SYMMETRIES have played a major role in particle physics

Gauge symmetries
(Spin 1 particles)

Chiral symmetries
(Spin ½ particles)

can "protect" masses

SUSY could play this role for Spin 0 particles (Higgs bosons) but even more:

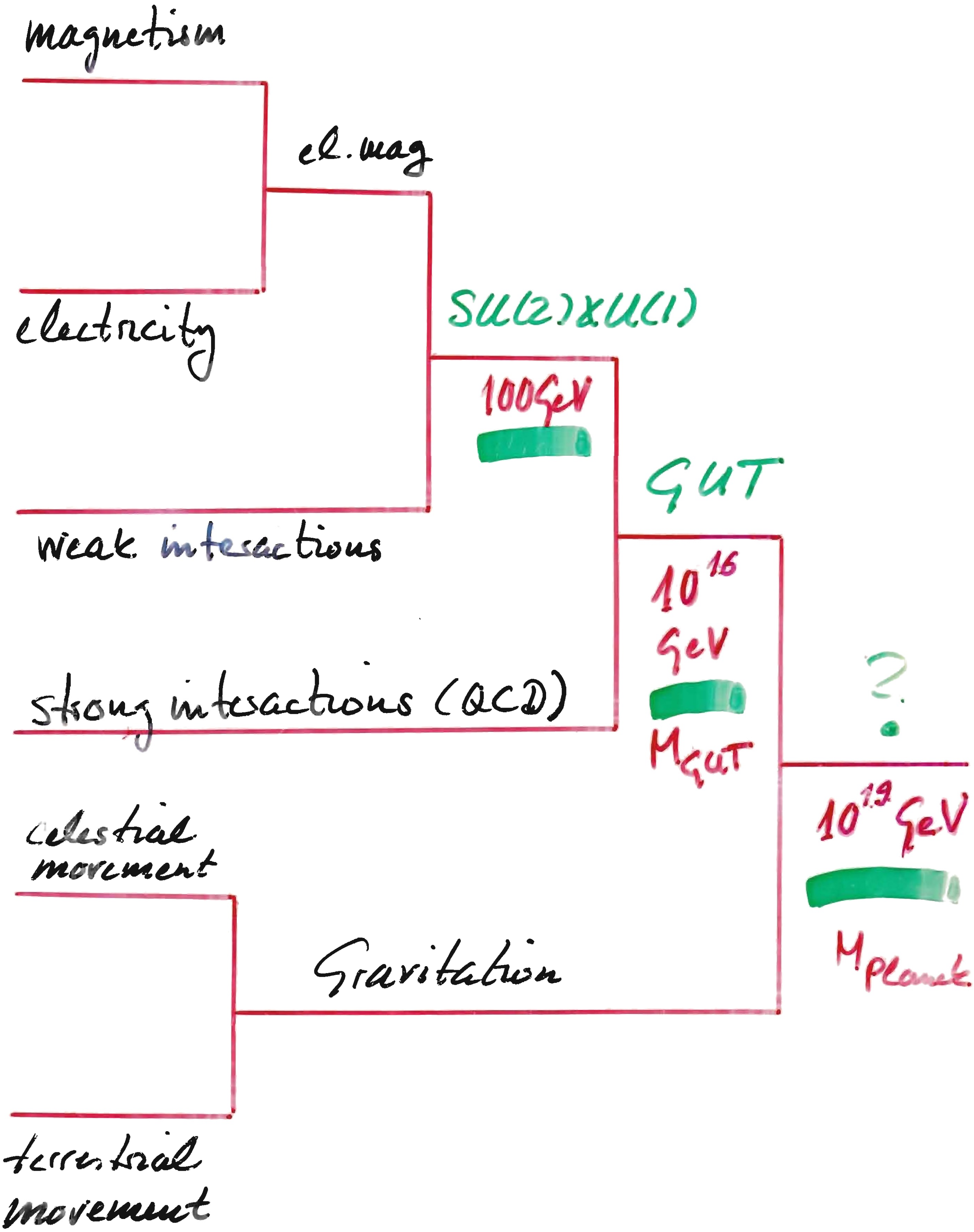
SUSY gauge symmetry

≡ SUPERGRAVITY

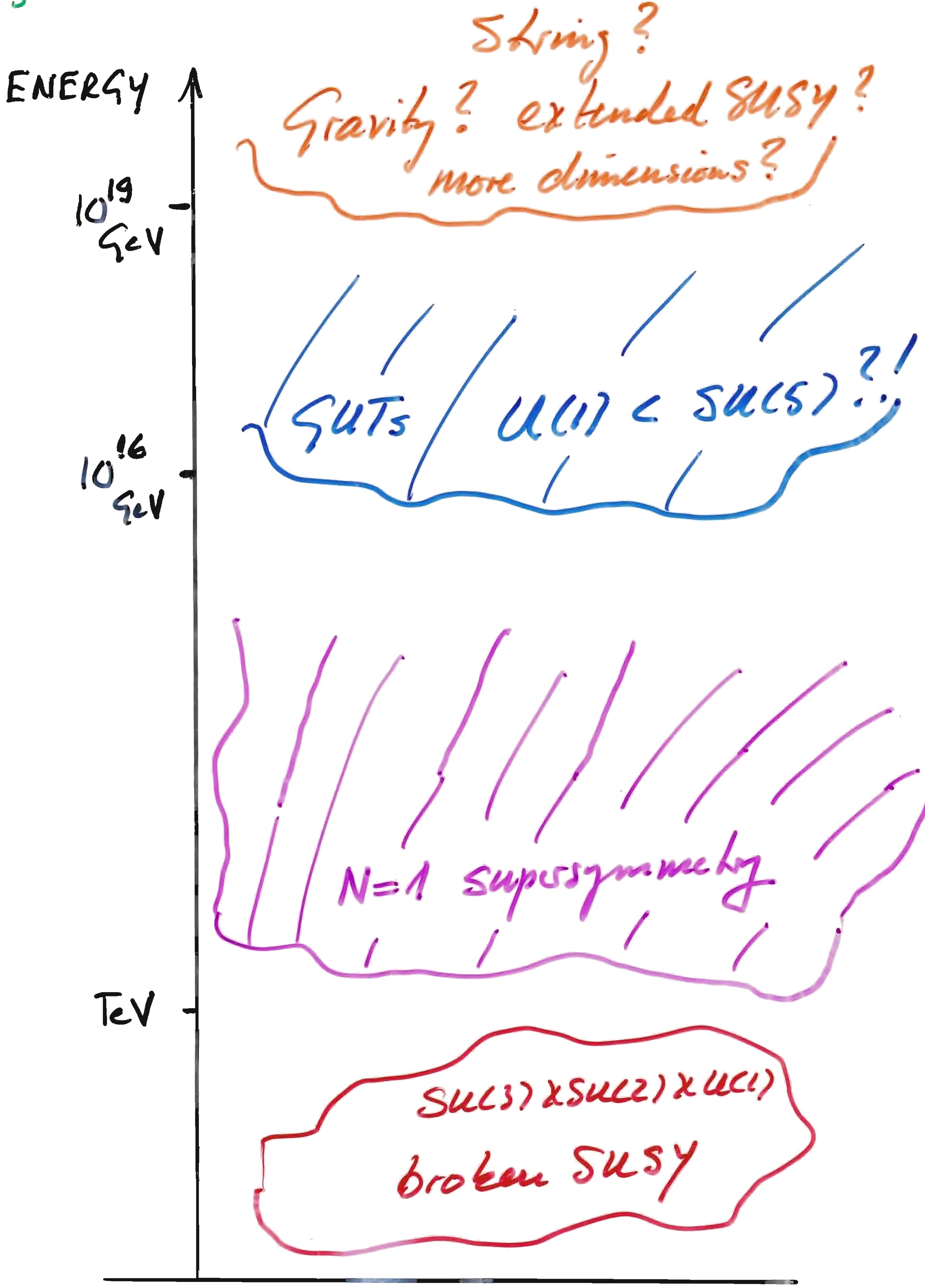
contains gravity
automatically!

