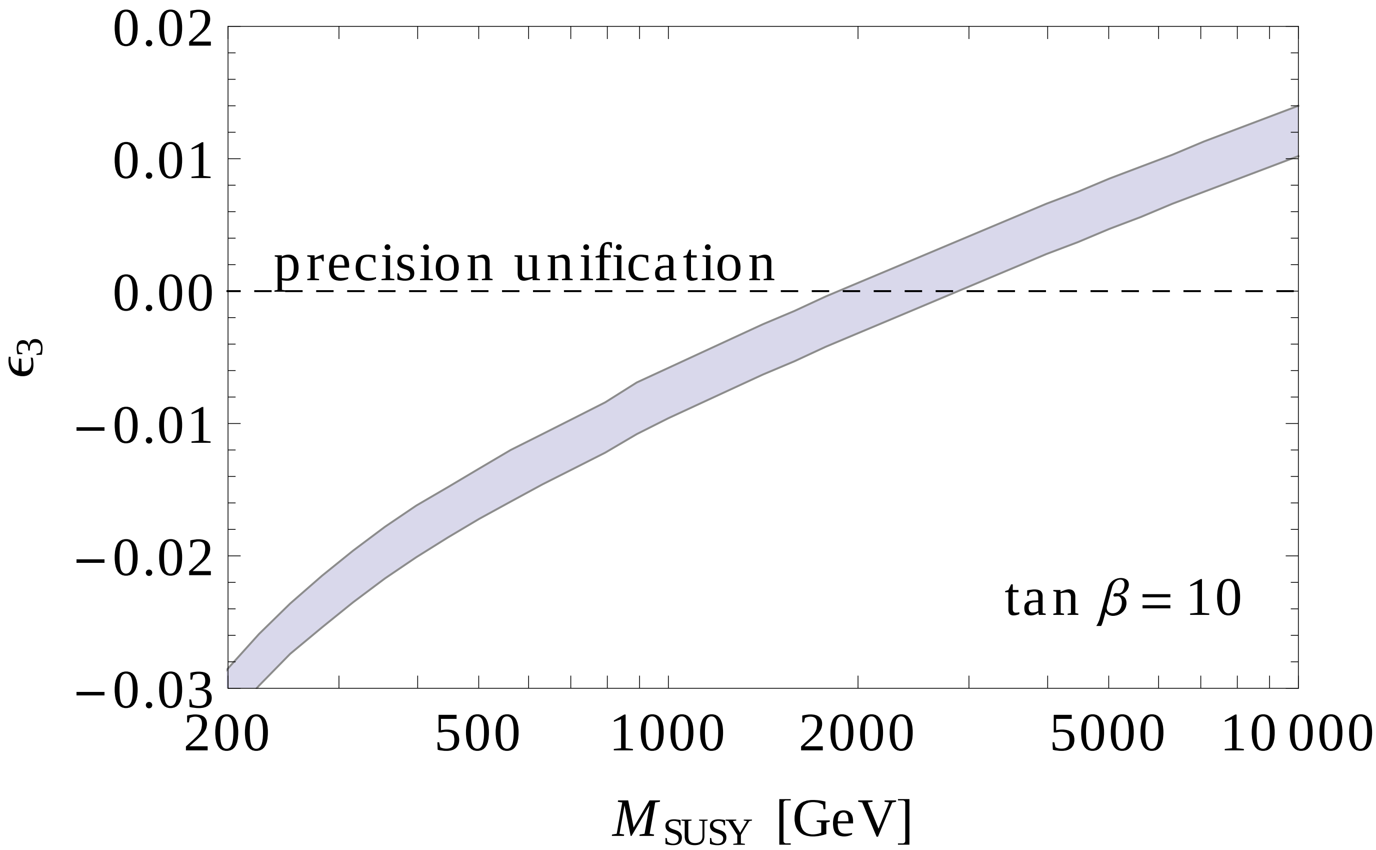
Low scale thresholds:

$$\frac{1}{g_{i,\mathsf{Thr}}^2} = \frac{b_i^{\mathsf{MSSM}} - b_i^{\mathsf{SM}}}{8\pi^2} \ln\left(\frac{M_{\mathsf{SUSY}}}{M_Z}\right)$$

The measure for gauge unification:

$$\epsilon_3 = \frac{g_3^2(M_{\mathsf{GUT}}) - g_{1,2}^2(M_{\mathsf{GUT}})}{g_{1,2}^2(M_{\mathsf{GUT}})}$$

Unification versus M_{SUSY}



M_{SUSY} should thus be in the few-TeV range.

