# Stability of non-BPS Orientifolds

von

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Ich versichere, daß ich diese Arbeit selbständig verfaßt und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt sowie die Zitate kenntlich gemacht habe.

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Korreferent: Prof. Rainald Flume

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# Contents

# 1. Introduction

One major aim of theoretical physics is to formulate a theory in which all fundamental forces can be explained in a unified way. Three of the four known fundamental interactions are described by quantum field theories. They are consistently formulated as non-abelian gauge field theories in the "standard model of elementary particle physics"(SM) (see for example [11] for an introduction and references therein). Although the standard model has some short comings which are considered to be fairly severe, for instance an unexplained hierarchy of scales, a complete lack of understanding of the cosmological constant, and the origin of the input parameters, it describes the known elementary particles and interactions with high precision. In order to solve some problems of the SM, supersymmetric extensions were proposed, although presently no supersymmetric partner of any known particle has been observed (for a review on supersymmetric models see [30]). Only the fourth known fundamental force, gravity, cannot be described as a quantum field theory in four dimensions.

String theory is the most promising theory for a unified description of all fundamental interactions (for reviews and references to the original literature see [22, 23, 37]). At the moment string theory seems to be the only scheme to quantise gravity consistently. The quantisation of gravity is inevitable if we want to explain gravity within the same framework as the theories of the other known fundamental interactions. The advantage of string theory is the fact that the fundamental constituents are not longer seen as point particles, as it is the case in quantum field theories, but as one-dimensional extended objects. This solves a lot of problems which arise, when we try to quantise gravity. If we try to quantise gravity in a conventional scheme, divergencies arise which cannot be absorbed by any renormalisation procedure. However, a lot of new problems are forced upon us, if we want to use string theory as the fundamental theory of nature. They arise, surprisingly, by the enormous amount of symmetry string theories possess. The most obvious difficulty we encounter is that string theory can only be consistently defined in a critical spacetime dimension. The critical dimension is determined by cancellation of the conformal anomaly on the worldsheet of the string. Only in the critical spacetime dimension it is possible to define string interactions consistently. The critical spacetime dimension for the bosonic string is 26, while for the superstring the critical dimension is ten. Because a purely bosonic theory is not a promising candidate for a fundamental theory of nature (all matter fields which are observed presently are fermionic), we will mainly consider superstring theories in what is to follow.

There exist five consistent superstring theories, namely the type I, type IIA, type IIB and the heterotic SO(32) and $E_8 \times E_8$ theories. They are connected by a web of dualities and are thought of as different limits of an underlying eleven-dimensional theory, often called "M-theory" [41, 53] or "modern string theory". In this thesis we will exclusively deal with type II and type I theories. The type II theories have $\mathcal{N}=2$ supersymmetry in ten dimensions which amounts to $\mathcal{N}=8$ supersymmetry after dimensional reduction to four dimensions. Such a large amount of supersymmetry is experimentally ruled out. Therefore, if string theory should be a promising candidate for a unifying theory of the observed physical phenomena we have to reduce the number of visible dimensions and we have to break supersymmetry by some mechanism. The type I theory and the heterotic theories have $\mathcal{N}$=1 supersymmetry in D=10 dimensions and therefore after dimensional reduction $\mathcal{N}$=4 SUSY in D=4. This is still too much. The path which is most often used is to compactify string theories with $\mathcal{N}$=1 supersymmetry in D=10 on a six-dimensional manifold with SU(3) holonomy which breaks supersymmetry down to $\mathcal{N}$=1 in D=4 [23]. These manifolds are called *Calabi-Yau threefolds*. Often so-called *orbifolds* are considered, as well [15, 16, 24, 31]. Orbifolds are spaces with points of singular curvature. They can be viewed of as limits of (non-singular) Calabi-Yau manifolds. We use these singular limits because string theories are mathematically better tractable on orbifolds than on general Calabi-Yau spaces.

In type I and type II theories exist $p$-dimensional extended objects, called *Dp-branes* ($p = -1 \ldots 9$)[1], which are characterised by the fact that open strings can end on them. Dp-branes are charged under Ramond-Ramond potentials. If we consider only Dp-branes of the same dimension with positive charge, they form BPS-states in the string theory and conserve half of the supersymmetry. On the world volume of the D-branes the open strings give rise to a gauge theory. With the appropriate grouping of D-branes it is in principle possible to incorporate the standard model or some GUT gauge group in this gauge theory. However, in supersymmetric D-brane models we still need some supersymmetry breaking mechanism because the branes give rise to $\mathcal{N}$=4 supersymmetric field theories in D=4. In order to reduce the amount of supersymmetry, D-branes are often placed in an orbifold background. A lot of literature on this approach to string phenomenology does exist (see for example [2, 4, 17] and references therein).

However, in this thesis we will investigate another promising strategy. If we introduce D-branes with opposite R-R charges, so called anti-D-branes or $\overline{\mathrm{D}}$-branes, the supersymmetry on the brane anti-brane system is broken completely. Hence, this construction admits to break supersymmetry already at the string scale in the gauge sector, while supersymmetry in the gravity sector, which resides in the closed string part of the spectrum, is conserved. Usually, these constructions are combined with orbifolds to break supersymmetry in the gravity sector to $\mathcal{N}$=1 in D=4. In most of the considered models it is possible to enlarge the compactification radii such that the hierarchy problem does not appear, as the string scale can be lowered appropriately (this way of solving the hierarchy problem was proposed in [7]). However, the drawback of this construction is that D-$\overline{\mathrm{D}}$-systems are unstable and tend to annihilate into the supersymmetric vacuum [48]. Therefore, to render these theories

[1] The $-1$-brane is an instanton.

consistent we have to find mechanisms to stabilise systems with branes and anti-branes (see for example [49, 28, 19, 42, 6]).

In this thesis we investigate the stability of different orientifold models with branes and anti-branes. Firstly, we introduce D-branes in chapter 2 and to exemplify symmetry breaking in a supersymmetric setting, we comment on gauge symmetry breaking if we move apart stacks of branes. We explain the fact that this mechanism of supersymmetric gauge symmetry breaking is not possible in D-$\overline{\text{D}}$-systems. In chapter 3 we will introduce the notion of orbifolds and orientifolds and exemplify the orientifold construction on the easiest model there is, namely the type I string theory, seen as an orientifold of type IIB. In chapter 4 we will discuss various orientifold models with branes and anti-branes. We consider three cases of orientifold models. The first case is a model which is connected to a supersymmetric setting by addition of brane anti-brane pairs in a consistent manner. It will turn out to be unstable. We derive the spectra of different settings of branes and anti-branes and consider the movement of the branes in the compact directions. This model was introduced in [5] but only comments on the instability were made. The second model which we discuss was introduced in [5], as well. It possesses branes and anti-branes stuck at orientifold fixed points. We calculate the spectrum of the model. In order to see if it is possible to vary the shape of the compactification tori, the moduli stemming from the closed string sector are investigated. The relevant moduli to decrease the radii of the tori are found to be present in the spectrum, therefore it is possible for the fixed points to approach each other and the model can develop tachyons between branes and anti-branes and thus show instabilities. This analysis has not been done in the literature. The remaining two models are non-supersymmetric and stable but they can undergo phase transitions in which the low energy effective field theory on the branes changes drastically. This class of models was introduced in [40] in an orientifold background and first considered in [45, 46, 47]. The phase transitions for one model are recalculated and the phase transitions for the second model are derived. We will see that there are stable non-supersymmetric orientifold models in four dimensions which do not decay but can undergo phase transitions when the compactification radii are varied. Chapter 5 contains our conclusions. To make the text better readable, all longer calculations can be found in the appendix.

# 2. D-Branes and Anti-D-Branes

We will shortly review the notion of D-branes [34, 39, 35, 8, 25] in string theory, focusing mostly on the difference between D-branes and anti-D-branes and their interactions. In order to introduce D-branes via T-duality we first explain toroidal compactification.

## 2.1. Toroidal compactification

Superstring theory is consistent only in ten spacetime dimensions [22, 51]. If superstring theory is the appropriate theory to describe nature, it is necessary to compactify six dimensions. The easiest compactification procedure is toroidal compactification where six dimensions become periodically identified.

First, let us consider the compactification of one dimension, $x^9$ for definiteness. For a scalar field $X$ this implies

$$X^9(\sigma,\tau) \,\widehat{=}\, X^9(\sigma,\tau) + 2\pi R, \tag{2.1}$$

where $0 \leq \sigma \leq 2\pi$ denotes the position along the string and $-\infty \leq \tau \leq \infty$ denotes the time parameter on the string world-sheet. Under the identification of equation (2.1) string states must be single valued, which implies that the translation operator $\exp(2\pi i R p)$ acting on the string must leave the state invariant [37]. $p$ is the momentum operator, the generator of translation. Thus, the eigenvalues $k$ of the momentum operator $p$ must be quantised in the compact direction

$$k^9 = \frac{n}{R}, \qquad n \in \mathbb{Z}. \tag{2.2}$$

This effect is well known from field theory with compact dimensions. The quantised momentum is called the *Kaluza-Klein momentum.* There is an additional effect for the closed string, which now may wind around the compact dimension, that is

$$X^9(\sigma + 2\pi) = X^9(\sigma) + 2\pi R w, \qquad w \in \mathbb{Z}. \tag{2.3}$$

$w$ is called the winding number of the string. These states are known as “winding states”. The mode expansion of the string is not affected by the periodicity condition, however, the mass shell and level matching conditions are altered by the compactification. They now read

$$m^2 = -k^\mu k_\mu = (p_L^9)^2 + \frac{4}{\alpha'}(N - 1/2) \tag{2.4}$$
$$= (p_R^9)^2 + \frac{4}{\alpha'}(\tilde{N} - 1/2),$$

which is equivalent to

$$m^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 1), \tag{2.5}$$
$$0 = nw + N - \tilde{N}, \tag{2.6}$$

where $p_L$ and $p_R$ are the left and right moving momenta

$$p_L^9 = \sqrt{\frac{2}{\alpha'}}\alpha_0 = \frac{n}{R} + \frac{wR}{\alpha'}, \tag{2.7}$$
$$p_R^9 = \sqrt{\frac{2}{\alpha'}}\tilde{\alpha}_0 = \frac{n}{R} - \frac{wR}{\alpha'}. \tag{2.8}$$

and $N$ and $\tilde{N}$ are the occupation numbers from the left and right moving sector of the string.

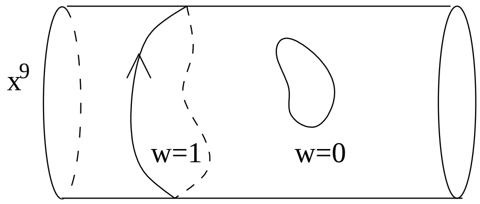


Figure 2.1.: A string with $w = 1$ winding once around the compact dimension and a string with $w = 0$.

To consider the spectrum of the theory, we split up the states in a compactified and an uncompactified part. For an arbitrary radius this gives from the Neveu-Schwarz (or NS) sector

$$\psi^\mu_{-1/2}\tilde{\psi}^\nu_{-1/2}|\,0,k\,\rangle, \tag{2.9}$$
$$(\psi^\mu_{-1/2}\tilde{\psi}^9_{-1/2} + \psi^9_{-1/2}\tilde{\psi}^\mu_{-1/2})|\,0,k\,\rangle, \tag{2.10}$$
$$(\psi^\mu_{-1/2}\tilde{\psi}^9_{-1/2} - \psi^9_{-1/2}\tilde{\psi}^\mu_{-1/2})|\,0,k\,\rangle, \tag{2.11}$$
$$\psi^9_{-1/2}\tilde{\psi}^9_{-1/2}|\,0,k\,\rangle, \tag{2.12}$$

where $\psi^I_{-1/2}, I = 0,\ldots,9$ denotes the oscillators of the left moving NS sector and $\tilde{\psi}^I_{-1/2}$ denotes the oscillators from the right moving NS sector.

The first mode (2.9) provides the 9-dimensional graviton, the dilaton and the 9-dimensional anti-symmetric tensor. The second mode, (2.10), transforms as a vector under the Lorentz group of the uncompactified dimension. In field theory, this is known as the Kaluza-Klein vector. The third part, (2.11), is a vector from the antisymmetric tensor while the last one, (2.12), transforms as a scalar, which is the modulus for the radius of the compactified dimension.

Regarding the Ramond-Ramond (or R-R) sector, from an n-form $F_{I_1\ldots I_n}$ with the indices $I_n = 0,\ldots 9$ we obtain the $n$-form $F_{\mu_1\ldots\mu_n}$ with $\mu_i \neq 9$ and the form $F_{\mu_1\ldots\mu_{n-1}9}$, which is a $(n-1)$-form from the perspective of the uncompactified dimensions. This procedure generalises to the case of more than one compact dimension. The moduli space becomes larger and exhibits a fascinating structure [55] however this will not be investigated further.

## 2.2. T-duality for closed strings

We will start by introducing the notion of T-duality for *closed* strings and then turn our attention to the case of open strings. This will give us a good understanding how D-branes arise in string theory. Since supersymmetry does not introduce additional features to T-duality, it is enough for our purposes to study the case of the bosonic string. The discussion can then be extend to the superstring case. For this discussion we will use a different parameterisation of the world-sheet. We use use a Euclidean metric with $\sigma_2 = i\tau$ and than combine the $\sigma_i$ coordinates to $z := \exp(-i\sigma_1 + \sigma_2)$ and $\bar{z} = \exp(i\sigma_1 - \sigma_2)$. To start with, we recall the mass formula and the level-matching condition for a bosonic string with one coordinate compactified on a circle [37]:

$$m^2 = \frac{n^2}{R^2} + \frac{w^2R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2), \tag{2.13}$$

$$0 = nw + N - \tilde{N}, \tag{2.14}$$

where $n$ is the Kaluza-Klein momentum, $w$ the winding number of the string. Upon interchanging

$$R \leftrightarrow R' = \frac{\alpha'}{R}, \qquad n \leftrightarrow w$$

the mass spectrum is left invariant. As it will be explained in section 3, the interchanging $n \leftrightarrow w$ is equivalent to

$$p_L \to p_L, \qquad p_R \to -p_R. \tag{2.15}$$

If we transform the coordinate $X(z,\bar{z}) = X_L(z) + X_R(\bar{z})$ to

$$X'(z,\bar{z}) = X_L(z) - X_R(\bar{z}), \tag{2.16}$$

the field $X'$ has the same physical properties as the field $X$ has (i.e. OPE's[1] and energy-momentum tensor are unchanged). The only difference between the two fields is the sign

[1] OPE: operator product expansion, in conformal field theories OPE's are used to extract physical quantities as for example correlation functions. All (perturbative) information of conformal field theories is encoded in the OPE's. For details see any introduction to conformal field theory, for example [37, 18].

change in (2.15), which induces an additional sign change for the oscillators. This implies that the redefinition of coordinates (2.16) changes the compactification radius of the string from $R$ to the radius $R'$ but the spectrum is left invariant. This transformation is known as *T-duality* for closed strings. T-duality can be viewed as a world-sheet parity operation acting only on the right moving sector.

## 2.3. D-branes

To introduce the notion of D-branes we will focus on the open bosonic string case, and later on we will address the case of the superstring. The mode expansion for the open bosonic string with *Neumann* boundary conditions $\partial_{\sigma^1} X = 0$ (which we will denote by *N boundary conditions* in the following) is given by

$$X^\mu(z,\bar{z}) = x^\mu - 2i\alpha' p_0^\mu \sigma^2 + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-n\sigma^2} \cos(n\sigma^1). \tag{2.17}$$

We can decompose the field for the open string into a right moving part and a left moving part

$$X^\mu(z,\bar{z}) = X_L^\mu(z) + X_R^\mu(\bar{z}), \tag{2.18}$$

with the mode expansions

$$X_L^\mu(z) = \frac{x^\mu}{2} + \frac{x'^\mu}{2} - i\alpha' p_0^\mu \ln z + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu z^{-n} \tag{2.19}$$

$$X_R^\mu(\bar{z}) = \frac{x^\mu}{2} - \frac{x'^\mu}{2} - i\alpha' p_0^\mu \ln \bar{z} + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu \bar{z}^{-n}. \tag{2.20}$$

$\alpha_{-n}^\mu$ are the creation operators for an excitation of the $n$-th level, whereas $\alpha_n^\mu$ are the annihilation operators and $\alpha_0^\mu = \sqrt{2\alpha'} p^\mu$. We now consider a bosonic field $X$ compactified on a circle of radius $R$. Since open strings do not have winding excitations, they do not have a quantum number $w$. If we now consider the limit $R \to 0$ the states with nonzero momentum still have infinite mass, even though there is no new continuum of light winding states. Thus the open strings live only in $D-1$ dimensions after $R \to 0$. This is in fact puzzling if we remember that the closed string still lives in $D$ dimensions after $R \to 0$ and a theory with open strings always has to contain closed strings for consistency. The resolution to this puzzle is that the points along the open string still vibrate in $D$ dimensions, while its endpoints (which distinguish it from the closed string) are constrained to live on a $D-1$ dimensional hypersurface. To see this in more detail, we write the open string mode expansion in its T-dual form

$$
\begin{aligned}
X'^{25}(z,\bar z) = X_L^{25}(z) - X_R^{25}(\bar z) = \\
x'^{25} - i\alpha' p^{25}\ln\left(\frac{z}{\bar z}\right) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{25}(z^{-n}-\bar z^{-n}) = \\
x'^{25} - 2\alpha' p^{25}\sigma^1 - \sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{25}e^{-n\sigma^2}\sin(n\sigma^1).
\end{aligned}
\tag{2.21}
$$

The zero mode sector does not have any dependence on the Euclidean time $\sigma^2$. Therefore its momentum at the endpoints vanishes. Since the oscillators give no contribution at $\sigma^1 = 0, \pi$ this equation shows that the endpoints of the string do not move in the $X'^{25}$ direction. This can also be seen from a change of the boundary conditions

$$
\partial_{\sigma^1} X = 0 \rightarrow i\partial_{\sigma^2} X' = 0. \tag{2.22}
$$

Equation (2.22) interchanges N boundary conditions $\partial_n X \equiv \partial_{\sigma^1} X = 0$ which impose that the derivative normal to the world-sheet vanishes with *Dirichlet* (or D) boundary conditions $\partial_t X \equiv -i\partial_{\sigma^2} X = 0$ in the T-dual description. D boundary condition force the derivative tangent to the world-sheet to vanish. The endpoints of the string have to lie on the same hyperplane:

$$
\begin{aligned}
X'^{25}(\pi) - X'^{25}(0) = \int_0^\pi d\sigma^1 \partial_1 X'^{25} = -i\int_0^\pi d\sigma^1 \partial_2 X^{25} \\
= -2\pi\alpha' p = -2\pi\alpha'\frac{n}{R} = -2\pi n R',
\end{aligned}
\tag{2.23}
$$

with $\partial_i = \partial\sigma^i$. Therefore, the distance between the endpoints of the open string is an integral multiple of the periodicity $2\pi R'$ of the dual dimension. It can be shown [37] that all strings have to end on the same hyperplane. Thus, the physics of compactifying open strings on a small circle is the same as that of compactifying them on a large circle with the ends restricted to live on a hyperplane. Such a hyperplane is called a *Dirichlet p-brane* where $p$ indicates the number of spatial dimensions the brane spans. If we have open strings with Neumann boundary conditions only, we have for the bosonic string a D25-brane. The same procedure can be generalised if we compactify more than one dimension. T-duality in a direction parallel to the brane turns it into a D$(p-1)$-brane, whereas T-duality in a direction transverse to the brane turns it into a D$(p+1)$-brane.