Spin 3/2 gauge particle
(gravitino)

Are all interactions based
on a gauge principle?



magnetism
el. mag
electricity
SU(2)xU(1)
100 GeV
weak interactions
GUT
10^{16} GeV
M_{GUT}
strong interactions (QCD)
?
10^{19} GeV
M_{Planck}
celestrial movement
Gravitation
terrestrial movement



symmetry restoration at high energies?

4.4 STANDARD MODEL

3 "FAMILIES" OF

QUARKS $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$

and LEPTONS $\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$

\+ GAUGE INTERACTIONS

$$SU(3) \times SU(2) \times U(1)$$

8 GLUONS → $SU(3)$

W^+, W^-, W^0 → $SU(2)$

B^0 → $U(1)$

W^0, B^0 → Z, γ

\+ SCALAR

"HIGGS BOSON" $\begin{pmatrix} h^0 \\ h^- \end{pmatrix}$

QUARKS AND LEPTONS

$$\mathcal{U} = \begin{pmatrix} u \\ d \end{pmatrix} \hat{=} (3, 2, 1/6)$$

$$\bar{u} \hat{=} (\bar{3}, 1, -2/3)$$

$$\bar{d} \hat{=} (\bar{3}, 1, +1/3)$$

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \hat{=} (1, 2, -1/2)$$

$$\bar{e} \hat{=} (1, 1, 1)$$

electric charge $Q = T_3 + Y$

3 Families

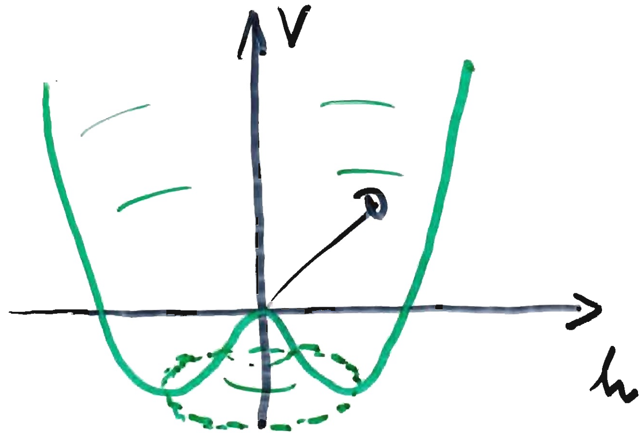
ABSENCE OF ANOMALIES

$$\sum_i Y_i = 0 \; ; \; \sum_i Y_i^3 = 0 \quad \text{etc.}$$

HIGGS POTENTIAL

$$V = \mu^2 h h^\dagger + \lambda (h h^\dagger)^2$$

$\mu^2 < 0 \longrightarrow$



$$\langle h \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \qquad v^2 = -\mu^2 / \lambda$$

$v \approx 250$ GeV

$$M_{W^\pm} = \frac{1}{2} g_2 v \qquad M_\gamma = 0$$

$$M_Z = \frac{1}{2} v \sqrt{g_1^2 + g_2^2}$$

i.e $SU(2)_L \times U(1)_Y \longrightarrow U(1)_Q$

$$e = g_2 \sin\theta_W = g_1 \cos\theta_W$$

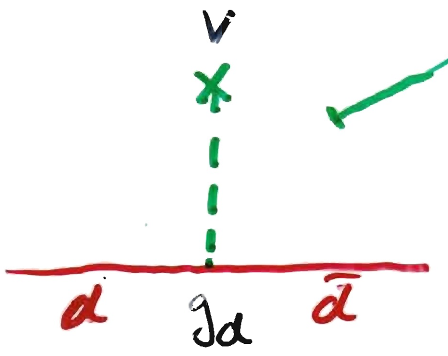
YUKAWA COUPLINGS

$$L_y = g_d U h \bar{d} + g_e L h \bar{e} + g_u U h^{\dagger} \bar{u}$$

LEAD TO MASSES FOR QUARKS AND LEPTONS

EXAMPLE:

$$g_d U h \bar{d} = g_d (u h^- \bar{d} + d h^0 \bar{d})$$



$$\Rightarrow \underbrace{g_d v}_{m_u} \bar{d} d$$

IN FACT: $(g_d)_{ij}$ $\quad i,j = 1,2,3$

is MATRIX

$\Rightarrow$ CKM MIXINGS

YUKAWA COUPLINGS

$$\mathcal{L}_y = g_d U h \bar{d} + g_e L h \bar{e} + g_u U h^{\dagger} \bar{u}$$

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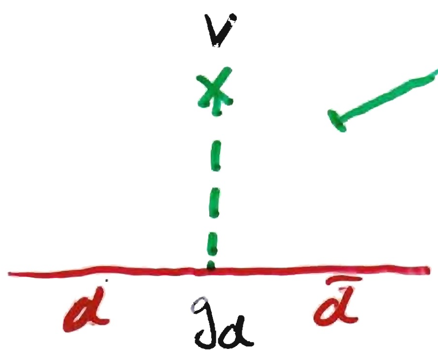


$$\Rightarrow \underbrace{g_d v}_{m_u} \bar{d} d$$

IN FACT: $(g_d)_{ij}$ $i,j = 1,2,3$

is MATRIX

$\Rightarrow$ CKM MIXINGS

PARAMETERS OF STANDARD MODEL

GAUGE SECTOR: g_1, g_2, g_3

$(\alpha, \alpha_{strong}, \sin\theta_W)$

Θ angle (STRONG CP)

HIGGS SECTOR: μ^2, λ

$(v^2 = -\mu^2/\lambda \simeq 250\ \text{GeV};\ m_h^2 \simeq -2\mu^2)$

FERMION MASSES: $m_{u,d,s,c,b,t}$

$m_{e,\mu,\tau}$

MIXINGS: 3 angles, 1 phase

"Accidental" SYMMETRIES

Baryon and Lepton numbers

Beyond the SM. Why?