The Susy-Scale

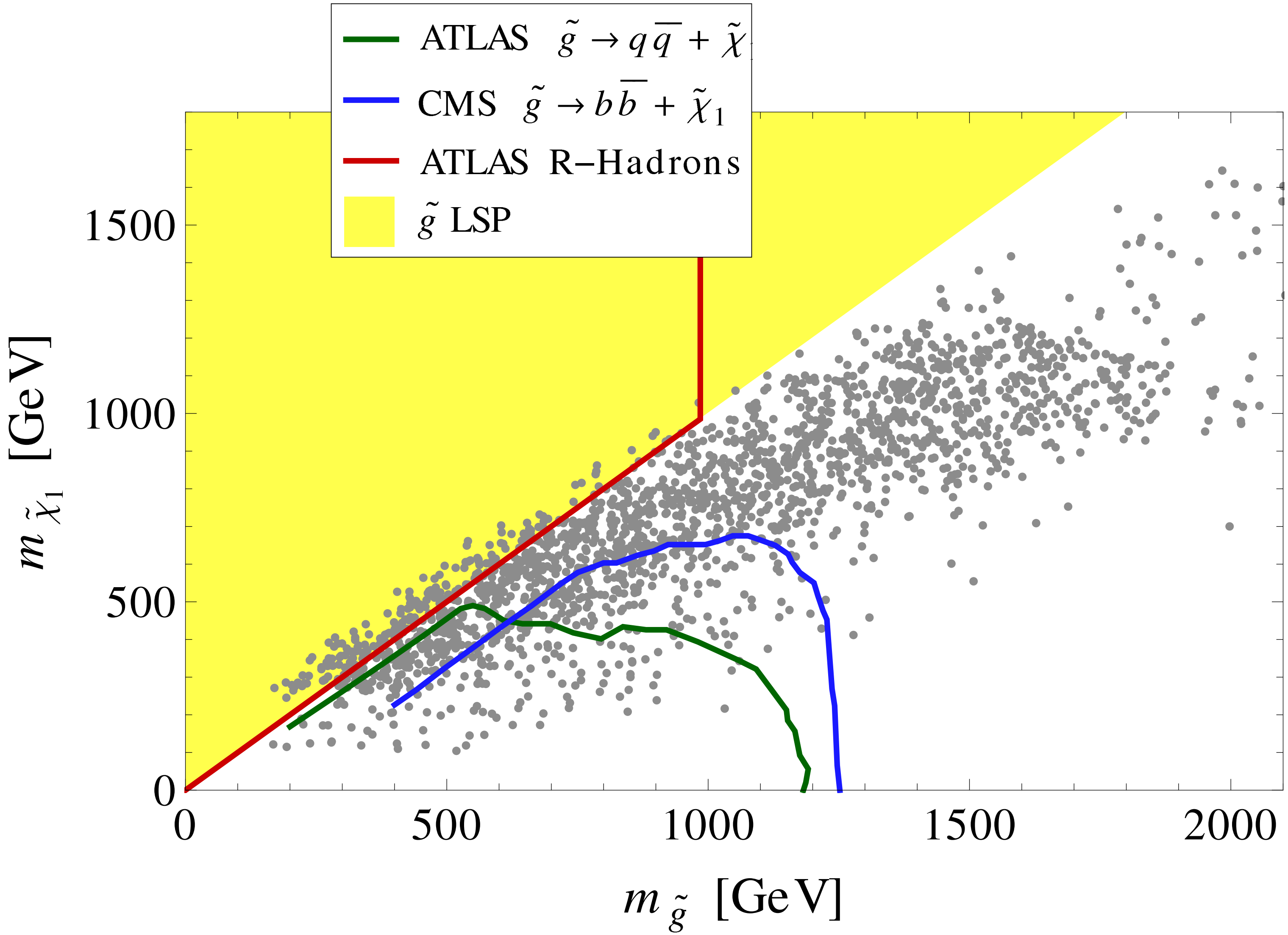
If all supersymmetric partners have the same mass M, then $M_{SUSY} = M$.
For non-universal masses we have an effective scale:

$$M_{\text{SUSY}} \sim \frac{m_{\widetilde{W}}^{32/19}\, m_{\widetilde{h}}^{12/19}\, m_{H}^{3/19}}{m_{\widetilde{g}}^{28/19}}\, X_{\text{sfermion}}$$