Now we consider the case that more than one D-brane is present at the same point in spacetime. In this case open strings have the freedom to end on different branes. We denote this by two additional labels on an open string state. An open string state now has the form

$$
|\psi; ij\rangle, \tag{2.24}
$$

where $\psi$ denotes the world-sheet degrees of freedom and the additional indices $i$ and $j$ incorporate the information that the string stretches from D-brane $j$ to D-brane $i$. Now let us consider a general open string state, which we denote by

$$|\psi; a\rangle. \tag{2.25}$$

It can consist of a linear combination of states which start and end on different branes. A basis for a general state is then given by

$$|\psi; a\rangle = \sum_{i,j=1}^{n} |\psi; ij\rangle \lambda_{ji}^{a}. \tag{2.26}$$

The $n \times n$ matrices $\lambda_{ji}^{a}$ have to be Hermitian, to ensure that the amplitudes of string interactions are still unitary. If we choose the normalisation $\mathrm{Tr}(\lambda^a \lambda^b) = \delta^{ab}$, they form a representation of a U$(n)$ gauge group. The matrices $\lambda_{ji}^{a}$ are called the *Chan-Paton* factors or CP-factors of the string. Thus a U$(n)$ gauge group is associated with the endpoints of the string. Now let us consider $n$ D25-branes, for definiteness. The massless state

$$\alpha_{-1}^{\mu} |0; ij\rangle \lambda_{ji} \tag{2.27}$$

transforms as a vector and can therefore be regarded as the vector boson of the U$(n)$ gauge symmetry on the D-brane. This argument can be made more rigorous if we consider the amplitudes of interacting strings and extract the low energy limit. The results can then be reproduced by the action for a U$(n)$ gauge theory. The calculations can be found for example in [37]. The same procedure works for the superstring, as well, where the effective theory on the world-volume of the brane is a supersymmetric U$(n)$ gauge theory.

## 2.4. Wilson lines

Our analysis so far considered open strings which ended on the same hyperplane. This condition can be relaxed, when we introduce the notion of *Wilson-lines*. Consider for example open strings with one compactified dimension $X^{25}$ and a $U(1)$ gauge theory on that dimension with the constant background field

$$A_{25}(x^M) = -\frac{\theta}{2\pi R} = -i\Lambda^{-1}\frac{\partial \Lambda}{\partial x^{25}}, \qquad \Lambda(x^{25}) = \exp\left(-\frac{i\theta x^{25}}{2\pi R}\right), \quad M = 0, \dots, 25, \tag{2.28}$$

where $\theta$ is independent of $x$. Locally we could always gauge away the field $A_{25}$. However, as the gauge parameter $\Lambda$ does not respect the spacetime periodicity, the background will have a physical effect. The gauge invariant quantity describing this effect is the Wilson line

$$W_q = \exp\left(iq \oint A_{25} dx^{25}\right) = \exp(-iq\theta). \tag{2.29}$$

$W_q$ is gauge invariant because a proper gauge transformation must be single valued over the circle. If we write down the Lagrangian for this theory and compute the canonical momentum we obtain

$$p^{25} = \frac{2\pi n + q\theta}{2\pi R}. \tag{2.30}$$

Generalising to a string with U($n$) CP-factors the constant gauge field $A_{25}$ can be written as

$$A_{25} = -\frac{1}{2\pi R}\mathrm{diag}(\theta_1, \ldots, \theta_n), \tag{2.31}$$

where $A_{25}$ is in the diagonal subgroup of U($n$), the U(1)$^n$. Since the gauge field couples to the Chan-Paton factor of a state as $[A_M, \lambda]$, a string with Chan-Paton state $|\, ij\,\rangle\lambda_{ji}$ has charge +1 under $U(1)_j$ and charge −1 under $U(1)_i$ and is neutral under the remaining ones. From equation (2.30) we then obtain the general result

$$p^{25} = \frac{2\pi n + \theta_j - \theta_i}{2\pi R}, \tag{2.32}$$

and the open string spectrum is found to be

$$m^2 = \frac{(2\pi n + \theta_j - \theta_i)^2}{4\pi^2 R^2} + \frac{1}{\alpha'}(N-1). \tag{2.33}$$

Therefore in the T-dual picture turning on a Wilson line modifies equation (2.3) to

$$\Delta X'^{25} = X'^{25}(\pi) - X'^{25}(0) = -(2\pi n + \theta_j - \theta_i)R', \tag{2.34}$$

as $p^{25}$ is modified by equation (2.32). Thus in the T-dual picture the Wilson line determines the position of the D-branes along the compactified dimension.

## 2.5. D-branes and gauge symmetry breaking (world-volume perspective)

From now on, we return to the case of superstrings and we work in ten spacetime dimensions. As explained in [8], D-branes in type II string theory can be introduced as topological defects. Open strings are then constrained to live on the D-branes, whereas the type II strings can propagate in the bulk.

The spectrum on a Dp-brane can be viewed as the dimensionally reduced vector multiplet from ten dimensions with a $p+1$-dimensional massless vector $A_\mu$ and $9-p$ scalars $\Phi^m$ from the NS sector. The R sector contributes the supersymmetric partners. The vector field corresponds to a U(1) gauge field on the D-brane while the scalars correspond to the position of the brane in the transverse space.

Now consider two parallel Dp-branes, separated by some distance along the compact directions. This gives rise to a second copy of the above spectrum. If the branes reside at the same point, we obtain an enhanced gauge symmetry as new degrees of freedom arise from massless strings that can stretch between the two branes. Consider the open string state

$$|\, \psi\,; ij\,\rangle\lambda_{ji} \qquad i,j = 1,2. \tag{2.35}$$

If the Chan-Paton degrees of freedom are contracted with a hermitean matrix, these matrices transform in the adjoint representation[2] of U(2). Therefore the effective action on the world-volume of the brane is a supersymmetric U(2) gauge theory [57]. We observe that the complete multiplet transforms in the adjoint representation. This implies that the scalars which correspond in the string theory picture to the transverse coordinates of the brane become matrix valued. Therefore we obtain four massless scalars per transverse direction in our example. In the classical limit with unbroken supersymmetry the matrices of the scalars can be diagonalised simultaneously [57], while in the case with broken supersymmetry and quantum corrections we obtain non-commutative coordinates, because the matrices associated to the scalars do not longer commute. So far, all branes were coincident in the compact dimensions. What happens if we move them apart? The strings $|\psi, ii\rangle$ which start and end on the same brane remain massless while strings $|\psi, ij\rangle$ $i \neq j$ which stretch between the branes obtain a mass $m^2 \propto T|Y|^2$, where $T$ is the string tension and $Y = (Y^{p+1}, \ldots, Y^9)$ the distance vector between the branes. Strings stretching between the branes are charged under the U(1) gauge fields living on each brane, thus the world volume vector bosons with charge $(\pm 1, \mp 1)$ obtain a mass, breaking the U(2) gauge symmetry down to $\mathrm{U}(1) \times \mathrm{U}(1)$. The scalars and fermions in the $(12) + (21)$ sector obtain the same mass. If we consider for definiteness two D3-branes at the same point, the spectrum on the pair of branes consists of a vector multiplet of D=4, $\mathcal{N}$=4 supersymmetry, consisting of a vector, six scalars and their fermionic partners, all transforming in the adjoint representation of U(2). If we now increase the distance between the branes, two of the four vector multiplets become massive. These massive multiplets correspond to ultra-short BPS multiplets because of the U(1) charges on each of the branes which have to be conserved. If this charge would not exist, we had a massive $\mathcal{N}$=4 multiplet with $2^{2\cdot 4} = 256$ states. Because of the conserved charge, the multiplets from the strings stretching between the branes consist of one massive vector, 5 scalars and their massive fermionic partners giving $2^4$ states in all, which is exactly the same number of degrees of freedom as for a massless multiplet. If we repeat the above analysis for more than two branes, the same mechanism applies. Generally, if from a stack of $n$ branes we move away $m$ branes, we break the $\mathrm{U}(n)$ gauge theory down to a $\mathrm{U}(n-m) \times \mathrm{U}(m)$ gauge theory.

## 2.6. R-R charges

In this section we address the case of the closed string sector of type II superstrings. Both type IIA and type IIB superstrings have identical fields in the NS-NS sector, namely the graviton $G_{\mu\nu}$, the dilaton $\Phi$ and the antisymmetric tensor $B_{\mu\nu}$, but the closed string spectrum differs in the R-R sector. We obtain the following potentials for the two different

[2] This notation is inprecise as the U(1)-factor does not have an adjoint representation; what we mean by this is the adjoint representation of SU(2) with states charged under the U(1).

string theories

$$\text{type IIA:} \quad A_{(1)}, A_{(3)}, A_{(5)}, A_{(7)}, A_{(9)} \tag{2.36}$$
$$\text{type IIB:} \quad A_{(0)}, A_{(2)}, A_{(4)}, A_{(6)}, A_{(8)}, A_{(10)} \tag{2.37}$$

where $A_{(p)}$ is a $p$-form potential, therefore $A_{(p)} = \star A_{(8-p)}$ and $A_{(4)}$ is self-dual[3]. Their field strengths are $dA_{(p)} = F_{(p+1)}$ and they have to satisfy the Maxwell equation

$$d \star F_{(p+1)} = 0 \tag{2.38}$$

If we analyse the vertex operators from the R-R sector of the string, as is done in [37], we find that the closed string vertex operators correspond to field strengths rather than to potentials, therefore the strings carry no R-R charge [8].

The natural objects charged under a $p+1$-form are $p$-branes, extended objects with $p$ spatial dimensions. In string theory these are exactly the Dirichlet $p$-branes, this can be understood as follows: If we calculate the cylinder amplitude for the exchange of a closed string between two D-branes (presented in appendix A) we obtain in the limit $t \to 0$ the amplitude for the exchange of massless strings, where $t$ is the modular parameter. It reads

$$\begin{aligned}\mathcal{A}_{\text{closed, massless}} &= 8(1-1)V_{p+1}\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-\frac{p+1}{2}} t^4 e^{-\frac{t|Y|^2}{2\pi\alpha'}} \\ &= 2\pi(1-1)V_{p+1}(4\pi^2\alpha')^{3-p}G_{9-p}(|Y|),\end{aligned} \tag{2.39}$$

where

$$G_d(|Y|) = \frac{1}{4\pi^{d/2}}\int_0^\infty \frac{dt}{t^{\frac{4-d}{2}}} e^{-t|Y|^2} \tag{2.40}$$

is the propagator for a massless scalar field in $d$ dimensions and $V_{p+1}$ is the (regularised) volume of the Dp-brane. The difference $(1-1)$ accounts for the contributions of NS-NS and R-R exchange and their cancellation is ensured by supersymmetry[4]. If we split the above amplitude into the part from the NS-NS sector and the part from the R-R sector we obtain

$$\mathcal{A}_R = -\mathcal{A}_{NS} = -2\pi V_{p+1}(4\pi^2\alpha')^{3-p}G_{9-p}(|Y|). \tag{2.41}$$

Let us now consider the field theoretic analogue of this calculation. We consider R-R forms coupled to p-branes by the action [37]

$$S = \frac{1}{2}\int F_{(p+2)} \wedge \star F_{(p+2)} + iT_p \int_{\text{brane}} A_{(p+1)}, \tag{2.42}$$

with $F_{(p+2)} = dA_{(p+1)}$ and $T_p$ the tension of the brane. The amplitude for the exchange of $A_{(p+1)}$ between two branes with the distance $|Y|$ in the transverse $9-p$ dimensional space is given by

$$\mathcal{A}_{\text{field theory}} = (iT_p)^2 V_{p+1} G_{9-p}(|Y|). \tag{2.43}$$

[3] The ten-form potential of type IIB and the nine-form potential of type IIA are not dual to any other potential and describe non-propagating fields.

[4] The string theory calculation is performed in appendix (A)

In this formula we have absorbed some normalisation constants, which are not relevant for our discussion, in the definition of the field strengths $F_{(p+2)}$. By comparing equation (2.43) with equation (2.41), we see that the parts for the R-R exchange coincide, provided we set $T_p^2 = 2\pi(\pi^2\alpha')^{3-p}$, so D-branes are charged under R-R forms [34]. A thorough investigation [26, 35] reveals that the charge of the D-branes obeys the Dirac quantisation condition.

In order to couple the R-R fields of the closed strings to the potentials present in the different type II theories, the type IIA theory must contain Dp-branes for $p = 0, 2, 4, 6, 8$ and type IIB must contain Dp-branes for $p = -1, 1, 3, 5, 7, 9$. By T-duality arguments one can conclude that a D8-brane must be included in type IIA theory and the presence of the D9-brane in type IIB string theory can be derived by analysis of the type I divergences [8].

## 2.7. D-branes and BPS states

Massive BPS states appear in theories with extended supersymmetry. The supersymmetry algebra for extended supersymmetry with $N$ supersymmetry generators without a central charge is given by[5]

$$\left\{Q^A_\alpha, \overline{Q}^B_\beta\right\} = -2\delta^{AB}P_\mu\Gamma^\mu_{\alpha\beta} \qquad A = 1, \ldots, N, \qquad \mu = 0, \ldots, D-1, \tag{2.44}$$

$$\left[P^\mu, Q^A_\alpha\right] = 0, \tag{2.45}$$

where $P^\mu$ is the momentum operator, $Q^A_\alpha$ the generator of the supersymmetry transformation and $\overline{Q}^B_\beta = Q^{\dagger B}_\gamma \Gamma^0_{\gamma\beta}$. For massless states we have $N$ creation operators, therefore the multiplets of this algebra have $2^N$ states; for massive representations we have two copies of the fermionic oscillator algebra and therefore massive representations have $2^{2N}$ states [56].

We can now extend this algebra by adding a *central charge*. The most general extended supersymmetry algebra for a point particle which is allowed by Lorentz invariance is:

$$\left\{Q^A_\alpha, \overline{Q}^B_\beta\right\} = -2\delta^{AB}P_\mu\Gamma^\mu_{\alpha\beta} - 2iZ^{AB}\delta_{\alpha\beta} \qquad A, B = 1, \ldots, N \tag{2.46}$$

$$\left[P^\mu, Q^A_\alpha\right] = \left[Z^{AB}, Q^C_\alpha\right] = \left[P^\mu, Z^{AB}\right] = \left[Z^{AB}, Z^{CD}\right] = 0 \tag{2.47}$$

where $Z^{AB}$ must be antisymmetric. If we consider a massive particle at rest, the supersymmetry algebra becomes:

$$\left\{Q^A_\alpha, Q^{\dagger B}_\beta\right\} = 2m\delta^{AB}\delta_{\alpha\beta} + 2iZ^{AB}\Gamma^0_{\alpha\beta}. \tag{2.48}$$

For an eigenstate of the charges $Z^{AB}$ we can use a basis in which

$$Z^{AB} = \begin{pmatrix} 0 & q_1 & 0 & 0 & \\ -q_1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & q_2 & \\ 0 & 0 & -q_2 & 0 & \\ & \vdots & & & \ddots \end{pmatrix} \tag{2.49}$$

[5]We use the conventions of [37]

with $q_i \geq 0$. If we now diagonalise $Z^{AB}$ we obtain a real expression on the right hand side of (2.48). The anti-commutator on the left hand side of equation(2.48) is proportional to the Hamiltonian and thus nonnegative. The eigenvalues $2(m \pm q_i)$ of he right hand side should therefore also be larger than zero, implying the *Bogolmonyi-Prasad-Sommerfeld (BPS)* bound

$$m \geq q_i.$$

This bound requires the massless states to be neutral under the charge $Z^{AB}$. Now let us consider the case where there are eigenvalues $q_1, \ldots, q_N$ with

$$q_1 = q_2 = \cdots = q_k = m$$

and

$$q_{k+1}, \ldots, q_N < q_k.$$

This implies that $2k$ pairs of fermionic oscillators have to annihilate the states. This gives a short or BPS representation with $2^{2(N-k)}$ states. If all the $q_i$'s and $m$ are equal the result is an ultrashort representation of dimension $2^N$ as in the case of the massless representation.