* deviation in $(g-2)_\mu$
* Candidate for Dark Matter
* Baryon asymmetry
* neutrino masses
* neutrino see-saw + scale
* unification of couplings

Meaningful UV-completion

* quadratic divergencies
* Landau poles

STANDARD MODEL IS

VERY SUCCESSFUL

BUT DOES NOT SEEM TO BE

THE COMPLETE STORY

ALL MASSES COME FROM

$v \sim \langle h \rangle \sim \sqrt{-\mu^2/\lambda}$

BUT WHERE DOES v COME FROM

RADIATIVE CORRECTION TO

HIGGS MASS

$$\delta m^2 = \lambda \int d^4q \, \frac{1}{q^2 - m^2} \approx \lambda q^2 \Big|_0^\infty$$

WE DO NOT UNDESTAND

V IS OF ORDER OF 100 GeV

RATHER THAN $M_{Planck} \approx 10^{19}$ GEV

(HIERARCHY PROBLEM)

POSSIBLE SOLUTIONS:

* NO FUNDAMENTAL SCALARS
* SUPERSYMMETRY

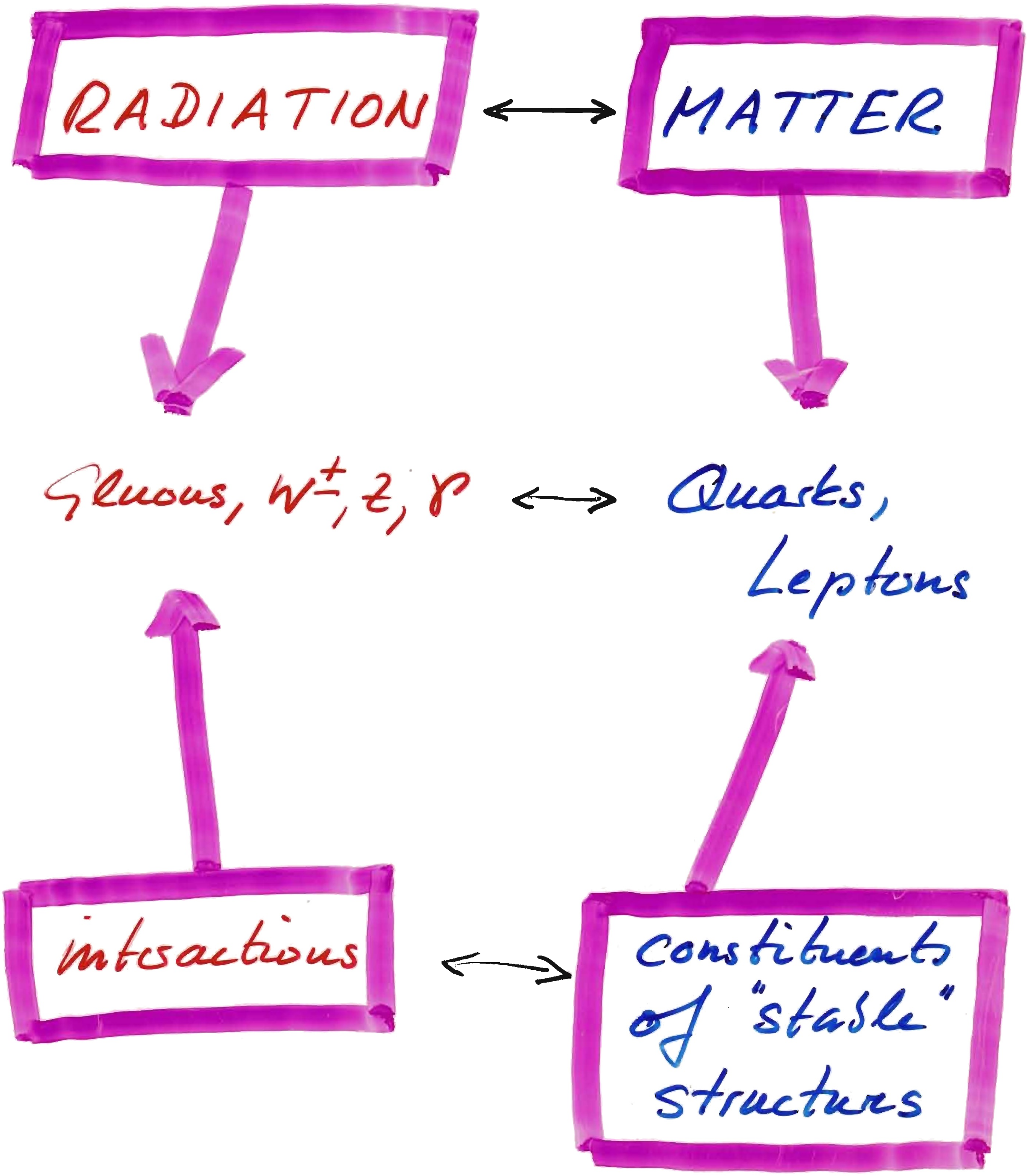
$$\delta m^2 \approx \lambda (m_B^2 - m_F^2) \equiv 0$$

BROKEN SUSY:

$$\delta m^2 \approx \lambda M_{susy}^2 \qquad M_{susy} \sim TeV$$

SUPERSYMMETRY = SUSY

relates **bosons** to **fermions**



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SUPERSYMMETRY

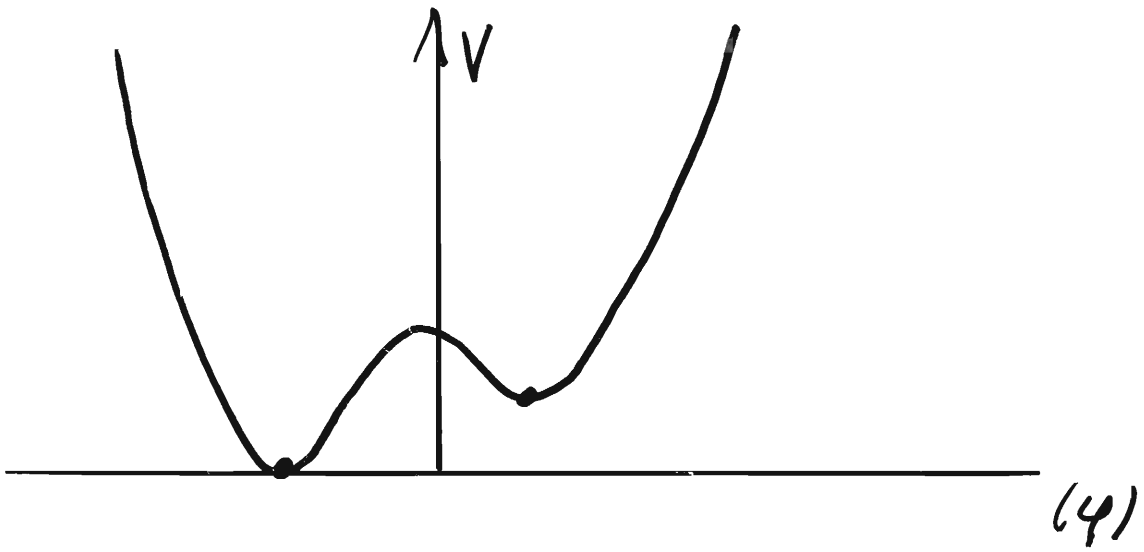
Bosons $\longleftrightarrow$ Fermions

$$Q_\alpha \,(\text{Boson}) \doteq (\text{Fermion})_\alpha$$

$$\{ Q^+_\alpha , Q_\beta \} = \gamma^\mu_{\alpha\beta} P_\mu$$

$P_0 = H =$ Hamiltonian

$\longrightarrow E_{vacuum} \geq 0$



Multiplets: $Q_\alpha \hat{=}$ "Spin 1/2"