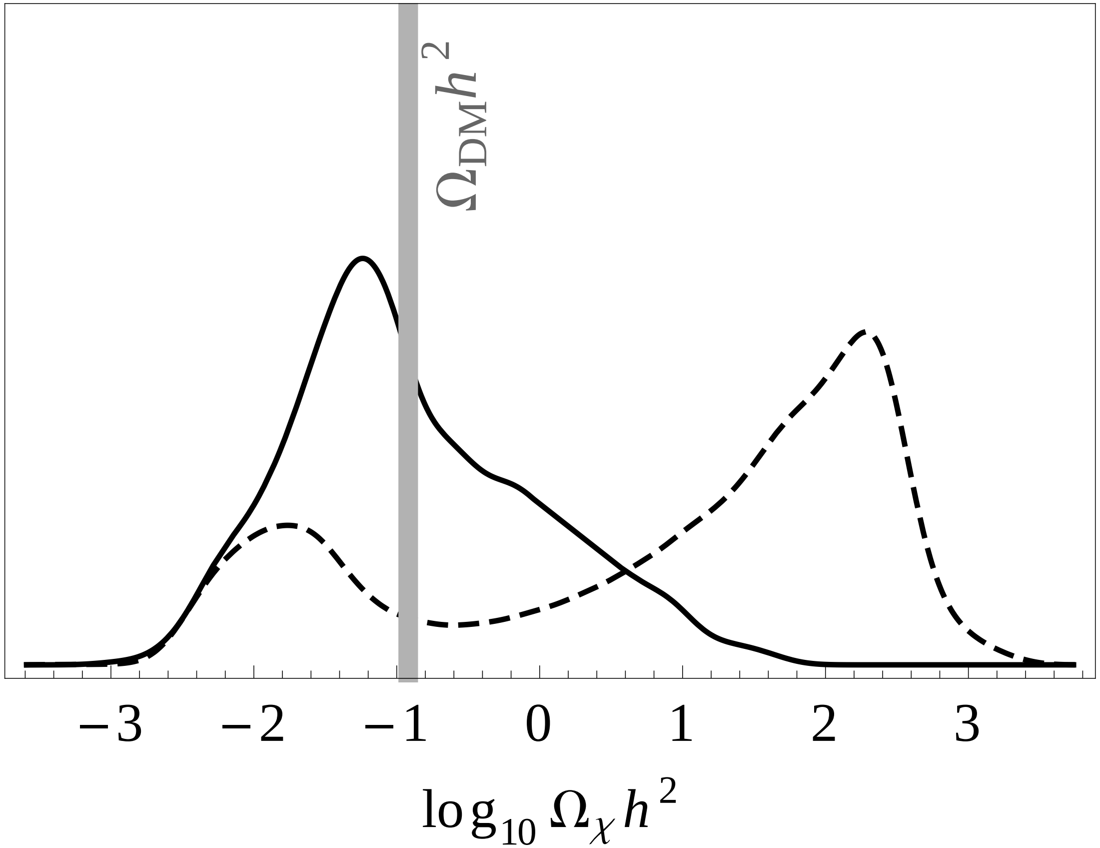
with

$$X_{\text{sfermion}} = \prod_{i=1...3} \left(\frac{m_{\widetilde{L}^{(i)}}^{3/19}}{m_{\widetilde{D}^{(i)}}^{3/19}}\right) \left(\frac{m_{\widetilde{Q}_{\mathrm{L}}^{(i)}}^{7/19}}{m_{\widetilde{E}^{(i)}}^{2/19}\, m_{\widetilde{U}^{(i)}}^{5/19}}\right)$$

LHC limits are weak



Dark Matter Relic Density



Distribution of thermal neutralino relic density for the benchmark sample with (solid) or without (dashed) the assumption of precision gauge coupling unification.