This is exactly what happens if we separate two D-branes: In the ten-dimensional picture the lowest oscillator level has (mass)$^2$

$$m^2 = p^2 + \left(\frac{\Delta Y^i}{2\pi\alpha'}\right)^2, \tag{2.50}$$

where $\Delta Y^i$ denotes the distance of the branes. In the world-volume theory this mass reappears as the central charge. For this to be a central charge and thus to give a BPS state, the length of the string must be proportional to some conserved charge. This charge we are looking for is induced by the anti-symmetric $B$ field of the closed string spectrum which couples to the open string. We argued in paragraph (2.5) that the charge is associated to the $U(1)$ charges on the brane world-volume, as there are no other conserved charges in the field theory description. Examining the coupling between the U(1) charges and $B$ field more closely, we see that a variation of the $B$ field induces boundary terms at the boundary of the open string world-sheet, and these have to be absorbed in the open string field $A_\mu$. This is the connection between the $B$ field and the $U(1)$'s on the brane world-volume. If we restate the BPS condition in terms of the field theory this will give us an expression involving the gauge fields $A_\mu$ and the expectation value of the Higgs which is proportional to $Y^2$.

We consider next type I string theory with one dimension, $X^9$ compactified on a circle. Taking the $R_9 \to 0$ limit and considering the T-dual theory leads to a space with D8-branes. If we look at the bulk theory far away from the D-branes the local physics is that of a closed string theory. The theory is oriented, because in the T-dual description the orientifold projection relates the string state with its image behind the orientifold plane [37]. Thus locally the bulk theory is IIA.

We can consider a scattering process with a closed string scattering from a D-brane. Computing the amplitude of this process requires a world-sheet with boundary. Now the open string boundary conditions are only invariant under D=10, $\mathcal{N}=1$ supersymmetry: The left moving world-sheet current for spacetime supersymmetry $S_\alpha(z)$ flows into the boundary and the right moving current $\widetilde{S}_\alpha(\bar{z})$ flows out of it. If we denote by $Q_\alpha$ and $\widetilde{Q}_\alpha$ the conserved charges, corresponding to the currents $S_\alpha(z)$ and $\widetilde{S}_\alpha(\bar{z})$, only the sum of the charges $Q_\alpha + \widetilde{Q}_\alpha$ of left and right moving currents is conserved. Under T-duality this becomes

$$Q_\alpha + (\beta^9 \widetilde{Q})_\alpha \tag{2.51}$$

where $\beta^9$ denotes the action of T-duality on the spinorial SUSY charges. Therefore, if a closed string scatters from a D-brane, only half of the supersymmetry is conserved. The same happens for the case of more than one T-dualised direction. Since a D-brane conserves exactly half of the supersymmetry it is a BPS state. The supersymmetry algebra for a p+1 dimensional extended object is given by [14, 54, 37]

$$\left\{Q_\alpha, \overline{Q}_\beta\right\} = -2\left(P_M + (2\pi\alpha')^{-1} Q^{NS}_M\right)\Gamma^M_{\alpha\beta}, \tag{2.52}$$

$$\left\{\widetilde{Q}_\alpha, \overline{\widetilde{Q}}_\beta\right\} = -2\left(P_M - (2\pi\alpha')^{-1} Q^{NS}_M\right)\Gamma^M_{\alpha\beta}, \tag{2.53}$$

$$\left\{Q_\alpha, \overline{\widetilde{Q}}_\beta\right\} = -2\sum_p \frac{T_p}{p!} Q^R_{M_1\ldots M_p}(\beta^{M_1}\cdots\beta^{M_p})_{\alpha\beta}, \tag{2.54}$$

The charges with the label $NS$ are exactly the ones we described above, while those with the label $R$ come from the R-R fields living on the D-brane.

## 2.8. Anti-D-Branes

An anti-D-brane is a D-brane with R-R charge of opposite sign compared to that of a usual D-brane. Therefore, if we assign to a Dp-brane the charge $+1$ under the R-R $p+1$-form potential the anti-Dp-brane ($\overline{\text{Dp}}$-brane) has charge $-1$ under this potential.

As long as there is only one kind of Dp-brane in the system and the background is flat, there exists no possibility to distinguish branes from anti-branes. This changes drastically as soon as we have $\overline{\text{Dp}}$-branes and Dp-branes or orientifold planes present in the model. Let us first consider coincident Dp-branes and $\overline{\text{Dp}}$-branes. Their R-R charges cancel so there is no BPS condition to be satisfied and the system is unstable. This instability can be deduced from the presence of a tachyonic mode: On the strings stretching between the D-brane and the $\overline{\text{D}}$-brane the GSO[6] projection is reversed. Therefore the tachyonic ground state of the NS-sector is not longer projected out. We can understand the reversal of the GSO projection as follows: The R-R exchange between the two branes has the opposite sign compared to the R-R exchange between two 'normal' D-branes, because the charge has opposite sign on the anti-D-brane. The sign change in the closed R-R sector corresponds

[6]In order to render superstring theories consistent it is necessary to impose a projection on states with definite fermion number. This projection is called the *Gliozzi-Scherk-Olive* or *GSO* projection. We hope that the reader is familiar with these matters. Details can be found in any introductory text on superstring theory.

to a reversed GSO projection in the open string NS sector. To understand this, consider the expression

$$\int d\psi \langle \psi | \hat{A} | \psi \rangle. \tag{2.55}$$

For a bosonic state $|\psi\rangle$ this would amount to the trace of the operator $\hat{A}$. If $|\psi\rangle$ represents *fermionic states* this expression amounts to $\mathrm{Tr}[(-1)^{\hat{F}}\hat{A}]$, where $\hat{F}$ is the *fermion number operator*, as is explained in [37]. Therefore, the partition function for fermionic states performed after a Wick-rotation to imaginary time becomes

$$\begin{aligned} \mathcal{Z} &= \int [d\psi]_P \exp(-S_E) && (2.56)\\ &= \int d\psi \int [d\psi]_{\psi,0}^{\psi,U} \exp(-S_E) && (2.57)\\ &= \int d\psi \langle \psi, U | \psi, 0 \rangle && (2.58)\\ &= \mathrm{Tr}\left[\exp(-\hat{H}U)(-1)^{\hat{F}}\right]. && (2.59) \end{aligned}$$

The $P$ denotes that we sum over all *periodic* paths, which is the same as performing the path integral of paths which connect the state $\psi$ at time $T = 0$ with the state $\psi$ at $T = U$ and summing over all states $\psi$, which is what is denoted in (2.57). Therefore the path integral over periodic paths gives an additional $(-1)^{\hat{F}}$ in the trace of the partition function. To obtain the trace without an additional insertion, we have to sum over all antiperiodic paths

$$\int [d\psi]_A \exp(-S) = \mathrm{Tr}\left[\exp(-\hat{H}U)\right]. \tag{2.60}$$

We consider now the partition function for the cylinder. If we regard this as the tree channel exchange of closed strings, in Fig. (2.2a), the time parameter $\tau$ runs vertically and $\sigma$ parameterising the position on the string runs around the cylinder. For this configuration,

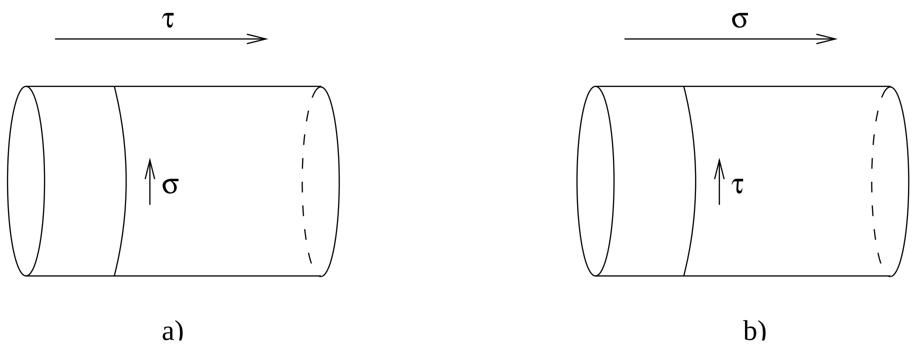


Figure 2.2.: The cylinder amplitude. a) corresponds to closed string tree level, b) corresponds to open string loop level.

periodic boundary conditions in $\sigma$ imply R-R strings and antiperiodic boundary conditions imply NS-NS strings. (NS-R and R-NS strings are not present.) If we sum over periodic paths in the time direction we obtain the $(-1)^{\hat{F}}$ insertion in the trace, while we obtain no additional insertion if we sum over the antiperiodic paths. The tree level cylinder amplitude of the closed string is linked with the one-loop amplitude of the open string by a modular transformation of the world-sheet parameters [37]. In the the open string loop channel, shown in figure (2.2b), we can rephrase the previous periodicities in the following way:

$$
\begin{array}{llll}
 & \text{closed} & & \text{open} \\
(\sigma,\tau) & & \rightarrow (\tilde{\tau},\tilde{\sigma}) & \\
(\mathrm{p,p}): & (-1)^{\hat{F}}\ \text{R-R} & \rightarrow (\mathrm{p,p}): & (-1)^{\hat{F}}\ \text{R} \\
(\mathrm{p,a}): & \text{R-R} & \rightarrow (\mathrm{p,a}): & (-1)^{\hat{F}}\ \text{NS} \\
(\mathrm{a,p}): & (-1)^{\hat{F}}\ \text{NS-NS} & \rightarrow (\mathrm{a,p}): & \text{R} \\
(\mathrm{a,a}): & \text{NS-NS} & \rightarrow (\mathrm{a,a}): & \text{NS}
\end{array}
$$

Therefore, the change of the sign on the closed string R-R sector induces a change of the sign in front of the $(-1)^{\hat{F}}$ NS contribution. The R-R part with $(-1)^{\hat{F}}$ in the trace vanishes identically. From this we see that reversing the sign of the R-R exchange forces the opposite GSO projection in the open string D-$\overline{\mathrm{D}}$ sector.

In the sector with reversed GSO projection, the tachyonic NS ground state is not projected out. The presence of tachyons signals an instability of the system. It has been conjectured by Sen [48] (see also [42]) that a D-$\overline{\mathrm{D}}$ system decays into the vacuum if it is not protected by any additional conserved charge. Indeed, if we place a brane and an anti-brane at different points in the space, they feel an attractive force [9], as can be deduced from the cylinder amplitude. This calculation is done in appendix B. Therefore, simply placing branes and anti-branes at different points in the bulk does not stabilise the system. Although a Dp-brane and an $\overline{\mathrm{Dp}}$-brane conserve each half of the supersymmetry, combining them breaks supersymmetry completely.

## 2.9. Tachyon condensation

In this section we will shortly review some issues on tachyon condensation in the D-$\overline{\mathrm{D}}$system. The basic assumption of this section is the idea formulated by Banks and Susskind [9] and worked out by Sen [48] that a brane and an anti-brane can annihilate into the vacuum such that only the supersymmetric theory remains. Different techniques were applied to prove this conjecture, such as (Super)string field theory [32] and the theory of non-commutative solitons [13]. It turns out that some difficulties arise if it is tried to describe this effect by an “ordinary” low energy field theory calculation. We will present some arguments why this approach might not be the most promising route to describe tachyon condensation.

Let us consider a system of two Dp-branes and one $\overline{\mathrm{Dp}}$-brane, for definiteness. The massless spectrum consists of one dimensional reduced D=10, $\mathcal{N}$=1 vector multiplet in the D-brane D-brane sector which which gives rise to a U(2) gauge field theory and another vector multiplet on the $\overline{\text{D}}$-brane which gives rise to a U(1) gauge field theory. In the D-$\overline{\text{D}}$-sector the GSO projection is reversed and the tachyonic NS ground state and eight fermions from the R sector are present after imposing the GSO projection. The states in the D-$\overline{\text{D}}$-sector transform under the bifundamental representation of the gauge groups. If the branes are sufficiently close, we are led to the assumption that the low-energy behaviour of the system can be described by a non-supersymmetric $\mathrm{U}(2)_D \times \mathrm{U}(1)_{\overline{D}}$ gauge field theory. By comparison with the mechanisms described in section 2.5 we infer that it should be possible to assign a v.e.v. to the tachyon field and break the $\mathrm{U}(2)_D \times \mathrm{U}(1)_{\overline{D}}$ gauge theory down to a $\mathrm{U}(1)_D$ gauge field theory. This theory should correspond to the gauge field theory on one Dp-brane, the other Dp-brane annihilated with the $\overline{\text{Dp}}$-brane. Within this process supersymmetry should be restored because the remaining D-brane possesses a supersymmetric spectrum. The tachyonic NS ground state transforms under the bifundamental representation of the gauge groups. Therefore it possesses the correct quantum numbers to account for the scalars which are needed to give a mass to the gauge bosons. However, we found it not possible to formulate a Lagrangian which could restore supersymmetry on the remaining brane. In our investigation it turned out that it was not possible to include the correct mass terms for the fermions which could restore supersymmetry.

Another question arises, if we try to formulate the brane anti-brane annihilation of one Dp-brane with a $\overline{\text{Dp}}$-brane. On the combined system we obtain a non-supersymmetric $\mathrm{U}(1) \times \mathrm{U}(1)$ gauge theory. It is possible to break the gauge group down to U(1), however it is not possible to break the gauge group completely [50, 58]. From the string theory point of view the question arises, to which gauge theory the remaining U(1) corresponds. If the branes annihilated, no open strings are left which could give rise to this gauge theory.

Another argument why this approach does not seem to be the most promising is the following: If branes and anti-branes reside at the same point the mass of the tachyon is given by [49]

$$m_T^2 = -\frac{1}{2\alpha'}.$$

If we perform the conventional field theory limit, we decouple massive modes, which corresponds to the limit $\alpha' \to 0$. If we perform this limit when a tachyon is present in the spectrum, this corresponds to the limit $m_T \to -\infty$. However, this does not seem to be the region in which the conventional low energy field theory limit is valid. One way to overcome this problem is to include a strong $B$ field in the analysis. On the branes this gives rise to non-commutative geometry but the tachyon mass can be shifted to a finite value.

# 3. Orbifolds and Orientifolds

In this section we will briefly review the construction of orbifolds and orientifolds in string theory [31, 21, 1, 37].

## 3.1. Orbifolds

Consider a type II string theory compactified on a torus $T^d$, with $d$ even. We can define a two-torus as a lattice $\Gamma$ on a complex plane. For this we take two vectors $e_1, e_2$ that are $\mathbb{R}$-linearly independent and impose the following condition: $z \widehat{=} z + me_1 + ne_2, m, n \in \mathbb{Z}$, which identifies points that differ only by lattice shifts. Formally speaking, we build up equivalence classes $[z]$ of points only differing by lattice shifts. We can denote this by

$$\mathcal{T} = \mathbb{C}/\Gamma.$$

This construction gives a fundamental cell onto which all points in the complex plane can be mapped. Since the sides of this fundamental cell are identified, the cell $\mathcal{T}$ can be viewed as a torus.

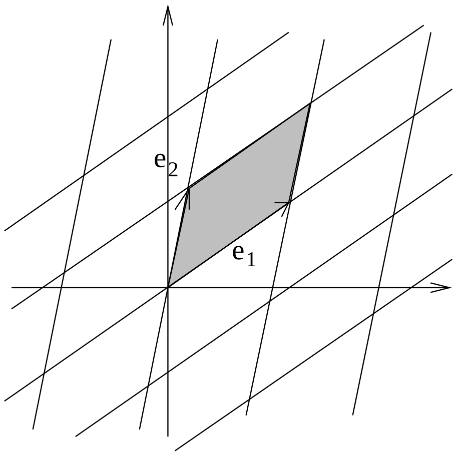


Figure 3.1.: A lattice in the complex plane spanned by the vectors $e_1$ and $e_2$. The shaded region depicts the fundamental cell of the torus.

This procedure is analogous to the compactification of a line on a circle. In the following, we will consider a special class of lattices which are invariant under some discrete rotation to be defined below. As an example take an orthogonal lattice $e_1 = 1$, $e_2 = i$. This lattice is invariant under a reflection $\theta$ with respect to the origin,

$$\theta : z \to -z.$$

Points which are mapped onto each other are identified,

$$z \,\widehat{=}\, -z.$$

This can be viewed as moding the lattice $\Gamma$ by a discrete symmetry group $G$ (in our example $\mathbb{Z}_2$). This construction defines a space which is smooth, except for singular points which are invariant with respect to the symmetry operation and therefore are mapped onto themselves (up to lattice shifts). The resulting object is called an "orbifold" $\mathcal{O}$ [15, 16].

$$\mathcal{O} = \mathcal{T}/G = (\mathbb{C}/\Gamma)\,/G$$

Now we construct a string theory this space. Although it is not obvious that a string model can be defined on such a singular space, it turns out that such a construction is possible [15, 16]. The string wavefunctions have to be equal at identified points. Therefore a new sector is introduced in the theory with the following boundary conditions

$$\phi(\sigma + 2\pi, \tau) = h \cdot \phi(\sigma, \tau) \qquad h \in G, \tag{3.1}$$

where $\phi$ denotes an arbitrary field on the world-sheet. This means that the closed string boundary conditions are only satisfied up to an orbifold group transformation. Strings in this sector are called *twisted states*. The twisted states are the analogue of winding states from toroidal compactification.

### 3.1.1. Construction of states

In order for the theory to be consistent, the states in the spectrum have to be invariant under the symmetry group $G$. Therefore, we have to project onto $G$-invariant states and include twisted states

$$\phi(\sigma + 2\pi, \tau) = h \cdot \phi(\sigma, \tau).$$

In the following we analyse the case of six compactified dimensions and point out the differences for the case of four compactified dimensions. We will consider the dimensions $0, \ldots, 3$ as uncompactified and the dimensions $4, \ldots, 9$ as compactified.