→ adjacent spin

chiral multiplet

$(\varphi \,,\, \psi)$

spin 0   spin 1/2 ← (example: electron)

Vector multiplet

$(\lambda \,,\, \gamma)$

spin 1/2   Spin 1 ← (example: photon)

this is $N=1$ supersymmetry

extended SUSY $N > 1$
gives bigger multiplets

$N = 4$ SUSY (finite)

$N = 8$ supergravity

are mathematically much more beautiful.

but cannot describe our (low energy) phenomena!

- no chiral fermions

- no unbroken asymptotically free nonabelian gauge groups

even $N=1$ broken

$Q_\alpha B = F_\alpha$ and $[Q_\alpha, H] = 0$

multiplets are degenerate in mass

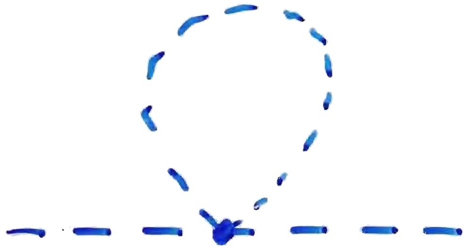
Where is (bosonic) partner of the electron?

$\Delta = m_{Boson} - m_{Fermion} = M_{susy}$

is magnitude of susy breakdown

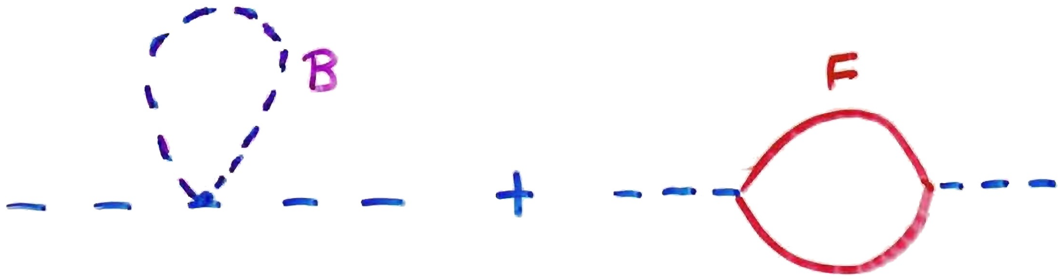
Size of M_{susy}?

remember in standard model the hierarchy problem



$$\delta\mu^2 = \lambda \int_0^{\Lambda^2} dq^2$$

in SUSY:



cancel exactly $\delta\mu^2 = 0$!

(nonrenormalization theorems)

cancellation exact if

$$\text{mass}_B = \text{mass}_F \quad !$$

in broken susy

$$\delta\mu^2 \approx \lambda \left(mass_B^2 - mass_F^2 \right)$$

$$\sim \lambda\, M_{susy}^2$$

Since the mass of the Higgs boson should be stabilized in the 100 – 1000 GeV region

we therefore conclude that susy "solves the hierarchy problem" provided

$$M_{susy} \sim O(\text{TeV})$$

spectrum of MSSM

gauge bosons in vector multiplets
(spin 1, spin 1/2)

gauge bosons ↔ gauge fermions (gauginos)

in adjoint of $SU(3) \times SU(2) \times U(1)$

quarks and leptons in
chiral multiplets (spin 1/2, spin 0)

Partner of quarks (squarks)
and leptons (sleptons) are
spin-0 scalar bosons

Higgs boson

$$h = \begin{pmatrix} h^0 \\ h^- \end{pmatrix} \hat{=} (1, 2, -1/2)$$

18 has same $SU(3)\times SU(2)\times U(1)$ quantum numbers as lepton doublet

$\bar{h}$ = selectron ??

unfortunately NO!

need fermionic partner (higgsino) to fill chiral supermultiplet

chiral fermion $(1, 2, -1/2)$

$\longrightarrow$ gauge anomaly

need 2^{nd} Higgs multiplet

$$\bar{h} = \begin{pmatrix} \bar{h}^+ \\ h^0 \end{pmatrix}$$

chiral supermultiplet $(1, \bar{2}, +1/2)$

$(\frac{1}{2}, 1)$ vector multiplets

gauge bosons	gauginos
g, W, Z, γ	$\tilde{g}, \tilde{W}, \tilde{Z}, \tilde{\gamma}$

$(0, 1/2)$ chiral multiplets

quarks, leptons	scalar partners
q, ℓ	$\tilde{q}, \tilde{\ell}$

Higgs fermion	Higgs boson
$\begin{pmatrix} \tilde{h}^0 \\ \tilde{h}^- \end{pmatrix}$	$h = \begin{pmatrix} h^0 \\ h^- \end{pmatrix}$
2nd Higgs fermion	**2nd Higgs boson**
$\begin{pmatrix} \tilde{\bar{h}}^- \\ \tilde{\bar{h}}^0 \end{pmatrix}$	$\bar{h} = \begin{pmatrix} \bar{h}^- \\ \bar{h}^0 \end{pmatrix}$

The superpotential (W)