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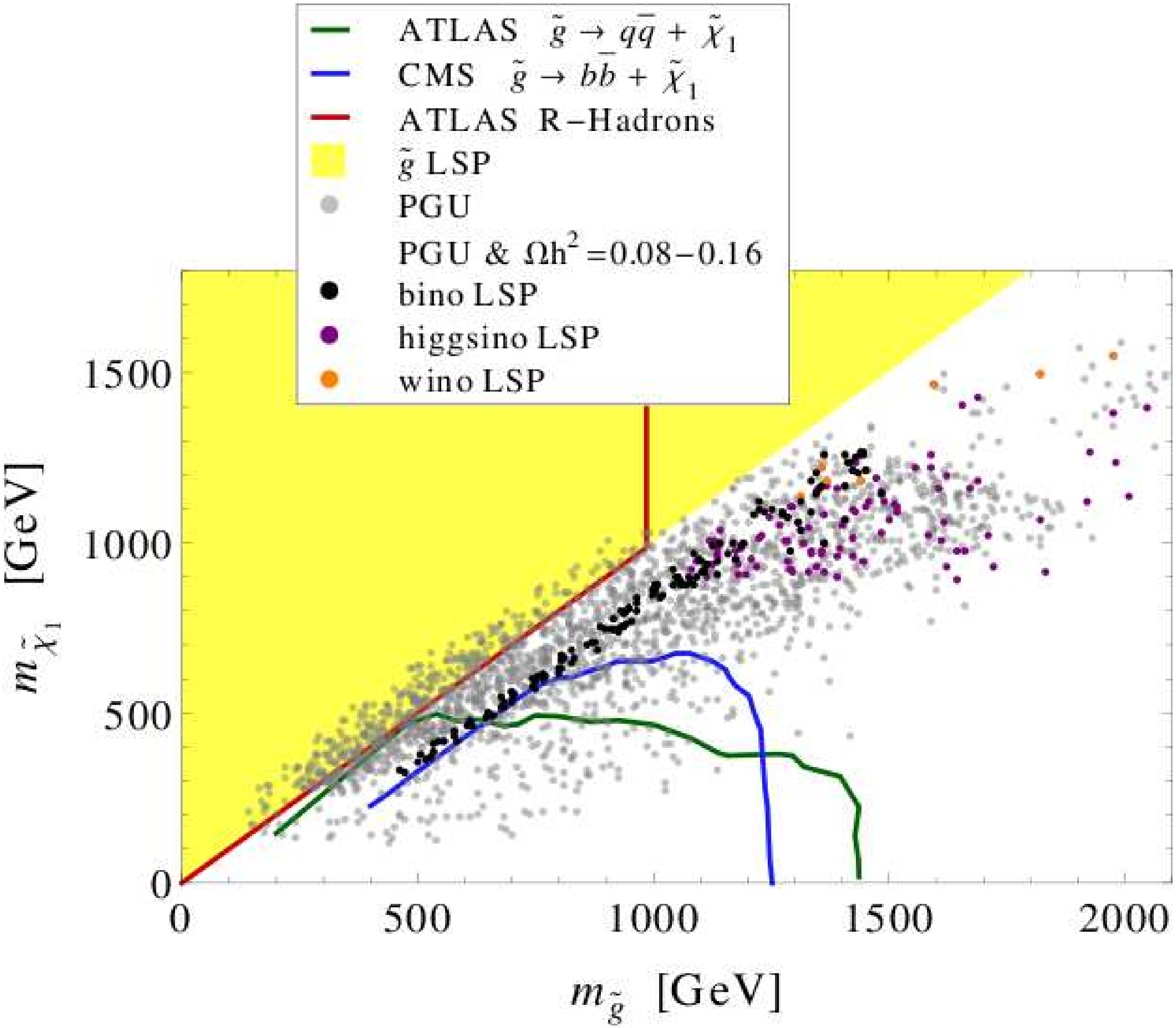
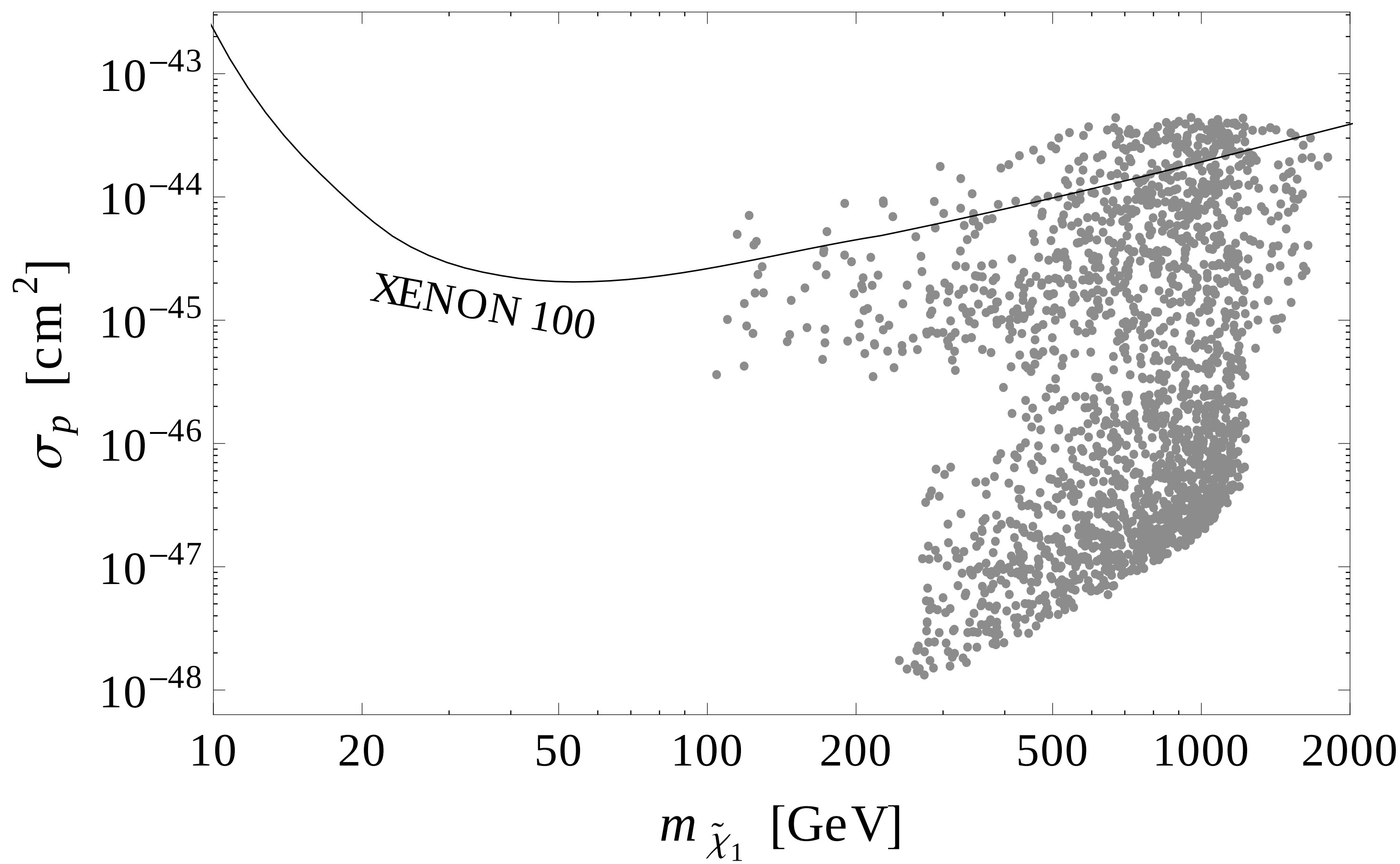


Limits from direct detection



Direct detection experiments might check the scheme.

Conclusions

The quest for unification of fundamental interactions

- requires new physics beyond the Standard Model: like e.g. supersymmetry and extra dimensions

Basic questions could be answered

- family as a 16-dim spinor of $SO(10)$
- $SO(10)$ as the grand-grand daughter of E_8
- extra dimensions explain repetition of families

Consequences:

- we need new experimental input to test the ideas!

The LHC shows us where to go