Let us consider the case that the orbifold group $G$ is given by $\mathbb{Z}_N$ with a generator $\theta$ which has the property $\theta^N = 1$. It is always possible to choose the coordinate axes such that

$$\theta = \exp[2\pi i (v_2 J_{45} + v_3 J_{67} + v_4 J_{89})] \tag{3.2}$$

with the $J_{ij}$ being the generator of rotation in the compact $i$-$j$ plane. Further, in the compact directions we define the linear combinations

$$Z^i \equiv \frac{1}{\sqrt{2}}(X^{2i} + iX^{2i+1}), \qquad i \in 2, 3, 4, \tag{3.3}$$

and

$$Z^{\bar{\imath}} \equiv \overline{Z^i} = \frac{1}{\sqrt{2}}(X^{2i} - iX^{2i+1}). \tag{3.4}$$

With this choice of coordinates the periodicity condition is

$$Z^i(\sigma + 2\pi) = \exp(2\pi i v_i) Z^i(\sigma).$$

So far we looked at the bosonic world-sheet fields. We consider now the case of their fermionic partners. On the fermionic world-sheet fields the twist acts as

$$\psi^i(\sigma + 2\pi) = \exp[2\pi i(v_i + \nu)]\psi^i(\sigma),$$

where $\nu = 0$ in the R sector and $\nu = \frac{1}{2}$ in the NS sector. We take the fermionic fields to be complexified analogously to equation (3.3) and equation (3.4). After imposing this periodicity conditions on the Laurent expansions of the world-sheet fields, one can see that the oscillators have the following mode numbers

$$\begin{array}{ll} \alpha^i : n + v_i, & \alpha^{\bar{\imath}} : n - v_i, \\ \tilde{\alpha}^i : n - v_i, & \tilde{\alpha}^{\bar{\imath}} : n + v_i, \\ \psi^i : n + v_i + \nu, & \psi^{\bar{\imath}} : n - v_i + \nu, \\ \tilde{\psi}^i : n - v_i + \nu, & \tilde{\psi}^{\bar{\imath}} : n + v_i + \nu, \end{array} \tag{3.5}$$

with $\nu = \frac{1}{2}$ in the NS sector and $\nu = 0$ in the R sector. The $\alpha$'s correspond to left moving bosonic oscillators and the $\psi$'s to left moving fermionic oscillators. The oscillators with a tilde are the corresponding right-moving oscillators. As $\theta^N = 1$, we can write the $v_i$ as $v_i = n_i/N, n_i \in \mathbb{Z}$. The spacetime transformation of the massless states is given by:

$$\theta : \alpha^i_{-1}|\,0\,\rangle \to e^{2\pi i v_i}\alpha^i_{-1}|\,0\,\rangle, \tag{3.6}$$

$$\theta : \psi^i_{-\frac{1}{2}}|\,0\,\rangle_{NS} \to e^{2\pi i v_i}\psi^i_{-\frac{1}{2}}|\,0\,\rangle_{NS}, \tag{3.7}$$

$$\theta : |\mathbf{s}\rangle_R \to e^{2\pi i v_i s_i}|\mathbf{s}\rangle_R, \tag{3.8}$$

where $|\mathbf{s}\rangle_R$ denotes the spinor representation of the R ground state. $\mathbf{s}$ is a vector with four entries $s_i = \pm\frac{1}{2}$ [37].

To preserve $\mathcal{N}$=2 supersymmetry in D=4, the $v_i$ have to satisfy

$$\sum_{i=2}^{4} v_i = 0 \quad (\text{mod } 2). \tag{3.9}$$

This can be explained in the following way: The R ground state is degenerate and transforms as a spinor representation under the Lorentz group. If we consider a type II theory with unbroken supersymmetry it has $\mathcal{N}$=8 supersymmetry in four dimensions. We want part of the eight supersymmetry generators $Q_i$ to be left unbroken by the orbifold group action. The generators of supersymmetry transform as

$$Q_{\mathbf{s}} \to e^{2\pi i \mathbf{v}\cdot\mathbf{s}} Q_{\mathbf{s}}$$

where $Q_{\mathbf{s}}$ is the supersymmetry generator in the spinor representation. In ten dimensional spacetime the spinor $\mathbf{s}$ has five entries $s_i = \pm\frac{1}{2}$. If some supercharges should be left unbroken this gives the equation

$$\frac{1}{2}\sum_{i=2}^{4} v_i s_i = 0 \pmod 2.$$

This equation leads with an appropriate choice of signs for the $s_i$ to equation (3.9). Namely, if we choose the spinors with the entries $(s_0, s_1, +\frac{1}{2}, +\frac{1}{2}, +\frac{1}{2})$ to be conserved. This automatically gives the spinor $(s_0, s_1, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$ to be conserved, as well. Thus, from the original $2^5 = 32$ supercharges $2^3 = 8$ are conserved. Therefore we obtain $\mathcal{N}$=2 supersymmetry in D=4.

### 3.1.2. Construction of the massless spectrum

The massless spectrum of an orbifold consists of states from the untwisted sector which are invariant under the orbifold group action and states from the twisted sectors.

#### The untwisted sector

The massless untwisted NS states in the uncompactified dimensions $\psi^{\mu}_{-\frac{1}{2}}|\,0\,\rangle_{NS}$ are invariant under the orbifold projection. Left and right movers transform as vectors under the spacetime Lorentz group. Tensoring right and left movers we obtain the graviton, the dilaton and the antisymmetric tensor field in four dimensions. The left handed NS states in the compactified dimensions[1] $\psi^{\pm i}_{-\frac{1}{2}}|\,0\,\rangle_{NS}$ are the scalar matter fields which transform with a phase $e^{\pm 2\pi i v_i}$. Right movers transform in the same way. Thus, a closed NS string state transforms as

$$e^{2\pi i \pm v_i \mp v_j}\psi^{\pm i}_{-\frac{1}{2}}\tilde{\psi}^{\mp j}_{-\frac{1}{2}}|\,0\,\rangle_{NS}.$$

For $\theta$ invariant states, the condition is: $\pm v_i \mp v_j \in \mathbb{Z}$. In a more formal language, we can rephrase this condition as [1]

$$(k - \tilde{k})\cdot v \in \mathbb{Z} \tag{3.10}$$

where $k, \tilde{k} = (0, \underline{\pm 1, 0, 0})$ are SO(8) vector weights which correspond to the bosonised world-sheet fermions, and similarly for states in the R sector: $\frac{1}{2}(\pm v_i \mp v_j) \in \mathbb{Z}$. In the NS-R and R-NS sector, $\theta$ invariant states are obtained in a similar way.

#### The twisted sector

Next we construct the twisted sector. In the $n$-th twisted sector, the ground state energy is $E_0 = \frac{1}{2}\sum_i n|v_i|(1 - n|v_i|)$ [37, 1]. The mass formula and the level matching condition in

[1] For convenience we use the notation $\psi^{-i}_n \equiv \psi^{\bar{\imath}}_n$

the twisted sector is

$$\alpha' m_L^2 = -\frac{1}{2} + \frac{1}{2}(r + nv)^2 + E_0 \tag{3.11}$$

$$\alpha' m_R^2 = -\frac{1}{2} + \frac{1}{2}(\tilde{r} - nv)^2 + E_0 \tag{3.12}$$

$$m_L^2 \stackrel{!}{=} m_R^2, \tag{3.13}$$

where $r, \tilde{r}$ are SO(8) weights. To incorporate the GSO projection for type IIB we impose

$$\sum_{i=0}^{3} r_i = odd, \tag{3.14}$$

and similar for $\tilde{r}$. In the NS-NS sector both $r$ and $\tilde{r}$ are vector weights while in the R-R sector they are spinorial weights. The NS-R and R-NS sector are obtained by the appropriate combination of vector and spinor weights. For the type IIA theory equation (3.14) has to be modified appropriately to take the different GSO projection in the R sector into account. Next we have to consider the multiplicities of the twisted states. In all cases considered in this thesis these are just the numbers of points fixed under the twist. Explicit examples are worked out in the appendices D, E, F.

## 3.2. Orientifolds

In this section we generalise the notion of an orbifold to the notion of an orientifold. We will exemplify the constraints on orientifolds in the type I string theory, which can be seen as an orientifold of type IIB string theory.

### 3.2.1. World-sheet parity and type I theory

We consider type IIB string theory. The massless spectrum of the type IIB theory consists of the graviton $G_{\mu\nu}$, the ani-symmetric tensor $B_{\mu\nu}$ and the dilaton $\Phi$ from the NS-NS sector, the zero-form $A$, the two-form $A_{\mu\nu}$ and the self-dual four-form $A^{(+)}_{\mu\nu\rho\sigma}$ from the R-R sector and two gravitinos transforming in the **56** and two spinors transforming in the spinorial $\mathbf{8}_s$ representation of the spacetime little group SO(8) from the R-NS and NS-R sectors. This theory has the same chirality on the left and right moving part of the string, therefore the world-sheet parity operator $\Omega$, interchanging left and right movers on the world-sheet, is an exact symmetry[2]. $\Omega$ acts on closed string oscillators as follows:

$$\alpha_n^\mu \to \Omega \alpha_n^\mu \Omega^{-1} = \tilde{\alpha}_n^\mu, \tag{3.15}$$

$$\tilde{\alpha}_n^\mu \to \Omega \tilde{\alpha}_n^\mu \Omega^{-1} = \alpha_n^\mu, \tag{3.16}$$

$$\psi_r^\mu \to \Omega \psi_r^\mu \Omega^{-1} = \tilde{\psi}_r^\mu, \tag{3.17}$$

$$\tilde{\psi}_r^\mu \to \Omega \tilde{\psi}_r^\mu \Omega^{-1} = \psi_r^\mu. \tag{3.18}$$

[2] For type IIA we would have to combine $\Omega$ with a spacetime parity reversal.

Therefore the action of $\Omega$ on the massless fields of type IIB theory is

$$\left.\begin{aligned} g_{\mu\nu} &\to g_{\mu\nu} \\ B_{\mu\nu} &\to -B_{\mu\nu} \\ \phi &\to \phi \end{aligned}\right\} \text{NS-NS sector} \tag{3.19}$$

$$\left.\begin{aligned} A &\to -A \\ A_{\mu\nu} &\to A_{\mu\nu} \\ A^{(+)}_{\mu\nu\rho\sigma} &\to -A^{(+)}_{\mu\nu\rho\sigma} \end{aligned}\right\} \text{R-R sector} \tag{3.20}$$

where the $A$'s denote the fields in the R-R sector and the $(+)$ denotes that the four-form field is self-dual.

From this action of $\Omega$, we can infer that in the NS-NS sector the symmetric representations of the Lorentz group, namely the graviton and the dilaton survive, whereas in the R-R sector the antisymmetric two-index tensor survives. Under an $\Omega$ projection, R-NS and NS-R sector are mapped onto each other, which implies that in the fermionic sector one copy of the $\mathbf{8}_s + \mathbf{56}$ is invariant under the projection.

If we project on states which are invariant under $\Omega$, we obtain the closed string spectrum of the type I string theory. To consider only the closed string part of this theory is not consistent, because it gives rise to an anomalous field theory [38]. Therefore we have in addition to include the open string sector of type I. The GSO projection in the open string NS sector gives rise to a gauge vector, transforming in the $\mathbf{8}$ of the spacetime little group SO(8) and gauginos in the $\mathbf{8}_s$ from the R sector. The massless spectrum of the theory consists of a graviton, an antisymmetric tensor and the dilaton from the closed string sector together with a gauge field from the open string sector and their supersymmetric partners from R-NS and NS-R sector. These states can be grouped into the supergravity multiplet and a vector multiplet of $\mathcal{N}$=1 D=10 supersymmetry.

In the open string sector, $\Omega$ reverses the orientation of the string: $\sigma \to \pi - \sigma$. From the mode expansion it follows that $\Omega$ acts on the oscillators as [21]:

$$\Omega : \alpha^\mu_n \to \pm(-1)^n \alpha^\mu_n, \tag{3.21}$$

$$\Omega : \psi^\mu_r \to \pm e^{-\pi i r} \psi^\mu_r, \tag{3.22}$$

where the upper sign is for NN boundary conditions and the lower sign for DD boundary conditions. From equation (3.22) we see that the $\Omega$-eigenvalues of the oscillators in the NS sector are $\pm i$. In order to cancel this factor, we define the action of $\Omega$ on the NS vacuum $|\,0\,\rangle_{NS}$ to be $\Omega|\,0\,\rangle_{NS} = +i|\,0\,\rangle_{NS}$ [21, 28]. This definition implies the correct transformation property for the massless states from this sector:

$$\Omega : \psi^\mu_{-\frac{1}{2}}|\,0\,\rangle_{NS} = -\psi^\mu_{-\frac{1}{2}}|\,0\,\rangle_{NS}. \tag{3.23}$$

As $\Omega$ reverses the direction of the string, it interchanges the endpoints and we must determine the action of $\Omega$ on the Chan-Paton degrees of freedom. For simplicity we derive the the transformation properties for the bosonic string only, the considerations for the superstring are analogous. The $\Omega$ operator transforms an open string state to

$$\Omega : |N,k;ij\rangle\lambda_{ji} \rightarrow \omega_N|N,k;j'i'\rangle\lambda'_{i'j'} \tag{3.24}$$

where $N$ again denotes the occupation numbers of the string state, $k$ its momentum and $\omega_N$ denotes the eigenvalue of $\Omega$ on the string state. This implies for the Chan Paton matrices

$$\Omega : \lambda^a_{ij} \rightarrow \lambda^a_{ji} = s^a \left(\lambda^a_{ij}\right)^T, \tag{3.25}$$

where $s^a$ denotes the eigenvalue of the $\Omega$ action on the CP-factor.

In order to project on states which are invariant under $\Omega$, we take a basis for the $\lambda^a_{ij}$ in which each matrix is either symmetric and therefore has the eigenvalue $s^a = +1$ under $\Omega$, or antisymmetric with eigenvalue $s^a = -1$. In this basis, the $\Omega$ eigenvalue $\omega$ of an open string state is given by $\omega = \omega_N s^a$. Therefore, as can be seen from equation (3.21), for even mass levels ($\alpha' m^2 = 0, 2, 4, \dots$) $\lambda^a$ has to be antisymmetric if the state should be invariant under $\Omega$ and for odd mass levels ($\alpha' m^2 = -1, 1, 3, \dots$) $\lambda^a$ has to be symmetric. The massless gauge bosons correspond to states $\alpha^\mu_{-1}|\,0\,\rangle$ and thus have an antisymmetric Chan-Paton factor. Therefore, the gauge group is SO($n$), since the adjoint representation of SO($n$) consists of the antisymmetric real $n \times n$ matrices. The states at odd mass levels transform in the symmetric representation of SO($n$) and the states at even mass levels transform in the antisymmetric representation.

We can generalise the $\Omega$ projection in the following way. We combine $\Omega$ with a non-trivial U($n$) transformation on the Chan-Paton factors

$$\Omega : |N,k;ji\rangle\lambda_{ij} \rightarrow \omega_N|N,k,i'j'\rangle(\gamma_\Omega)_{j'j}\left(\lambda_{ij}\right)^T(\gamma_\Omega^{-1})_{ii'}. \tag{3.26}$$

As before, the eigenvalue of the complete string state under the action of $\Omega$ has to be $+1$. $\Omega$ has to square to the identity, which imposes the following constraints on the transformation matrices.

$$\Omega^2 : |N,k;ji\rangle\lambda^a_{ij} \rightarrow (\omega_N)^2|N,k,j'i'\rangle\left(\gamma_\Omega(\gamma_\Omega^T)^{-1}\lambda^a\gamma_\Omega^T\gamma^{-1}\right)_{i'j'}. \tag{3.27}$$

This implies

$$\gamma_\Omega^T = \pm\gamma_\Omega. \tag{3.28}$$

If we choose $\gamma_\Omega^T = +\gamma_\Omega$, it is always possible to rotate the basis of the CP-matrices in such a way that we can choose $\gamma_\Omega$ to be the identity matrix. In the antisymmetric case $\gamma_\Omega^T = -\gamma_\Omega$, it is always possible to rotate the basis of the CP-matrices such that we can choose $\gamma_\Omega$ to be

$$\gamma_\Omega = M := i\begin{pmatrix} 0 & \mathbb{1}_k \\ -\mathbb{1}_k & 0 \end{pmatrix} \tag{3.29}$$

As $\gamma_\Omega$ has to be invertible, the rank $n$ of the CP-factor must be an even number $n = 2k$. We choose a basis for the CP-factors such that

$$M(\lambda^a)^T M^{-1} = s^a\lambda^a$$

with $s^a = \pm 1$. If an open string state should be invariant under this $\Omega$ projection, the CP-factors have to satisfy

$$M(\lambda^a)^T M^{-1} = -\lambda^a \tag{3.30}$$

for even mass levels and

$$M(\lambda^a)^T M^{-1} = +\lambda^a \tag{3.31}$$

for odd mass levels. The gauge bosons belong to the even mass levels, and the matrices which satisfy condition (3.30) form the adjoint representation of the symplectic group. Therefore, the gauge group in the case of an $\Omega$ projection which acts via a matrix of the form (3.29) on the CP-factors is the symplectic group USp$(2k)$.