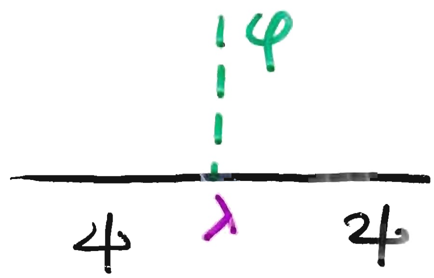
is a product of chiral multiplets

example $W = m\phi^2 + \lambda\phi^3$

with $\phi = (\varphi, \psi)$

gives mass term $m\psi\psi$

and Yukawa coupling $\lambda\varphi\psi\psi$



in SM we have Yukawa coupling

$$h^0 e \bar{e} \longrightarrow \boxed{H L \bar{E}}$$

$$W = \mu H \bar{H} + g_E L H \bar{E} + g_D Q H \bar{D} + g_u Q \bar{H} \bar{u} + Q L \bar{D} + L \bar{E} L + \bar{u} \bar{D} \bar{D}$$

need two Higgs supsfields
for quark and lepton masses

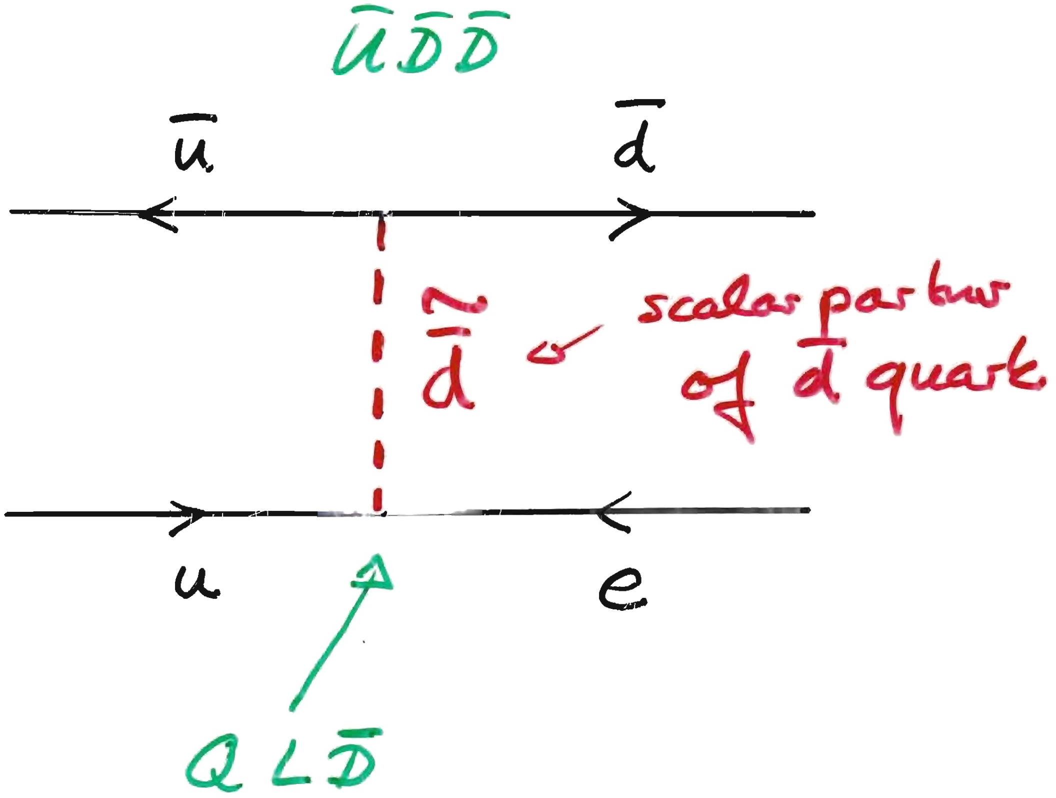
More terms than needed!

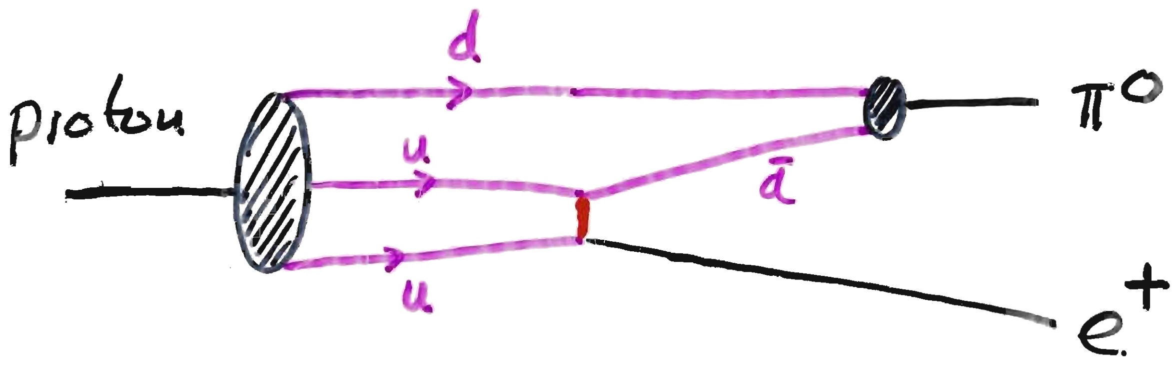
This is in contrast to standard model

where Baryon number and
Lepton number conservation
was automatic

(i.e consequence of other symmetries
+ renormalizability)

here L and H have the same quantum numbers!





proton unstable!

new symmetry

R – PARITY

R (particle) = +1

R (new SUSY partner) = −1

$h c \bar{e}$ allowed

$\bar{u} \bar{d} \tilde{\bar{d}}$ forbidden

Consequences:

- new SUSY particles only produced in pairs

- new stable particle

Lightest Supersymmetric Particle

(cold dark matter)

One model-independent statement

mass of lightest Higgs boson
is bounded from above

$$m_h \lesssim 150\,\text{GeV}$$

no "light" Higgs boson
$\longrightarrow$ no SUSY

at least in MSSM

(in SM $\quad v \sim \sqrt{-\mu^2/\lambda}$
in SUSY $\quad \lambda \sim g^2$)

how heavy are LSP and other SUSY-partners?

$M_{susy} = \mathcal{O}(\text{TeV})$

but SUSY partners could in principle be lighter

model building: **MSSM**

We need a consistent model with broken N=1 supersymmetry!

partner masses are strongly model dependent!

LSP could be a few GeV

but also a few × 10^2 GeV

SUSY GUTs

simplest case:

$$SU(5) \supset SU(3) \times SU(2) \times U(1)$$

↳ 24 gauge bosons

$$(8,1) + (1,3) + (1,1)$$

gluons → $(8,1)$; $W^\pm Z \gamma$ → $(1,3)$, $(1,1)$

$$+ (3,2) + (\bar{3},\bar{2})$$

mass of order $M_{GUT} \approx 10^{16}\,\mathrm{GeV}$

quarks and leptons:

$$\bar{5} \mathrel{\hat{=}} \begin{pmatrix} \bar{d} \\ \bar{d} \\ \bar{d} \\ e \\ \nu \end{pmatrix} \qquad 10 \mathrel{\hat{=}} \begin{pmatrix} 0 & \bar{u} & \bar{u} & u & d \\ & 0 & \bar{u} & u & d \\ & & 0 & u & d \\ & & & 0 & \bar{e} \\ & & & & 0 \end{pmatrix}$$

H_1 and H_2 in $5, \bar{5}$ representation

Yukawa couplings

$g_E\, L H \bar{E} + g_D\, Q H \bar{D} + g_u\, Q \bar{H} \bar{U}$

in $SU(3) \times SU(2) \times U(1)$ model

in $SU(5)$ only **two** !