If we now consider again the superstring, these considerations imply that type I theory must have open strings with gauge group SO(32). This can be deduced from the tadpole cancellation conditions. If we start with closed strings only, we obtain a divergent diagram from the tree level partition function of a closed string with an $\Omega$ insertion. This tadpole can be canceled by the introduction of open strings. The contributions of strings exchanged between two D9-branes and strings exchanged between a D9-brane and an orientifold plane cancel the tadpoles. This cancellation occurs exactly for

$$(n \mp 32) = 0 \tag{3.32}$$

with the minus sign for the SO$(n)$ case and with the plus sign for the USp$(n)$ case. Therefore, the type I string theory is only consistent if we introduce 32 D9-branes. In this calculation, the R-R charge of a single D9 brane is $+1$. $\pm 32$ is the R-R charge of the orientifold plane. This charge has to be canceled by the D9-branes. Therefore, we have to choose the minus sign in equation (3.32) in order to cancel the R-R charges of the orientifold plane with the R-R charges of the D9-branes and we obtain the gauge group SO(32) for the type I theory [38, 37]. The computations are carried out in appendix A. The case where $\overline{\text{D9}}$-branes are used is analysed in [52].

### 3.2.2. More general Orientifolds

Let us now consider more general orientifolds [21, 1, 5]. If we take an orbifold with orbifold group $G_1 = \mathbb{Z}_N$, we can combine the action of $\Omega$ with another discrete group $G_2 = \mathbb{Z}_M$, i.e. world sheet parity acts together with a discrete spacetime rotation. Since $\Omega^2 = 1$, it follows that for any two elements $h, h' \in G_2$ the product of the two elements has to belong to $G_1$, since $\Omega h \Omega h' \in G_1$. This is required by closure of the orientifold group. In all cases we consider in this thesis we can write the group by which we divide as

$$\widetilde{G} = \{1, \alpha^1, \ldots, \alpha^{N-1}, \Omega\beta^1, \ldots \Omega\beta^m\}, \tag{3.33}$$

where $\alpha$ is the generator of $G_1$ and $\beta$ is the generator of $G_2$. The condition $(\beta^k)^2 = \alpha^l$ for all must be satisfied for all $k$ to ensure closure of the group. We call $\widetilde{G}$ the "orientifold group"[3].

[3] In the literature sometimes $G_2$ is called the orientifold group and sometimes $\Omega G_2$ is called the orientifold group. But observe that the elements with non-trivial $\Omega$ only form a set as there is no identity element.

### Closed spectra

The spectrum of an orientifold of a type II string theory is the part of the spectrum of the orbifold that is invariant under the $\Omega G_2$-action. If $G_2$ consists only of the identity element we have to symmetrise in the NS-NS sector and to antisymmetrise in the R-R sector to derive the orientifold spectrum[4]. The NS-R and R-NS sectors are mapped onto each other. Therefore one gravitino survives, signaling $\mathcal{N}$=1 supersymmetry in the closed string sector. There exists only one sector that is new to orientifolds. Let $\Omega$ be accompanied with a spacetime rotation group $G_2$ that maps some orbifold fixed points onto each other. This is only possible if $G_1 \subset G_2$. A twisted state at a fixed point $P$ of $G_1$ is mapped under $G_2$ to another fixed point $P'$ of $G_1$. Therefore there can be states present in the spectrum which are $\Omega G_2$-invariant and distributed among different fixed points of $G_1$ An example of such a state occurs in appendix F. The orbifold group of this example is given by $G_1 = \mathbb{Z}_2$ and the group accompanied with $\Omega$ is $G_2 = \mathbb{Z}_4$. They are represented as 180° and 90° degree rotations, respectively on the first complex plane. The $\mathbb{Z}_2$ has four fixed points in that plane of which two are mapped onto each other under $\mathbb{Z}_4$. If we now consider a twisted state at one of the fixed points which is not invariant under the $\mathbb{Z}_4$ twist, its $\Omega$ image lives at the other fixed point. This distribution of twisted states among fixed points is new to orientifolds.

### Open spectra

Orientifolds contain open strings and D-branes, as the type I string theory does. The orientifold group determines which types of branes an orientifold model contains. If the orientifold group contains $\Omega$ without a spacetime rotation the model contains D9-branes. D5-branes are present whenever there is an order two element of the form $\Omega R_i R_j$ in the orientifold group, where $R_i$ acts as a reflection on the $i$th complex plane. The D5-branes fill out $D = 4$ spacetime and the complex torus which is left invariant under $\Omega R_i R_j$. D7-branes are expected if the Orientifold group contains an element of the form $\Omega' R_1$, where $R_1$ reflects only one complex direction. $\Omega'$ here does act as $\Omega(-1)^{F_L}$ where $F_L$ is the *left moving spacetime fermion number* [27, 28, 44]. This can be deduced as follows. In an orientifold with an $\Omega$ projection D9-branes are needed. Let us now perform T-duality in both directions of the $i$th complex plane. By this T-duality transformation $\Omega$ is transformed to $\Omega R_i(-1)^{F_L}$ and the D9-branes are transformed to D7-branes, because the N boundary conditions along the D9-brane are transformed to D boundary conditions, as is explained in section 2.3. If we then T-dualise along another complex plane, say the $j$th one, $\Omega$ is transformed to $\Omega R_i R_j$ and the D7-branes are mapped to D5-branes. For the derivation of the T-duality transformations on $\Omega$ see appendix C. We can visualise this fact in the

[4] In the twisted sector of $\mathbb{Z}_N$ orientifolds with even $N$ exists an ambiguity. It is possible to anti-symmetrise in the NS-NS sector and to symmetrise in the R-R sector in the order two twisted sectors [36, 33].

following diagram.

$$
\begin{array}{c}
D9+\Omega \text{ on } T^2\times T^2\times T^2\\
\Big\downarrow{\scriptstyle T_2}\\
D7+\Omega(-1)^{F_L}R_1 \text{ on } \tilde{T}^2\times T^2\times T^2\\
\Big\downarrow{\scriptstyle T_2}\\
D5+\Omega R_1R_2 \text{ on } \tilde{T}^2\times \tilde{T}^2\times T^2\\
\Big\downarrow{\scriptstyle T_2}\\
D3+\Omega(-1)^{F_L}R_1R_2R_3 \text{ on } \tilde{T}^2\times \tilde{T}^2\times \tilde{T}^2
\end{array}
\tag{3.34}
$$

The last step to D3-branes is included for completion but is not considered in this thesis. However, it is known that there are no conserved charges in type I theory that could live on D7-branes or D3-branes [58]. Therefore this theories really have to be considered as orientifolds of type IIB. From this analysis we can determine which type of branes are present in a given orientifold model. The analysis carries over to the case where supersymmetry on the branes is not conserved.

T-dualising in the $k$th direction combines $\Omega$ with a reflection in this direction. Therefore $\Omega$ relates fields at $x^k$ with fields at $-x^k$. If we have a compact direction with radius $2\pi R$ and perform a T-duality transformation along this direction the $\Omega$ projection transforms the circle to the line segment $0\le x\le \pi R'$ with orientifold fixed planes at the ends. Along the line segment the physics is locally as in an oriented theory because $\Omega$ relates the fields at two different points. Only at the fixed planes we obtain the physics of an unoriented theory. This explains why we obtain $\mathrm{U}(n)$ gauge groups in the orientifold models we are going to calculate.

The orientifold group must act on the Chan-Paton factors in a way consistent with the action on the world-sheet degrees of freedom [21, 1]. For some element $g\in G_1$ of the orientifold group the action is

$$
g: |\psi, ij\rangle\lambda_{ji} \to |g\cdot\psi, i'j'\rangle(\gamma_{g,p})_{j'j}\lambda^{pq}_{ji}(\gamma_{g,q})^{-1}_{ii'}, \tag{3.35}
$$

for some matrix $\gamma_g$ associated to $g$. The indices $p$ and $q$ denote the brane on which $\gamma$ acts. The matrices $\gamma_g$ must form a (projective) representation of the orientifold group. This is required by closure of the orientifold group. If some D-brane $M_i$ is mapped to $g\cdot M_i$ the non-zero elements of $\gamma_g$ have to be exactly those which connect these D-branes. For an element $\Omega h$ with $h\in G_2$ the action is given by

$$
\Omega h: |\psi, ij\rangle\lambda^{pq}_{ji} \to |\Omega h\cdot\psi, j'i'\rangle(\gamma^q_{\Omega h})_{j'j}\left(\lambda^{pq}_{ji}\right)^T(\gamma^p_{\Omega h})^{-1}_{ii'}, \tag{3.36}
$$

where $\gamma_g$ and $\gamma_{\Omega h}$ are unitary matrices.

There are some more constraints on the Chan-Paton matrices. As was shown by Gimon and Polchinski [21] there are no pure gauge twists, i.e. the matrix $\gamma_{\theta^0}$ associated to $1\in G_1$ has to be the unit matrix (up to a phase) and consistent with group multiplication we can define $\gamma_{\Omega g} := \gamma_g \gamma_\Omega$ if $g \in G_1$. Let us consider the case in which $G_1 = G_2 = \mathbb{Z}_N$. The more general cases which will appear in some models we are going to consider work analogous. Let the matrix $\gamma_{\theta^k} \equiv \gamma_k$ correspond to the twist $\theta^k$. We can choose:

$$\gamma_k = \gamma_1^k, \tag{3.37}$$

which leads to

$$\gamma_1^N = \pm 1. \tag{3.38}$$

As in the case for the type I example

$$\gamma_\Omega^T = \pm \gamma_\Omega, \tag{3.39}$$

or more generally from $(\Omega\theta^k)^2 = \theta^{2k}$

$$\gamma_{\Omega k} = \pm \gamma_{2k} \gamma_{\Omega k}^T. \tag{3.40}$$

These relations together with the unitarity of the $\gamma$ matrices imply

$$\gamma_k^* = \pm \gamma_\Omega^* \gamma_k \gamma_\Omega. \tag{3.41}$$

If different branes are present in one model the action of $\Omega^2$ on states between different kind of branes must be analysed. Consider the case, where D9-branes and Dp-branes are present. Our definition of $\Omega$ gives a phase factor $(\pm i)^{(9-p)/2}$ on 9-p states. If we choose the action of $\Omega$ on the CP-matrices of the D9-branes as $\gamma_{\Omega,9} = +\gamma_{\Omega,9}^T$, as it is required by tadpole cancellation, this implies the form of $\gamma_{\Omega,p}$ for $p \neq 9$. (If no D9-branes are present in a model usually the correct factors can be determined by T-duality.) Which conditions the Chan-Paton factors have to satisfy (and thus which gauge groups arise) is mentioned for each analysed example separately.

Let us now analyse the action of $\Omega$ on anti-D-branes. The change of sign of the R-R charge on the anti-brane implies a change of sign of the action of $\Omega$ on states in the open string R sector, as mentioned in section 2.8. Therefore the $\Omega$ projection acts with a relative minus sign on the open string NS sector and the open string R sector of a $\overline{\mathrm{D}}$-brane. This relative minus sign breaks manifestly supersymmetry in the spectrum of an orientifold with anti-branes.

### 3.2.3. Tadpole cancellation conditions

In type I theory the analysis of the cylinder amplitude reveals the following: The complete amplitude vanishes by supersymmetry but the part corresponding to the NS-NS sector and to the R-R sector each are divergent. This divergence signals the presence of tadpoles. Let us focus on the R-R tadpole. The type IIB theory contains $n$-form R-R potentials for all even $n$ with $n$ and $8-n$ equivalent by Hodge duality [25]

$$dA_{(n)} = F_{(n+1)} = \star F_{(10-n-1)} = \star dA_{(10-n-2)}.$$

$\Omega$ projects out all the multiplicities of 4, thus the 2-form, the 6-form and the 10-form potentials remain. The 10-form potential $A_{(10)}$ can exist in 10 dimensions but its field strength is identically zero. The coupling

$$\mu_{10} \int A_{(10)},$$

where $\mu_{10}$ denotes the coupling constant, can appear in the action but there is no kinetic term present therefore the propagator of this field is proportional to $1/0$ giving a divergence

$$\frac{\mu_{10}^2}{0}. \tag{3.42}$$

The tadpole from equation (3.42) is exactly the one which must be canceled if we calculate the Möbius strip and the Klein bottle and choose the gauge group SO(32).

Let us return to more general orientifolds. D-branes and orientifold planes are electric and magnetic sources for R-R fields [34]. The n-form field strength $F_{(n)}$ satisfies

$$dF_{(n)} = \star J_{(9-n)}, \quad \text{and} \quad d \star F_{(n)} = \star J_{(n-1)} \tag{3.43}$$

for some current $p$-form $J_{(p)}$. This is inconsistent with the Bianchi identity and the equations of motion. The field equations are only consistent if

$$\int_{\Gamma_k} \star J_{(10-k)} = 0$$

for all closed curves $\Gamma_k$. In flat ten dimensional spacetime the only non-trivial curves are points $\Gamma_0$, and we obtain the gauge group SO(32). In compact theories there are more constraints.

The divergencies can be determined by the vacuum amplitudes of the Klein bottle, the Möbius strip and the cylinder. From this computations we obtain conditions on the $\gamma$ matrices because the results include traces over the $\gamma$'s.

If we impose an $\Omega$ projection combined with a spacetime rotation on an orbifold there can arise tadpoles in the twisted sectors, as well. An element $\Omega h$ of $\widetilde{G}$ produces a tadpole in the $h^2$ twisted sector at $h$-fixed points. Thus the tadpole cancellation conditions imply more constraints in more general orientifolds.

# 4. Stability of Orientifolds with D-branes and anti-D-branes

We have reviewed orientifold constructions and D-branes to discuss the stability of orientifold models with Dp-branes and $\overline{\text{Dp}}$-branes. We will analyse four different orientifold models with branes and anti-branes in this thesis, from which two are unstable while the other two are stable. The stable models undergo phase transitions, when some moduli are varied.

All models are constructed such that supersymmetry is conserved in the closed string sector. In the open string sector supersymmetry is broken by the introduction of anti-branes. There are two possible ways to remove the tachyonic modes stretching between Dp-branes and $\overline{\text{Dp}}$-branes with the same dimension $p$. One way is to separate branes and anti-branes in the compact dimensions, such that the stretching modes between branes and anti-branes become massive. The second way to remove the tachyon is to project it out by some projection of the orientifold group. Both ideas are used in different models. The strategy to remove the tachyon by some projection of the orientifold group, will turn out to be better suited to stabilise the model.

## 4.1. A toy model

In the first model we take type IIB strings compactified on $T^2 \times T^2$ moded out by $\mathbb{Z}_3$ and $\Omega R \equiv \Omega'$, where $R$ reflects all compact directions. As we explain in section 3.2 this model needs D5-branes to cancel the tadpoles. The tadpole cancellation conditions imply that the total number of D5-branes has to be 32. This model can be made consistent by introducing only D5-branes, no $\overline{\text{D5}}$-branes are needed, but pairs of D5- and $\overline{\text{D5}}$-branes can be added in a consistent manner. The model in which exclusively D5-branes are used, possesses unbroken supersymmetry in the open string sector. We add brane-anti-brane pairs to the supersymmetric setting, to see how the spectrum of a general orientifold changes when we introduce $\overline{\text{D5}}$-branes. The spectra discussed in this section are derived in appendix D. The supersymmetric model was first discussed in [20] and [12], the non-supersymmetric extension was introduced in [5].

We derive the spectrum of the supersymmetric model with the largest possible gauge group. Other models can be obtained by placing some D5-branes at other $\mathbb{Z}_3$ fixed points than the origin. The gauge sector of the supersymmetric model with the maximal gauge group transforms under the group U(8) × SO(16). Matter multiplets transform in the antisymmetric representation of U(8), which is the (**28**,**1**), and in the fundamental representation of U(8) × SO(16), the (**8**,**16**). All branes can be placed at the origin, the result is the above group. Moving some branes to other fixed points results in smaller gauge groups[1].

We want to extend this model by adding brane anti-brane pairs. The difference of the number of branes and anti-branes $n_5 - n_{\bar{5}}$ has to remain 32, as this number is needed to cancel the charges of the orientifold planes. We consider the case where we place 4 D5-branes at the origin, together with 8 $\overline{\text{D5}}$-branes. To obtain a total of 40 D5-branes, we have to add 36 more D5-branes. We place 18 D5-branes at one fixed point in the compact space away from the origin and the remaining 18 at its $\Omega'$ image. As we place branes and anti-branes at the same point in the compact space, a tachyon is present in the D5-$\overline{\text{D5}}$ sector. Thus, to stabilise this model we have to choose the CP-matrices in such a way that the tachyon is projected out by the CP-matrices at the origin. We choose the compactification radii large enough, such that the NS ground state of strings stretching between the D5-branes at the fixed points and the $\overline{\text{D5}}$-branes at the origin are massive. Therefore we do not have to consider a tachyon in this sector. We obtain the gauge group SO(4) on the D5-branes and U(4) on the $\overline{\text{D5}}$-branes. The spectrum on the D5-branes is supersymmetric, containing a D=6, $\mathcal{N}$=1 vector multiplet of SO(4). The spectrum on the anti-branes is not supersymmetric, although the gauge sector contains the same number of fermionic and bosonic degrees of freedom. However, this is an accidental correspondence and can be explained as follows: The CP-matrices of the $\mathbb{Z}_3$ rotation break the original gauge group U(8) down to U(4) × U(4). The $\Omega'$ projection maps the representations of the U(4)'s onto each other and therefore only the degrees of freedom of one U(4) are relevant. This mapping relates in the gauge sector two diagonal blocks of the CP-factors. If we write the CP-factor as

$$\lambda = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

with $A, B, C, D$ being $4 \times 4$ matrices, $\Omega$ imposes

$$A = D^T$$

for the gauge bosons and

$$A = -D^T$$

for the associated fermions. Therefore, fermions and bosons transform in the adjoint representation of U(4). This is not longer the case in the matter sector on the $\overline{\text{D5}}$-branes. In this sector the off-diagonal blocks are relevant, which are mapped onto themselves. $\Omega'$ imposes $B = -B^T$ for the complex scalars, hence they transform in the anti-symmetric representation of the gauge group. For the fermions the $\Omega'$ projection implies $B = +B^T$

[1]One other possible model would be to set 8 D5-branes at the origin and 12 D5-branes at another fixed point '1' and the remaining 12 at its $\Omega'$ image '2'. As the CP-matrices at the fixed points '1' and '2' have to be traceless to satisfy the tadpole cancellation conditions, the gauge group would be $\mathrm{U}(4)^3 \times \mathrm{SO}(8)$. The CP matrices had to be $\mathbb{1}_8$ at the origin and $\mathrm{diag}(\mathbb{1}_4, e^{2\pi i/3}\mathbb{1}_4, e^{-2\pi i/3}\mathbb{1}_4)$ at the other fixed points.

and therefore the fermions transform in the symmetric representation[2]. Hence, the manifest breaking of supersymmetry in orientifolds is due to the action of $\Omega$ on the $\overline{\mathrm{D}}$-branes. The D5-branes at the other fixed points have supersymmetric $\mathrm{U}(6)^3$ spectra. The spectrum is derived in appendix D. The relevant point for our investigation is that complex scalars, transforming in the fundamental representation survive the projection. They can be identified as moduli for the position of the branes and thus it is possible to move the D5-branes from the fixed points into the bulk.

D-branes and $\overline{\mathrm{D}}$-branes feel an attractive force. This fact is explained in section 2.8. Since the D-branes are not stuck at the fixed points, they are free to move into the bulk, can approach the origin and lead to D-$\overline{\mathrm{D}}$ annihilation. The branes have to approach the origin in a way that is invariant under the orientifold projections and respects the tadpole cancellation conditions. The minimal amount of branes that can move away from the fixed points is three per fixed point, because the orbifold projection maps the branes onto each other. When we move $3n$ D5-branes away from the fixed points, the gauge group is reduced to $\mathrm{U}(6-n)^3$. This mechanism is explained in section 2.5. When the branes reach the origin, the CP-matrices for the branes at the origin have to be modified, in order to incorporate the additional branes. They have to fulfill the tadpole cancellation conditions and to form a representation of the orientifold group. Therefore the CP-matrices at the origin have to be modified in the following way[3]

$$\gamma_{1,5,0} = \mathbb{1}_4 \to \gamma'_{1,5,0} = \mathrm{diag}(e^{2\pi i \frac{1}{3}} \mathbb{1}_{2n}, e^{2\pi i \frac{2}{3}} \mathbb{1}_{2n}, \mathbb{1}_{4+2n}) \tag{4.1}$$

because the combination $e^{2\pi i \frac{1}{3}} + e^{2\pi i \frac{2}{3}} + 1 = 0$ does not change the trace and forms a representation of the $\mathbb{Z}_3$ twist. The CP-matrix for $\Omega'$ becomes

$$\gamma_{\Omega',5,0} = \mathbb{1}_4 \to \gamma_{\Omega',5,0} = \begin{pmatrix} 0 & \mathbb{1}_{2n} & 0 \\ \mathbb{1}_{2n} & 0 & 0 \\ 0 & 0 & \mathbb{1}_{4+2n} \end{pmatrix}. \tag{4.2}$$

We obtain always multiples of two in the various sectors at the origin because we have to take the branes from both fixed points into account, which have to be moved simultaneously since $\Omega'$ maps them onto each other. The gauge group with the additional $6n$ D5-branes becomes $\mathrm{U}(2n)_5 \times \mathrm{SO}(4+2n)_5 \times \mathrm{U}(4)_{\bar{5}}$.

This configuration is unstable, because the tachyon is not longer projected out. The tachyon transforms in the bifundamental representation of the unitary groups on the branes and on the anti-branes. This signals that brane anti-brane annihilation is possible. When all branes moved to the origin, all brane-anti-brane pairs which we added can annihilate and the original supersymmetric model is regained. This picture is certainly an oversimplification as it is not very likely that all branes *first* move to the origin and enlarge the gauge group and *afterwards* annihilate with the anti-branes. However, as it is not possible to explain the effect of D-$\overline{\mathrm{D}}$ annihilation quantitatively, this is the only way to get an idea

[2] The same holds for $C$ which is the transformation matrix of the CPT conjugate particle.

[3] $\gamma_{1,5,0}$ denotes the CP-matrix, corresponding to $\theta^1$, acting on the D5-branes, at the origin, see appendix D.

which phenomena occur[4]. For the case that all branes are placed at the origin, the spectrum is analysed in detail in appendix D. The gauge group is $\mathrm{U}(12)_{55} \times \mathrm{SO}(16)_{55} \times \mathrm{U}(4)_{\bar{5}\bar{5}}$. Note, that if each of the $\overline{\mathrm{D5}}$-branes annihilates with one of the D5-branes from the U(12) sector, the gauge group which remains is $\mathrm{U}(8) \times \mathrm{SO}(16)$, which is exactly the gauge group of the supersymmetric model.

The supersymmetric and the non-supersymmetric setting in this model are smoothly connected as nothing prevents the branes to move towards the origin. Therefore placing the branes at the fixed points and projecting out the tachyon does not stabilise this model.

## 4.2. A virtually stable model

In this model we investigate a non-supersymmetric extension of the $\mathbb{Z}_6'$ orientifold model[5]. The supersymmetric model was discussed in [1, 59] the non-supersymmetric model was introduced in [5].

We compactify type IIB string theory on a $T^6$ which we decompose as $T^2 \times T^2 \times T^2$. The orientifold group contains $\Omega$ and an order two element associated with $\Omega$ reversing the first two complex planes. Therefore, we need 32 D9-branes to cancel the $\Omega$ tadpoles and 32 $\mathrm{D5}_3$-branes[6] to cancel the tadpoles, stemming from $\Omega R_1 R_2$. In addition we add $2M$ D5-branes and $2M$ $\overline{\mathrm{D5}}$-branes at different fixed points. The tadpole cancellation conditions

$$n_9 = 32, \tag{4.3}$$
$$n_5 - n_{\bar{5}} = 32, \tag{4.4}$$
$$\mathrm{Tr}\gamma_{k,9} - 2\mathrm{Tr}\gamma_{k,5,L} + 2\mathrm{Tr}\gamma_{k,\bar{5},L} = 0; \qquad k = 1,5, \tag{4.5}$$
$$\mathrm{Tr}\gamma_{2,9} = -8, \tag{4.6}$$
$$\mathrm{Tr}\gamma_{4,9} = 8, \tag{4.7}$$
$$\sum_M \Big( (\mathrm{Tr}\gamma_{2,5,M})^2 - \big(\mathrm{Tr}\gamma_{2,\bar{5},M}\big)^2 \Big) + 16 \sum_L \big(\mathrm{Tr}\gamma_{2,5,L} - \mathrm{Tr}\gamma_{2,\bar{5},L}\big) = -64, \tag{4.8}$$
$$\sum_M \Big( (\mathrm{Tr}\gamma_{4,5,M})^2 - \big(\mathrm{Tr}\gamma_{4,\bar{5},M}\big)^2 \Big) - 16 \sum_L \big(\mathrm{Tr}\gamma_{4,5,L} - \mathrm{Tr}\gamma_{4,\bar{5},L}\big) = -64, \tag{4.9}$$

imply that the branes are stuck at the fixed points and thus cannot move into the bulk and annihilate, as in the model discussed previously. This can be seen from equation (4.5). The CP-matrices of the D9-branes are linked to the CP-matrices of the D5-branes at the fixed points in a non-trivial way. Therefore it is not possible to move some of the branes away from the fixed points without changing the CP-matrices for the D9-branes. Furthermore, the traces of the CP-matrices at different fixed points are related to each other by equation (4.8) and (4.9). There is another reason why it is in general not possible

[4]We could have started with more D5-branes sitting at the origin and then analyse the spectrum, this would have led to the same result.

[5]This model is denoted with a prime in the literature to distinguish it from another $\mathbb{Z}_6$ orientifold with a different (inequivalent) twist vector.

[6]A $\mathrm{D5}_i$ brane wraps the $i$th complex plane, while a $\mathrm{D7}_i$-brane is transverse to it.

to move the branes away from the fixed points. The motion would necessarily have to be $\mathbb{Z}_6$ invariant, but we are free to add a number of brane anti-brane pairs that cannot be moved in a $\mathbb{Z}_6$ invariant way. This implies that the branes cannot be moved away from the fixed points. However, we note that the additional D5-$\overline{\text{D5}}$-brane pairs are not needed to satisfy the tadpole cancellation conditions. Thus if the branes can come close to each other it is still possible to annihilate these pairs.

D5- and $\overline{\text{D5}}$-branes sit at different internal fixed points. As long as the radii of the tori are large enough, the stretching modes between D-branes and $\overline{\text{D}}$-branes are massive and the model is stable. We note that the $\overline{\text{D5}}$-branes cannot annihilate with the D9-branes. This can be seen from the fact that 5-9 strings have ND boundary conditions and are massless. (Another reason is that all D9-branes are needed for consistency of the model.)

As the branes are stuck at the fixed points, the only possibility to destabilise the model would be to move the fixed points. If the radii of the tori decrease, branes and anti-branes can approach each other. The 5-$\overline{5}$ strings eventually become tachyonic and the system is destabilised. In order to see whether this possibility exists, we have to investigate the closed spectrum, as the moduli for varying the geometry of the space reside in the closed string sector. More explicitly, we have to determine wether the states which correspond to the metric in the compactified dimensions survive the projection. The moduli coming from the metric can be used to vary the shape of the tori. We cannot expect that all moduli from the metric survive, as that would indicate that the form of the tori is not fixed. However, because the tori have to be $\mathbb{Z}_6$ invariant, this cannot be the case. On the other hand, we do not expect that the size of the tori is fixed by the orientifold projections. This means that the diagonal elements $G_{ii}$ have to survive the projections. The states which are invariant under the orientifold group in the untwisted closed sector are

$$\psi^{(\mu}_{-1/2}\tilde{\psi}^{\nu)}_{-1/2}|\,0\,\rangle_{NS}\widehat{=}G_{\mu\nu}+\Phi, \tag{4.10}$$
$$\psi^{2}_{-1/2}\tilde{\psi}^{\bar{2}}_{-1/2}+\psi^{\bar{2}}_{-1/2}\tilde{\psi}^{2}_{-1/2}|\,0\,\rangle_{NS}, \tag{4.11}$$
$$\psi^{3}_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2}+\psi^{\bar{3}}_{-1/2}\tilde{\psi}^{3}_{-1/2}|\,0\,\rangle_{NS}, \tag{4.12}$$
$$\psi^{3}_{-1/2}\tilde{\psi}^{3}_{-1/2}|\,0\,\rangle_{NS}, \tag{4.13}$$
$$\psi^{\bar{3}}_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2}|\,0\,\rangle_{NS}, \tag{4.14}$$
$$\psi^{4}_{-1/2}\tilde{\psi}^{\bar{4}}_{-1/2}+\psi^{\bar{4}}_{-1/2}\tilde{\psi}^{4}_{-1/2}|\,0\,\rangle_{NS}, \tag{4.15}$$

as is derived in appendix E. When we rewrite equations (4.11-4.15) such that we extract the real oscillators from the complexified description we see, that the trace of the graviton survives the projection

$$G_{2i\,2i}+G_{2i+1\,2i+1}=$$
$$\frac{1}{\sqrt{2}}(\psi^{i}_{-1/2}\tilde{\psi}^{\bar{\imath}}_{-1/2}+\psi^{\bar{\imath}}_{-1/2}\tilde{\psi}^{i}_{-1/2})|\,0\,\rangle_{NS}=(\psi^{2i}_{-1/2}\tilde{\psi}^{2i}_{-1/2}+\psi^{2i+1}_{-1/2}\tilde{\psi}^{2i+1}_{-1/2})|\,0\,\rangle_{NS}. \tag{4.16}$$

This is the modulus for the compactification radius [37]. Therefore, it is possible to vary the size of the tori. If the size of the compact dimensions becomes small, tachyons can develop, signaling the decay into the supersymmetric model. In the second complex plane some more degrees of freedom survive. They come from the fact that the orbifold projection acts as a $\mathbb{Z}_2$ twist in the second complex plane and a larger class of lattices has a $\mathbb{Z}_2$ invariance. Therefore, it is possible to change the shape of the lattice in the second complex plane. However, we are interested in the question wether it is possible to vary the size of the torus and for that question these moduli are not relevant.

If the brane-anti-brane forces are strong enough to shrink the compact torus, is a question which has not yet been investigated. The question whether brane anti-brane forces can decrease the radii of the internal tori is highly dynamical and out of scope of this thesis. We simply state that if the naive picture is correct and no stabilisation occurs, a model of this kind is unstable.

From this example we see, that it is not sufficient to separate D-branes and $\overline{\text{D}}$-branes. As long as there are no stronger reasons that $\overline{\text{D}}$-branes are needed for consistency, annihilation to a supersymmetric state can still occur. There are two ways to overcome this problem. One way would be to fix the radii of the internal tori at some large value, such that tachyons are absent, which is complicated, as there is only one radius which is singled out, that is the self-dual radius. All other radii are "equally good", from a static analysis none of them is preferred. Any preferences of special compactification radii are dynamical questions, which are in most cases beyond calculational control. The second way to stabilise orientifolds with anti-branes is to construct models that need D-branes and $\overline{\text{D}}$-branes for consistency. If a model is inconsistent without $\overline{\text{D}}$-branes, the anti-branes cannot annihilate without rendering the model inconsistent. These models have intrinsically non-supersymmetric open string sectors. They are not connected to any supersymmetric phase. This is the class of models we will analyse further.

## 4.3. Stable models with phase transitions

The models we discuss in this section are stable against a decay into a supersymmetric phase. The setup of these kind of models is chosen in a way such that they do contain twisted tadpoles but no untwisted tadpoles. I.e. they have zero charge under untwisted R-R forms but non-zero charge under twisted R-R forms. Systems of branes satisfying these conditions are necessarily non-supersymmetric because the untwisted charge of D-branes is always larger than zero. In order to cancel the untwisted charges of the D-branes, $\overline{\text{D}}$-branes have to be introduced. Because no supersymmetric solutions to the tadpole cancellation conditions exist, these models are stable against decay into a supersymmetric configuration. However, a given non-supersymmetric configuration can undergo phase transitions and transform into another (non-supersymmetric) configuration, when some of the geometric moduli are varied. At any value of the moduli at least one and sometimes several configurations of branes do exist which satisfy the tadpole cancellation conditions and are stable. At the points where phase transitions between different configurations appear, the low energy field theory changes drastically. Similar phase transitions have been studied in [45, 28, 42, 19, 43, 6].

We construct a model without untwisted tadpoles in the following way: If the orientifold group contains an element $\Omega h$, this element gives rise to a tadpole from a R-R field in the $h^2$-twisted sector at $h$-fixed points. Therefore, in order to construct a model without untwisted tadpoles, we have to ensure that the orientifold group does not contain elements $\Omega h$ with $h^2 = 1$, i.e. $\Omega$ must not be combined with an order two twist. We will consider a $\mathbb{Z}_{2N}$ action $\hat{\alpha}$ acting crystallographically on $T^4$ or $T^6$. We consider type IIB theory on $T^k/\mathbb{Z}_N$ ($k = 4, 6$), with $\mathbb{Z}_N$ generated by $\theta = \hat{\alpha}^2$. We mod out this theory by $\Omega\hat{\alpha}$. The orientifold group has the structure

$$\widetilde{G} = \{1, \Omega\hat{\alpha}, \hat{\alpha}^2, \dots, \Omega\alpha^{2N-1}\} \tag{4.17}$$

where $N$ must be even,as otherwise $\Omega\hat{\alpha}^N$ would be of order two and hence lead to untwisted tadpoles. The models considered here were introduced in [40].

### 4.3.1. The $T^4/\widetilde{G}$ model

The first model we discuss is type IIB on $T^4$ moded out by

$$\widetilde{G} = \{1, \Omega'\hat{\alpha}, \hat{\alpha}^2, \Omega'\alpha^3\}, \tag{4.18}$$

where $\hat{\alpha}$ generates a $\mathbb{Z}_4$ twist. $\Omega' = \Omega R_1 R_2$ acts as an orientifold projection combined with the reflection of all compact directions. For the case that all compactification radii are large, we need D5-branes and $\overline{\text{D5}}$-branes to cancel the tadpoles in the twisted sector. The supersymmetric closed spectrum consists of the D=6 $\mathcal{N}$=1 supergravity (SUGRA) multiplet, three tensor multiplets and two hypermultiplets from the untwisted sector and another ten tensor multiplets and six hypermultiplets from the twisted sector. The spectrum is derived in appendix F.

In the setting we start with, we place two stacks of eight D5-branes at two $\Omega'\hat{\alpha}$ fixed points in the internal space and another two stacks of eight $\overline{\text{D5}}$-branes at the other two $\Omega'\hat{\alpha}$ fixed points, see figure (4.1). The projections give rise to a symplectic gauge group USp(8) on the branes and the gauge group SO(8) on the anti-branes. The gauge bosons and their fermionic partners survive the $\theta$ projection, however, all matter multiplets are projected out. On the anti-branes gauge bosons and their associated fermions transform under different representations of the gauge group, because $\Omega'$ acts with an additional minus sign on the fermions.