$\bar{5} \cdot 10 \cdot H_2 \;+\; 10 \cdot 10 \cdot H_1$

corresponds to $L H \bar{E}$ and $Q H \bar{D}$ (the $\bar{5} \cdot 10 \cdot H_2$ term)

corresponds to $g_u Q \bar{H} \bar{U}$ (the $10 \cdot 10 \cdot H_1$ term)

applied to the 3rd family this implies

$m_{\text{bottom}} = m_{\text{tau}}$ at M_{GUT}

(Yukawa coupling unification)

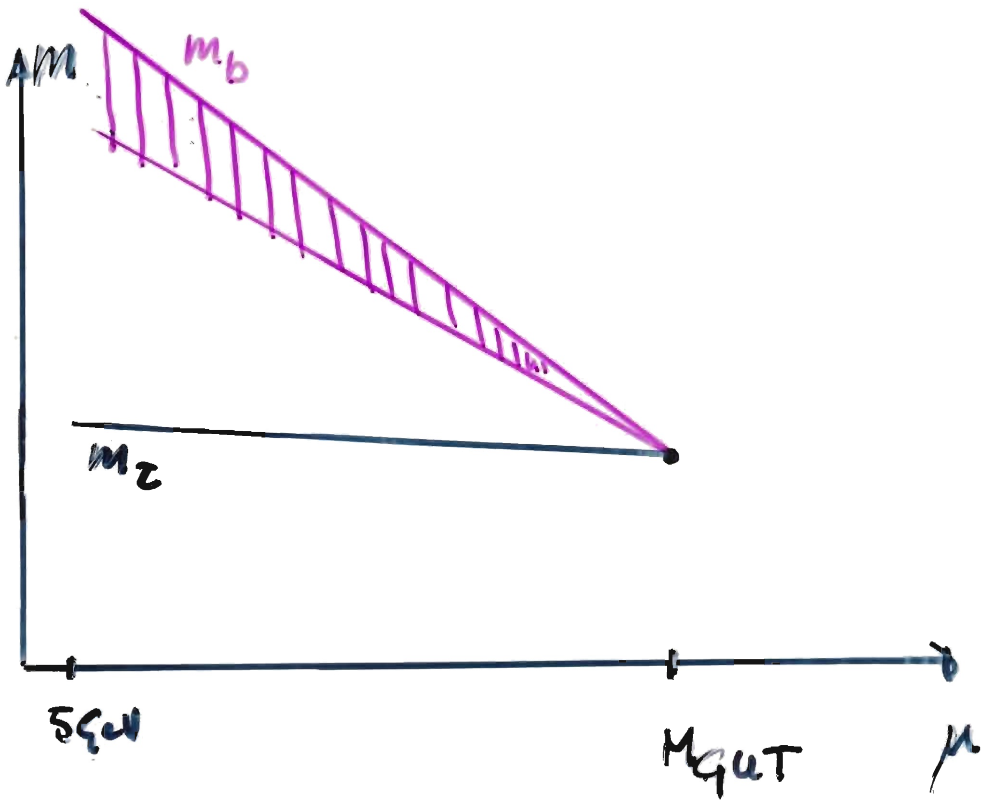
evolution of Yukawa couplings

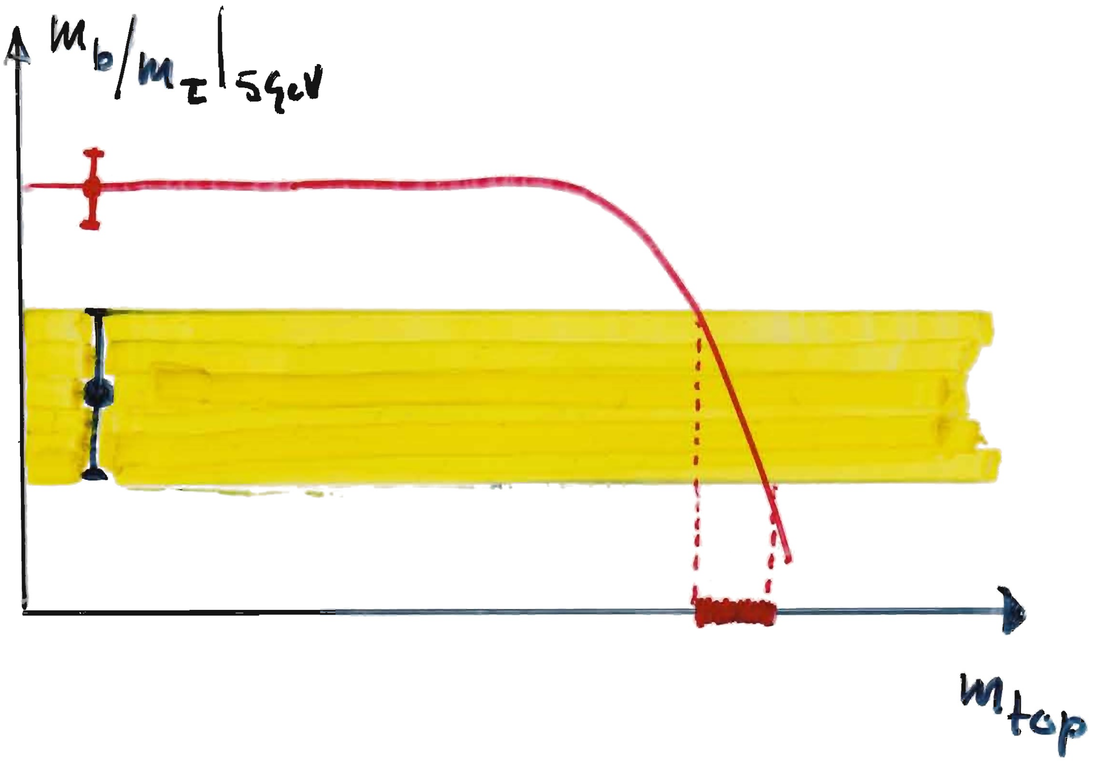
$$\mu \frac{\partial g_\tau}{\partial \mu} = -\frac{1}{8\pi^2} g_\tau \left(\frac{3}{2} g_2^2 + \frac{3}{2} g_1^2 \right)$$

$$\mu \frac{\partial g_b}{\partial \mu} = -\frac{1}{8\pi^2} g_b \left(\frac{8}{3} g_3^2 + \frac{3}{2} g_2^2 + \frac{7}{18} g_1^2 \right) +$$

$$+ \frac{1}{16\pi^2} g_b \, g_{top}^2$$

- $m_b > m_\tau$ at low energy due to strong interactions
- dependence on g_{top} (i.e. m_{top}) for large g_{top} (i.e $g_{top} \sim g_3$)

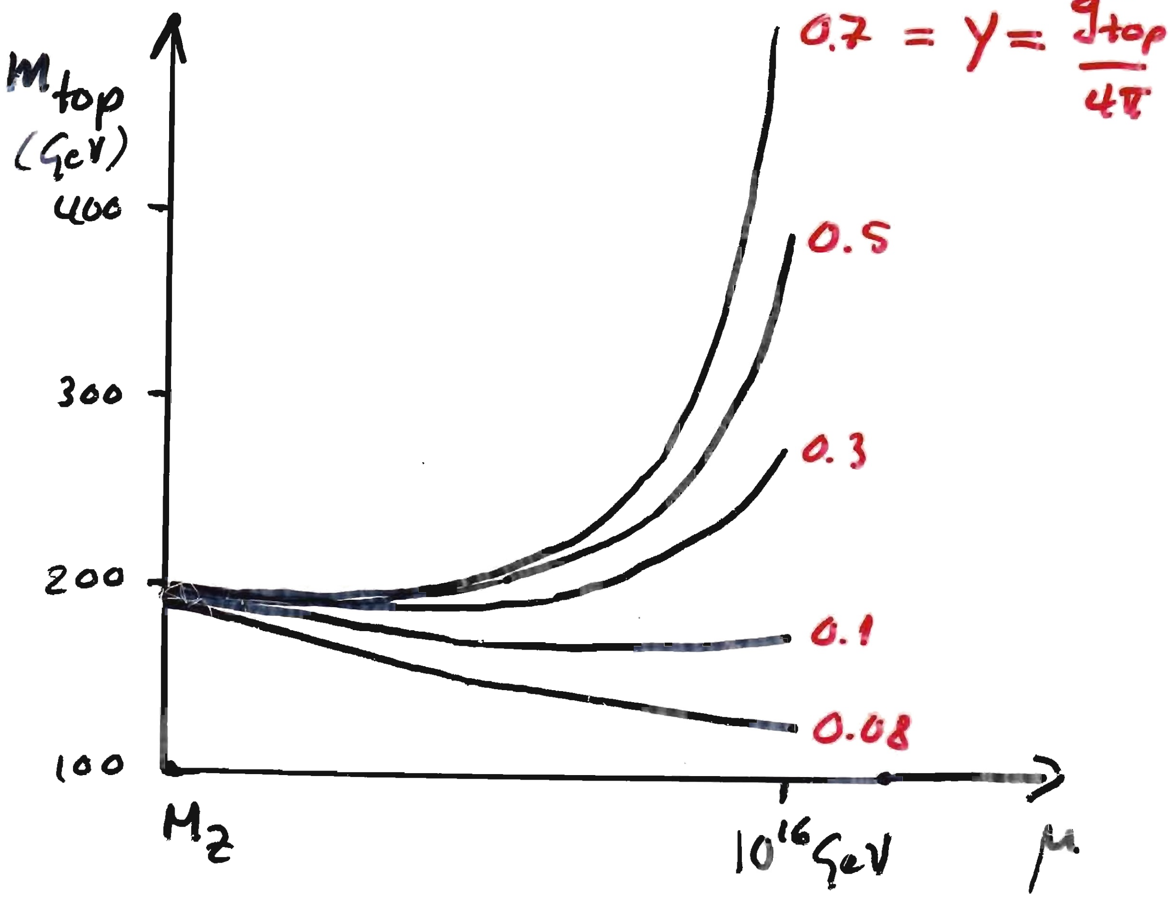




$$160\ \mathrm{GeV} \leq m_{top} \leq 180\ \mathrm{GeV}$$

evolution of m_{top}

$$\mu \frac{\partial g_{top}}{\partial \mu} \approx \frac{1}{8\pi^2} g_{top} \left(3 g_{top}^2 - \frac{8}{3} g_3^2 \right)$$



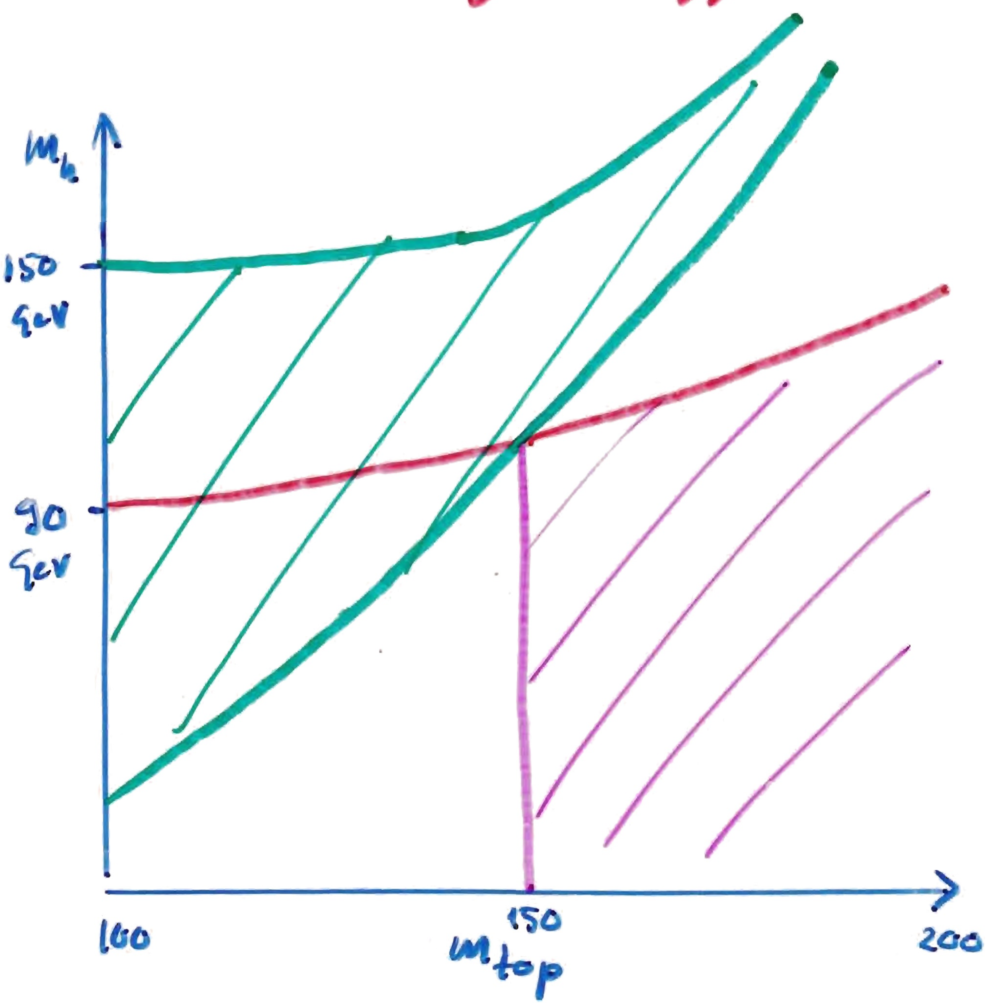
leads to (quasi) - infrared - fixed point