As we consider the case where the compactification radii are large, the $5\overline{5}$-strings are massive and therefore have not to be considered. For further analysis we determine the total tension of the branes which is for a single Dp-brane given by [25, 43, 37]

$$T_p = \frac{1}{(2\pi)^p}\frac{1}{g}, \tag{4.19}$$

where $g$ is the string coupling. Thus for our model we obtain a total tension of

$$T_5 = \frac{1}{(2\pi)^5}\frac{32}{g}. \tag{4.20}$$

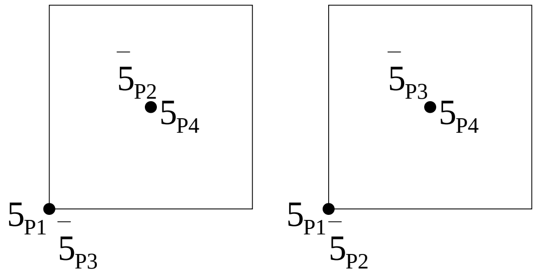


Figure 4.1.: The brane configuration of the $T^4/\widetilde{G}$ orientifold with D5-branes and $\overline{\text{D5}}$-branes

Let us now vary the radius of the first complex plane, while keeping the radius of the second complex plane at a fixed (large) value. The distance of the fixed points in the first complex plane is given by

$$Y^2 = 2\pi^2 R_1^2$$

The mass formula for the tachyonic NS vacuum mode is given by

$$m^2 = -\frac{1}{2\alpha'} + \frac{1}{4\pi^2\alpha'^2} Y^2.$$

Therefore, at $R_1 = \sqrt{\alpha'}$ the tachyonic mode becomes massless. This signals the possibility for the system to undergo a phase transition [45, 29]. Let us now set $\alpha' = 1$ for convenience. The critical radius in this units is given by $R_1 = 1$. The different phases of this model were analysed in [40]. We review this analysis.

Because the mass of the NS ground state becomes negative for $R_1 < 1$, this signals an instability of the model. But in contrast to the models we analysed before, this model cannot decay into a supersymmetric state. Therefore, the tachyon signals a phase transition to another non-supersymmetric brane configuration. At the critical radius the D5-$\overline{\text{D5}}$-branes combine into non-BPS $\widehat{\text{D6}}$-branes, wrapped on a non-supersymmetric cycle on the first complex torus. This class of phase transitions was analysed in [45]. He showed that all configurations of D-branes which appear in this class of model are connected at the critical radius via marginal deformations of the corresponding CFT's. The branes pass through the fixed points $z_1 = 0$ and $z_1 = \frac{1+i}{2}$. Since the $\widehat{\text{D6}}$-branes are not fixed under $\Omega'\hat{\alpha}$, we have to introduce their images under this projection as well. On each of the two $\Omega'\hat{\alpha}$ fixed points in the second torus we place four $\widehat{\text{D6}}$-branes and their image-branes under $\Omega'\hat{\alpha}$. Therefore we obtain a total of 16 $\widehat{\text{D6}}$-branes. The tension of a non-BPS p-brane is

$$T_{\hat{p}} = \frac{1}{(2\pi)^p}\frac{\sqrt{2}}{g}$$

and the branes wrap the first complex plane on a circle passing through $z_1 = 0, \frac{1+i}{2}$. Hence, the total tension is

$$T_{tot} = T_{\hat{6}} \times 16 \times 2\pi\sqrt{2}R_1 = 32 R_1 T_5. \tag{4.21}$$

Thus, the tensions match for $R_1 = 1$ and the transition is energetically possible. The spectrum of this configuration is analysed in the appendix. On the $\widehat{\text{D6}}$-branes, we have to take into account the sector with ordinary GSO projection as well as a sector with the reversed GSO projection. Therefore the tachyonic ground state is not projected out by the GSO projection and thus a priori present on the branes. We have to investigate, if it is projected out under the action of the orientifold group. Let us consider the fate of the tachyonic field along the $\widehat{\text{D6}}$-brane wrapped on a cycle in the first complex plane. We denote the coordinate along the brane with $y$. The tachyon field is a function of $y$ and time $t$. As was shown by Sen [49], the tachyon field $T(y,t)$ on the $\widehat{\text{D6}}$-brane transforms under the orbifold $\mathbb{Z}_2$ projection as

$$T(y,t) \to -T(-y,t)$$

if we decompose the field into its Fourier modes we obtain

$$T(y,t) = \sum_n T_n(t) e^{iny/(\sqrt{2}R_1)}. \tag{4.22}$$

Under $\theta$ the modes transform as

$$T_n(t) \to -T_{-n}(t). \tag{4.23}$$

Therefore, $T_0$ is projected out and for $n \neq 0$ the combination $T_n - T_{-n}$ survives the the $\theta$ projection. The effective $(\text{mass})^2$ of $T_n - T_{-n}$ is given by

$$m_n^2 = -\frac{1}{2} + \frac{n^2}{2R_1^2}. \tag{4.24}$$

Therefore for $R_1 > 1$ the system becomes unstable and decays back into the D5-$\overline{\text{D5}}$-brane system. For $R_1 < 1$ we obtain a tachyon from strings stretching between two different points, e.g. from strings between $(z_1, z_2) = (\frac{1+i}{2}R_1, 0)$ and $(R_1, 0)$. Therefore the system of $\widehat{\text{D6}}$-branes is only marginally stable at $R_1 = 1$. For other values of $R_1$ the system with $\widehat{\text{D6}}$-branes is unstable.

Let us now consider the case $R_1 < 1$. As was shown in [29], for $R_1 < 1$ the system transforms into a system of D7-$\overline{\text{D7}}$-branes, filling the uncompactified directions and the first complex plane. The D7-$\overline{\text{D7}}$-branes sit at $\Omega'\hat{\alpha}$ fixed points in the second complex plane. At this point a question arises. The D7-branes are charged under the R-R 8-form potential which is odd under $\Omega'\hat{\alpha}$ and thus is projected out. We would expect that D7-branes are inconsistent in this model. But as was explained in [58], in the case of a D7-$\overline{\text{D7}}$ brane system $\Omega'$ maps D7-branes on $\overline{\text{D7}}$-branes. Thus the object formed by a D7-brane and a $\overline{\text{D7}}$-brane is still an allowed state in the orientifolded theory. The combined state is neutral under the R-R 8-form, which is consistent with the fact that the 8-form is projected out.

We have to determine the action of the $\theta$ projection on the CP-states. We note that at $R_1 = 1$ the D7-$\overline{\text{D7}}$-system can coexist with the D5-$\overline{\text{D5}}$-branes. 5-7 strings have ND boundary conditions in the first complex plane, and the resulting phase $\pm i$ from the action of $\theta$ on the oscillators should be absorbed in the CP-matrices acting on the D7-$\overline{\text{D7}}$-branes. From this we can conclude that the CP-matrices have the form given in the appendix. In order to reproduce the correct twisted charges, we have to turn on Wilson lines in the first complex plane: The branes fill the first complex plane and thus would contribute to the twisted charges at the $\theta$ fixed points not fixed under $\Omega'\hat{\alpha}$. To cancel this contribution we introduce Wilson lines of the form

$$\gamma_{W_1,7} = \mathrm{diag}(\mathbb{1}_4, -\mathbb{1}_4), \quad \gamma_{W_1,\bar{7}} = \mathrm{diag}(\mathbb{1}_4, -\mathbb{1}_4). \tag{4.25}$$

Consider a state at $z_2 = \frac{1+i}{2}$. Under $\theta$ this state is mapped to itself but only up to two lattice shifts. From each lattice shift we "pick up" one Wilson line. The Wilson line factors are multiplied with the CP-matrices. Equation (4.25) implies that multiplication of the CP-matrices with one Wilson line renders the CP-matrices traceless, whereas the Wilson lines coming from two lattice shifts leave the CP-matrices invariant. Points not fixed under $\Omega'\hat{\alpha}$ pick up only one Wilson line, and thus the CP-matrices become traceless and the tadpole conditions at all fixed points are satisfied. This is sketched in figure 4.2.

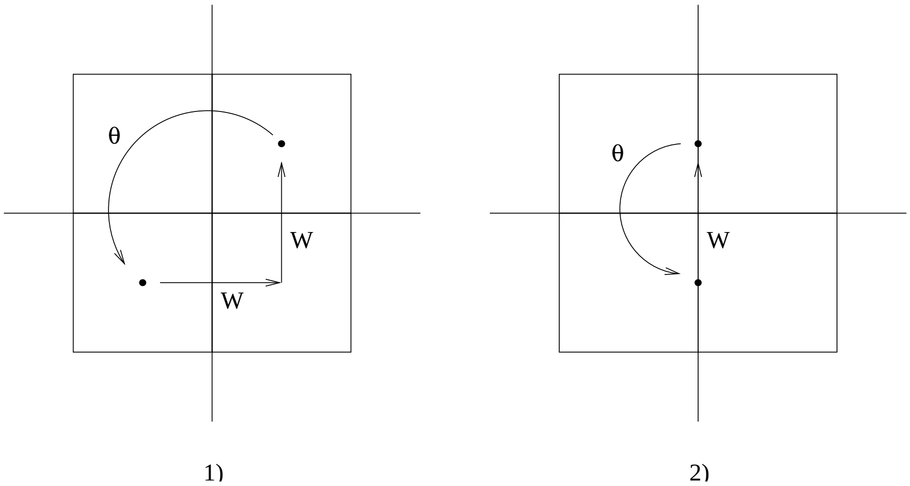


Figure 4.2.: Wilson lines and $\theta$ twist. In 1) the fixed point is mapped upon itself up to two lattice shifts, picking up two Wilson lines. In 2) the fixed point is mapped to itself up to one lattice shift, picking up one Wilson line.

The tension of a D7-brane is

$$T_7 = \frac{1}{4\pi^2} T_5, \tag{4.26}$$

therefore the total tension of the seven-branes at one fixed point is

$$16 \cdot (2\pi R_1)^2 T_7 = 16 \times R_1^2 T_5. \tag{4.27}$$

Thus, at $R_1 = 1$ the tensions are equal. The spectrum, derived in appendix F, consists of the gauge group $\mathrm{U}(4)^4$ plus fermions in the adjoint and in the antisymmetric representation. The different phase transitions of the model are sketched in figure (4.3).

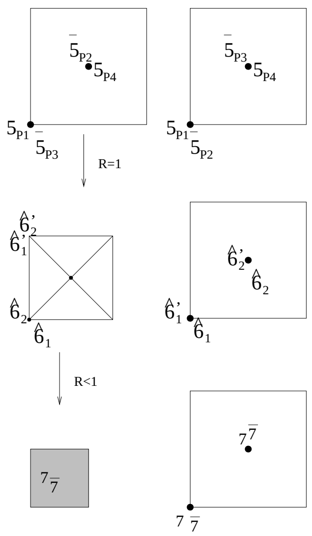


Figure 4.3.: The phases of the $T^4/\widetilde{G}$ orientifold. The lines in the second line denote, how the $\widehat{\mathrm{D}6}$-branes wrap the first torus and the shaded plane in the last line denotes that the first torus is compeletely wrapped by the branes.

We chose the original setup such that the distribution of branes was symmetric in the two complex planes. Hence we could perform the same analysis varying $R_2$ and keeping $R_1$ large. However, we obtain new transitions if we make both radii small. We start with $R_1 < 1$ and consider how the system changes for small $R_2$.

At $R_2 = 1$ the ground state of a string stretching between a D7-brane at $z_2 = 0$ and a $\overline{\mathrm{D}7}$-brane at $z_2 = \frac{1+i}{2} R_2$ becomes massless and leads to tachyons for $R_2 < 1$. We obtain a configuration of non-BPS $\widehat{\mathrm{D}8}$-branes, wrapping a non-supersymmetric cycle in the second complex plane, filling the non-compact space and the first complex plane completely. As in the case of the $\widehat{\mathrm{D}6}$-branes, we have to include the $\Omega'\hat{\alpha}$ image. This configuration is only stable at $R_2 = 1$ and is T-dual to the system of $\widehat{\mathrm{D}6}$-branes, described above. In order to understand this, we have to perform a T-duality in the first plane and then interchange the first and second complex plane.

For $R_1, R_2 < 1$ the stable configuration is a set of 16 D9- and 16 $\overline{\mathrm{D}9}$-branes with Wilson lines to reproduce the correct twisted charges. This system is T-dual to the system of D5- and $\overline{\mathrm{D}5}$-branes we started with and thus the spectra are identical.

Let us investigate how the massless spectrum of the theory changes in the different phases. Starting from the D5-$\overline{\mathrm{D}5}$ system we obtain the gauge group $\mathrm{USp}(8)^2_{55} \times \mathrm{SO}(8)^2_{\overline{55}}$ without any matter multiplets. The fermions associated to the gauge bosons on the $\overline{\mathrm{D}5}$-branes transform under the symmetric representation of SO(8). On the $\widehat{\mathrm{D}6}$-branes we obtain the gauge group $\mathrm{U}(4)\times\mathrm{U}(4)$ with fermions in the adjoint representation and fermions in the antisymmetric representation from the $\hat{6}\hat{6}'$ sector. On the system of D7-$\overline{\mathrm{D}7}$-branes we obtain the gauge group $\mathrm{U}(4)^4$, where each stack of branes at each of the two fixed points contributes $\mathrm{U}(4)^2$. It is interesting to notice that the rank of the gauge groups varies between the settings with D- and $\overline{\mathrm{D}}$-branes, in this case it has rank 16, and the setting with $\widehat{\mathrm{D}6}$-branes, which has a gauge group of rank eight. For the initial setting $\Omega'\hat{\alpha}$ mapped a state onto itself, whereas in the configurations for $R_1 \leq 1$ $\Omega'\hat{\alpha}$ related different sectors to each other. Therefore, in the initial setting the gauge group is a product of $\mathrm{USp}(2k)$ and $\mathrm{SO}(2k)$, whereas in the other settings it is a unitary subgroup of the maximal $\mathrm{U}(n)$. It would be interesting, to see more explicitly how the gauge groups are related. The model we started with and the final model are related in the following way: In the setting with D5-branes the branes are placed at $\Omega'\hat{\alpha}$ fixed points and thus the gauge groups are broken down to $\mathrm{SO}(8)^2$ and $\mathrm{USp}(8)^2$, respectively. This reduces the rank of the gauge group from the maximal possible value of 32 down to 16. All open string states are stuck at orientifold fixed planes. In the final setting the states are no longer constrained to the orientifold fixed planes and thus we obtain a unitary group $\mathrm{U}(4)^4$. However, now D7-branes are related to $\overline{\mathrm{D}7}$-branes and thus the rank of maximal possible gauge group is broken to one-half by this mechanism. In the intermediate case at $R_1 = 1$ this explanation fails. In this case the number of branes is smaller and the rank of the gauge group is reduced by one half of the original one. This cannot be explained with the same arguments as before, although the remarks why the gauge group is $\mathrm{U}(4)^2$ rather than some $\mathrm{SO}(n)$ or $\mathrm{USp}(n)$ group still applies.

However, in the approach we have taken to analyse the model, it is not possible to obtain a more detailed picture of the transformation of the gauge groups. We constructed the different phases by determining the kind of branes which could arise at the various radii and then arranged the CP-matrices to satisfy the tadpole cancellation conditions. In this approach it is not possible to keep track of the evolution of the gauge groups.

### 4.3.2. The $T^6/\widetilde{G}$ model

In this section a type IIB on $T^6/\widetilde{G}$ is discussed. The phase transitions are similar to the transitions in the $T^4/\widetilde{G}$, but were never analysed in detail before. The orientifold group is chosen to be

$$\widetilde{G} = \{1, \Omega'\hat{\alpha}, \hat{\alpha}^2, \Omega'\alpha^3\}, \tag{4.28}$$

where $\hat{\alpha}$ generates a $\mathbb{Z}_4$ twist and $\Omega' = \Omega R_1 R_2 R_3 (-1)^{F_L}$, i.e. $\Omega$ is combined with a reflection of all complex planes and $(-1)^{F_L}$, which is the left-handed spacetime fermion number considered in section 3.2.2. Another $\Omega'$ projection would lead to T-dual models.

The closed spectrum is supersymmetric and consists of the D=4 $\mathcal{N}$=1 SUGRA multiplet, six vector multiplets, the dilaton multiplet and eight additional chiral multiplets from the untwisted sector and another six vector and 26 chiral multiplets from the twisted sector.

To cancel the twisted tadpoles we place 16 $\mathrm{D}7_1$- and 16 $\overline{\mathrm{D}7}_1$-branes at different fixed points in the first complex plane. The branes fill the non-compact directions and the other two complex planes completely. We have to introduce Wilson lines to cancel the contributions to the tadpoles at $\theta$-fixed points not fixed under $\Omega'\hat{\alpha}$. The details of the spectrum are derived in appendix G. On the D7-branes we obtain a $\mathcal{N}$=1 vector multiplet of USp$(8)^2$ and chiral multiplets in the $(\mathbf{36},\mathbf{1})+(\mathbf{1},\mathbf{36})$ of USp$(8)^2$. On the anti-branes we obtain gauge bosons of SO$(8)^2$ and complex scalars transforming in the antisymmetric representation $(\mathbf{28},\mathbf{1})$ + $(\mathbf{1},\mathbf{28})$, whereas all fermions on the anti-branes transform under the symmetric $(\mathbf{35}+\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{35}+\mathbf{1})$ of SO$(8)^2$. As in the model we have considered before, this configuration is only stable as long as the compactification radii are large. To be more precise, because only the first complex plane is not wrapped completely, only the radius of the first torus is of importance for the phase transitions. If we shrink the torus to $R_1=1$, the massive NS ground state stretching between branes and anti-branes becomes massless and signals a phase transition to a non-BPS $\widehat{\mathrm{D8}}$-brane stretching between the fixed points.

The brane tension of a D7-brane is given by

$$T_7=\frac{1}{(2\pi)^7}\frac{1}{g} \tag{4.29}$$

and

$$T_{tot}=32\cdot T_7. \tag{4.30}$$

If we compute the tension of the $\widehat{\mathrm{D8}}$ branes we obtain

$$T_8=2\sqrt{2}\pi R_1\cdot 16\cdot\frac{\sqrt{2}}{(2\pi)^8}\frac{1}{g}=32R_1T_7, \tag{4.31}$$

thus the tensions match at $R_1=1$. As in the previous model the $\widehat{\mathrm{D8}}$-branes are not fixed under $\Omega'\hat{\alpha}$ and we have to include their images under this projection. To produce no twisted charges at the points not fixed under $\Omega'\hat{\alpha}$, Wilson lines along the second complex planes are introduced, breaking U(8) down to U(4)$\times$U(4). Apart from the gauge sector the spectrum consists of a chiral multiplet and two additional fermions, all transforming in the adjoint. The additional fermions come from the sector with the reversed GSO projection. The tachyonic zero mode is odd under the $\mathbb{Z}_2$ twist and is thus projected out. From the strings stretching between the branes and their $\Omega'\hat{\alpha}$ image, we obtain another 4 fermions per fixed point in the antisymmetric representation, giving a total of eight additional fermions from this sector. The complete spectrum is listed in appendix G.