160 GeV ≤ m_{top} ≤ 200 GeV

SUSY-GUT seems to require

heavy top quark

and light Higgs-boson



$m_{top} \gtrsim 150$ GeV is a consequence of

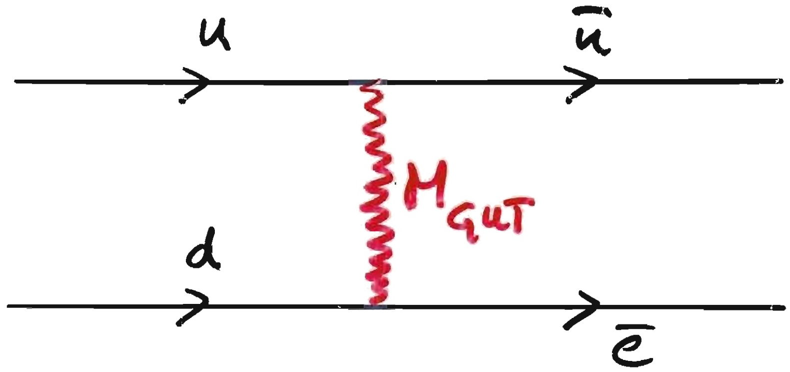
$b-\tau$ unification

Proton decay

Experiment:

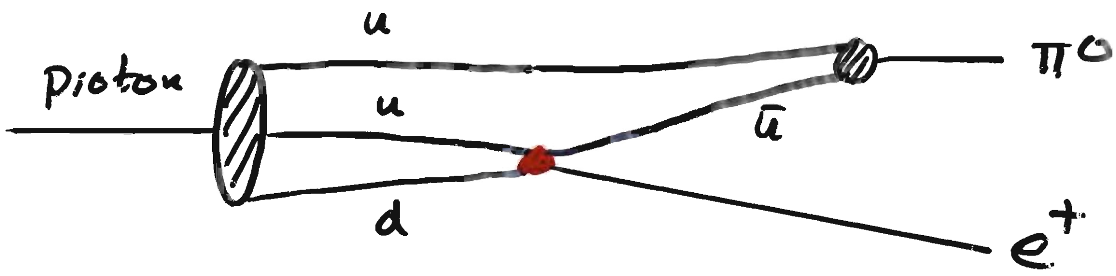
$p \longrightarrow e^+ \pi^0 \qquad \tau > 10^{33}\,\text{yr}$

$p \longrightarrow K^+ \bar{\nu} \qquad \tau > 10^{32}\,\text{yr}$



$\sim \frac{1}{M_{GUT}^2}$

(dimension 6 operator)

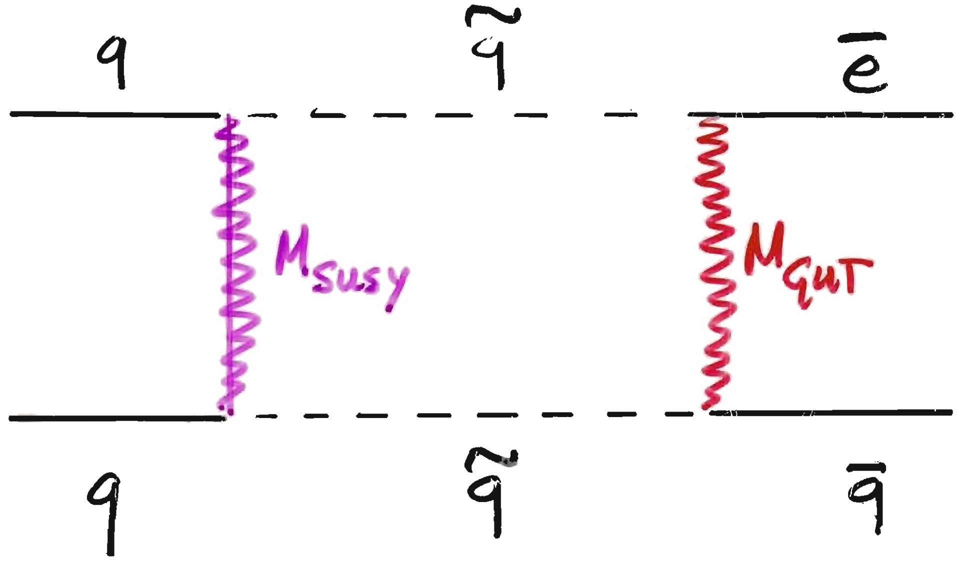


(GUT: $\tau \sim 10^{31 \pm 1}\,\text{yr}$)

SUSY-GUT $\tau \sim 10^{38 \pm 1}\,\text{yr}$

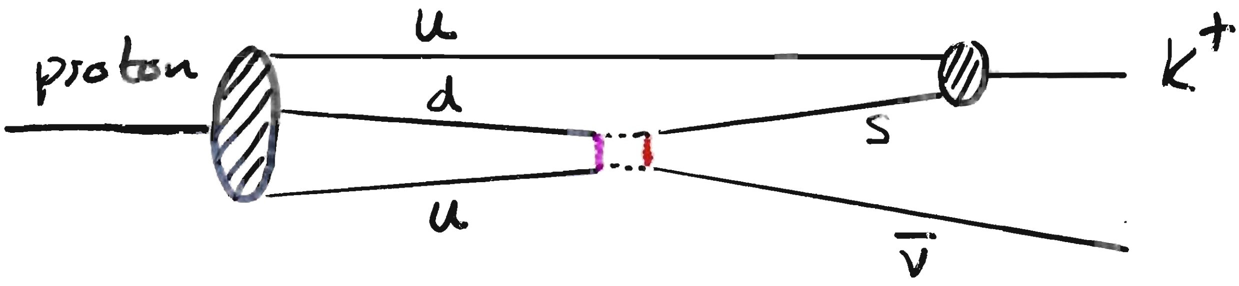
new mechanism in SUSY-GUT

(dim. 5-operator)



$$\sim \frac{1}{M_{SUSY} M_{GUT}}$$

dominant process:



rate is model dependent:

in simplest case

$$\tau_{p \to K^+ \bar{\nu}} \sim 10^{29 \pm 4}\ yr$$

GUTs

* need SUSY
* gauge and (partial) Yukawa unif.
* large m_{top}
* no "Landau pole"

More good things

* SO(10), E_6 .....
* neutrino see-saw

Some "problems"

* breakdown of GUT group (large representations)
* doublet-triplet splitting
* proton stability

Neutrino See-Saw

$$16 \longrightarrow 10 + \bar{5} + 1$$

$\bar{5}$: contains ν_L $\qquad$ 1: ν_R

Dirac mass: $\bar{\nu}_L \, \nu_R$

Majorana mass: $\bar{\nu}_L \, \nu_L$ (breaks L)

<u>mass matrix</u>

$$\begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}$$

<u>eigenvalues:</u> $M \,, \quad m_D^2 / M$

(if $m_D \ll M$)

$$m_D^2 = (100\,\text{GeV})^2 \qquad M = 10^{16}\,\text{GeV}$$

$$m = m_D^2/M \sim 10^{-12}\,\text{GeV} = 10^{-3}\,\text{eV}$$