In the region $R_1<1$ the branes transform into pairs of D9-$\overline{\mathrm{D9}}$-branes. Here the analogous discussions as for the D7-$\overline{\mathrm{D7}}$-branes in the previous section applies. The R-R 10-form potential is odd under the $\Omega'$ projection and is projected out. We can see this from T-dualising all the compact directions: We obtain the usual $\Omega$ projection (combined with the $\mathbb{Z}_4$ rotation which is not important for this issue). The ten form becomes a 4-form after the T-duality transformation, which is odd under $\Omega$ and therefore projected out. Thus, we have to consider D9-$\overline{\mathrm{D9}}$ pairs. The tachyon does not survive the $\theta$ projection. The form of the CP-matrices is obtained analogously to the previous model, however to cancel the charges of the D9-$\overline{\mathrm{D9}}$-brane pairs at the $\theta$ fixed points not fixed under $\Omega'\hat{\alpha}$, we have to include Wilson lines at the first and the second complex plane. We choose

$$\gamma_{W_1,9}=\mathrm{diag}(\mathbb{1}_8,-\mathbb{1}_8),\quad \gamma_{W_1,\bar{9}}=\mathrm{diag}(-\mathbb{1}_8,\mathbb{1}_8) \tag{4.32}$$

in the first complex plane. In the second complex plane we choose

$$\gamma_{W_2,9} = \mathrm{diag}(\mathbb{1}_8, -\mathbb{1}_8), \quad \gamma_{W_2,\bar{9}} = \mathrm{diag}(\mathbb{1}_8, -\mathbb{1}_8), \tag{4.33}$$

as before.

The resulting spectrum contains one vector and one chiral multiplet in the adjoint of $\mathrm{U}(8)^2$ on the branes, plus four additional fermions in the anti-symmetric representation from the D9-$\overline{\mathrm{D9}}$ sector.

This is the final state of this model for small $R_1$. Varying the other radii gives no new transitions because the branes wrap the other complex planes completely already at the beginning. The transitions are shown in figure (4.4).

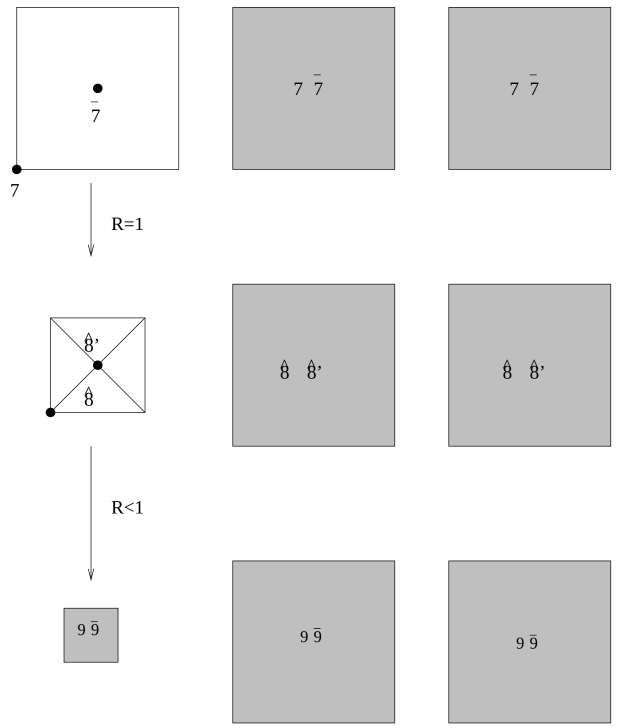


Figure 4.4.: The transitions for varying $R_1$.

From this analysis we conclude that there are orientifolds which are non-supersymmetric and stable against a decay into any supersymmetric state. As we have seen, the models undergo interesting phase transitions if the compactification radii are varied.

# 5. Conclusion and summary

Orientifolds with branes and anti-branes can be stabilised, although simply adding $\overline{\text{D}}$-branes in a way consistent with tadpole cancellation usually leads to unstable models. We have seen this effect in the $\mathbb{Z}_3$ model where D5-branes, initially placed at some fixed points, can move into the bulk and lead to D-$\overline{\text{D}}$ annihilation. Since $\overline{\text{D}}$-branes are not needed for consistency and the branes can be moved into the bulk this model is continuously connected to the supersymmetric setting. Unfortunately we cannot describe the breakdown of the gauge group, when the branes annihilate, but the intuitive picture that each annihilated brane anti-brane pair decreases the gauge group by a factor of $\text{U}(1)_5 \times \text{U}(1)_{\bar{5}}$ seems to be promising, although the idea to describe this procedure on the effective field theory level by a Higgs effect failed. The reasons were given in section 2.9.

The structure of the instabilities in the $\mathbb{Z}_6'$ model is more involved. The branes are stuck at fixed points but the tadpole cancellation conditions can be satisfied without anti-branes. The moduli to vary the compactification radii are present, therefore it is possible that branes annihilate with the anti-branes as the internal radii become small. The remaining model after D-$\overline{\text{D}}$ annihilation would be supersymmetric. In order to make this annihilation possible, the fixed points on which D-branes and $\overline{\text{D}}$-branes are stuck have to approach each other. We showed that it is possible to decrease the compactification radii, because the relevant moduli are present in the spectrum. However, in the approach we have taken it cannot be derived whether brane anti-brane forces are strong enough to shrink the compact tori, but from our analysis there are no obstructions to this assumption. If a stabilisation of the radii takes place, this has to be a dynamical effect, because at the static level there are no other forces present which would prefer larger compactification radii. This model shows that it is possible to satisfy tadpole cancellation conditions in either a supersymmetric and a non-supersymmetric way. The tadpole cancellation conditions in the $\mathbb{Z}_6'$ model link the CP-matrices at different fixed points. In that sense they are different from the tadpole cancellation conditions of the $\mathbb{Z}_3$ model where the conditions could be satisfied at each fixed point separately.

The other two models we considered are stable against the decay into a supersymmetric phase, because there exists no supersymmetric setting of D-branes that can satisfy the tadpole cancellation conditions. $\overline{\text{D}}$-branes have to be included and thus these models are intrinsically non-supersymmetric. This implies that these models belong to another class of orientifolds whose tadpole cancellation conditions cannot be satisfied in a way that preserves supersymmetry. Actually, the $\mathbb{Z}_4$ model in four dimensions was considered to be inconsistent in the earlier literature [1]. In contrast to supersymmetric models, this class of models undergoes phase transitions when the compactification radii are varied. The gauge groups on the branes change between the different phases. It would be interesting to understand exactly what happens to the gauge groups at the points where phase transitions take place. However, from the strategy we used to approach this problem it is not possible to deduce how the gauge groups are linked. We considered which classes of brane-anti-brane systems are stable at the different radii and then arranged the CP-matrices to satisfy the tadpole cancellation conditions. By this procedure it is impossible to see any relations between the gauge groups. In the literature it is not much discussed how the gauge groups in the different phases are related. Only in the original phase we obtain other gauge groups than $\mathrm{U}(n)$. This is related to the fact that only in this setting the CP-factors of the D-branes are mapped onto themselves under $\Omega'\hat{\alpha}$. In all other settings either D- and $\overline{\mathrm{D}}$-branes are interchanged or they are mapped onto some $\Omega'\hat{\alpha}$ image. In both models the ranks of the gauge groups agree for the different settings of branes and anti-branes. In the marginally stable setting with non-BPS branes, the rank of the gauge group is reduced by one-half.

All non-supersymmetric models we have analysed possess a larger number of fermionic degrees of freedom than bosonic degrees of freedom. This can be explained by the fact that the reversed GSO projection, which appears in D-$\overline{\mathrm{D}}$ sector, projects on the tachyonic ground state in the NS sector which has to be projected out by the orientifold group. In the R sector it projects on eight fermionic states with the opposite fermion number which do not have to be projected out. Thus, the number of states in the R sector is larger and they do not have to be projected out to render the model consistent. Therefore it is only possible to construct non-supersymmetric models with more fermions than bosons. From a phenomenological point of view this might be interesting, as there exists more known fermionic matter than bosonic matter. On the other hand the gauge groups and spectra of the models considered in this thesis are far from realistic. They are certainly not of phenomenological interest. But other models with branes and anti-branes with more realistic spectra can be constructed, as was shown in [4, 3]. However a lot of these models suffer the same stability problems which were discussed in this thesis. If some stable non-supersymmetric orientifolds with more realistic spectra can be constructed they might be of great phenomenological interest as supersymmetry is broken at the string scale. On the other hand, it is important to stabilise the compactification radii at large values to lower the string scale appropriately as otherwise the hierarchy problem cannot be solved.

There is another class of non-supersymmetric orientifold models we did not comment on in this thesis which appears to be stable. These orientifold models contain D9-branes and $\overline{\mathrm{D}5}$-branes [5, 4]. As explained in section 4.2, D9-branes cannot annihilate against $\overline{\mathrm{D}5}$-branes and thus this class of orientifolds is completely stable against decay into a supersymmetric state.

For further investigation it would be interesting to consider other orientifold groups than $\mathbb{Z}_4$. However, the action of the orientifold group on the vacuum, namely in the NS-sector with reversed GSO projection, is a problem that has not yet been discussed in the literature for other than the $\mathbb{Z}_4$ case.

# A. Tadpole calculations for type I

We want to review the derivation of the tadpole conditions for the type I theory. Before we start with that, we will define some helpful quantities [37, 21, 10, 17]:

## A.1. $\vartheta$ functions

All amplitudes we have to compute can be expressed in terms of the so-called *Jacobi $\vartheta$-functions* which have special properties under modular transformations. The definitions in this section are based on [37].

The basic $\vartheta$ function is:

$$\vartheta(\nu,\tau) := \sum_{n\in\mathbb{Z}} e^{\pi i n^2\tau + 2\pi i n\nu} \tag{A.1}$$

it has the product representation

$$\vartheta(\nu,\tau) = \prod_{n=1}^{\infty}(1-q^n)(1+zq^{n-1/2})(1+z^{-1}q^{n-1/2}) \tag{A.2}$$

with

$$q = e^{2\pi i\tau}, \quad z = e^{2\pi i\nu}.$$

We can define more general $\vartheta$ functions with *characteristics*:

$$\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(\nu,\tau) = e^{\pi i a^2\tau + 2\pi i a(\nu+b)}\vartheta(\nu + a\tau + b, \tau) \tag{A.3}$$

$$= \sum_{n\in\mathbb{Z}} \exp\left[\pi i(n+a)^2\tau + 2\pi i(n+a)(\nu+b)\right] \tag{A.4}$$

Another useful quantity is *Dedekind's $\eta$-function*:

$$\eta(\tau) = q^{1/24}\prod_{m=1}^{\infty}(1-q^m) \tag{A.5}$$

For completeness we list some other notations for some special cases which are common in the literature:

$$\vartheta\begin{bmatrix}0\\0\end{bmatrix}(\nu,\tau)=\vartheta_{00}(\nu,\tau)=\sum_{n\in\mathbb{Z}}q^{n^2/2}z^n, \tag{A.6}$$

$$\vartheta\begin{bmatrix}0\\1/2\end{bmatrix}(\nu,\tau)=\vartheta_{01}(\nu,\tau)=\sum_{n\in\mathbb{Z}}(-1)^n q^{n^2/2}z^n, \tag{A.7}$$

$$\vartheta\begin{bmatrix}1/2\\0\end{bmatrix}(\nu,\tau)=\vartheta_{10}(\nu,\tau)=\sum_{n\in\mathbb{Z}}q^{(n-1/2)^2/2}z^{n-1/2}, \tag{A.8}$$

$$\vartheta\begin{bmatrix}1/2\\1/2\end{bmatrix}(\nu,\tau)=\vartheta_{11}(\nu,\tau)=-i\sum_{n\in\mathbb{Z}}(-1)^n q^{(n-1/2)^2/2}z^{n-1/2}, \tag{A.9}$$

they have the product expansions

$$\vartheta\begin{bmatrix}0\\0\end{bmatrix}(\nu,\tau)=\prod_{m=1}^{\infty}(1-q^m)(1+zq^{m-1/2})(1+z^{-1}q^{m-1/2}), \tag{A.10}$$

$$\vartheta\begin{bmatrix}0\\1/2\end{bmatrix}(\nu,\tau)=\prod_{m=1}^{\infty}(1-q^m)(1-zq^{m-1/2})(1-z^{-1}q^{m-1/2}), \tag{A.11}$$

$$\vartheta\begin{bmatrix}1/2\\0\end{bmatrix}(\nu,\tau)=2q^{1/8}\cos\pi\nu\prod_{m=1}^{\infty}(1-q^m)(1+zq^m)(1+z^{-1}q^m), \tag{A.12}$$

$$\vartheta\begin{bmatrix}1/2\\1/2\end{bmatrix}(\nu,\tau)=-2q^{1/8}\sin\pi\nu\prod_{m=1}^{\infty}(1-q^m)(1-zq^m)(1-z^{-1}q^m). \tag{A.13}$$

Their modular transformations are

$$\vartheta\begin{bmatrix}0\\0\end{bmatrix}(\nu,\tau+1)=\vartheta\begin{bmatrix}0\\1/2\end{bmatrix}(\nu,\tau), \tag{A.14}$$

$$\vartheta\begin{bmatrix}0\\1/2\end{bmatrix}(\nu,\tau+1)=\vartheta\begin{bmatrix}0\\0\end{bmatrix}(\nu,\tau), \tag{A.15}$$

$$\vartheta\begin{bmatrix}1/2\\0\end{bmatrix}(\nu,\tau+1)=e^{\pi i/4}\vartheta\begin{bmatrix}1/2\\0\end{bmatrix}(\nu,\tau), \tag{A.16}$$

$$\vartheta\begin{bmatrix}1/2\\1/2\end{bmatrix}(\nu,\tau+1)=e^{\pi i/4}\vartheta\begin{bmatrix}1/2\\1/2\end{bmatrix}(\nu,\tau). \tag{A.17}$$

and

$$\vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix} (\nu/\tau, -1/\tau) = (-i\tau)^{1/2} e^{\pi i \nu^2/\tau} \vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix} (\nu, \tau), \tag{A.18}$$

$$\vartheta \begin{bmatrix} 0 \\ 1/2 \end{bmatrix} (\nu/\tau, -1/\tau) = (-i\tau)^{1/2} e^{\pi i \nu^2/\tau} \vartheta \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} (\nu, \tau), \tag{A.19}$$

$$\vartheta \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} (\nu/\tau, -1/\tau) = (-i\tau)^{1/2} e^{\pi i \nu^2/\tau} \vartheta \begin{bmatrix} 0 \\ 1/2 \end{bmatrix} (\nu, \tau), \tag{A.20}$$

$$\vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} (\nu/\tau, -1/\tau) = (-i\tau)^{1/2} e^{\pi i \nu^2/\tau} \vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} (\nu, \tau). \tag{A.21}$$

For $\nu = 0$, $t := i\tau$, which are the cases we need, these transformations are

$$\vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix} (0, 1/t) = \sqrt{t} \vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix} (0, t) \tag{A.22}$$

$$\vartheta \begin{bmatrix} 0 \\ 1/2 \end{bmatrix} (0, 1/t) = \sqrt{t} \vartheta \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} (0, t) \tag{A.23}$$

$$\vartheta \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} (0, 1/t) = \sqrt{t} \vartheta \begin{bmatrix} 0 \\ 1/2 \end{bmatrix} (0, t) \tag{A.24}$$

$$\vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} (0, 1/t) = -i\sqrt{t} \vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} (0, 1/t) \tag{A.25}$$

they satisfy Jacobi's "*aequatio identico satis abstrusa*"

$$\vartheta_{00}^4(0, t) - \vartheta_{01}^4(0, t) - \vartheta_{10}^4(0, t) = 0 \tag{A.26}$$

and

$$\vartheta_{11}(0, t) = 0 \tag{A.27}$$

Another notation, which is often used, is:

$$Z^a{}_b(t) = \frac{1}{\eta(t)} \vartheta \begin{bmatrix} a/2 \\ b/2 \end{bmatrix} (0, t) \tag{A.28}$$

and

$$f_1(q) = q^{1/12} \prod_{n=1}^{\infty} \left(1 - q^{2n}\right) = \eta(t^2), \tag{A.29}$$

$$f_2(q) = q^{1/12} \sqrt{2} \prod_{n=1}^{\infty} \left(1 + q^{2n}\right) = \left(\frac{\vartheta_{10}(t^2)}{\eta(t^2)}\right)^{1/2}, \tag{A.30}$$

$$f_3(q) = q^{-1/24} \prod_{n=1}^{\infty} \left(1 + q^{2n-1}\right) = \left(\frac{\vartheta_{00}(t^2)}{\eta(t^2)}\right)^{1/2}, \tag{A.31}$$

$$f_4(q) = q^{-1/24} \prod_{n=1}^{\infty} \left(1 - q^{2n-1}\right) = \left(\frac{\vartheta_{01}(t^2)}{\eta(t^2)}\right)^{1/2}. \tag{A.32}$$

For $\nu = 0$ and for $a \in ]-1/2, 1/2]$, the $\vartheta$ functions have the following product representation[1]:

$$\frac{\vartheta\begin{bmatrix} a \\ b \end{bmatrix}}{\eta}(\tau) = e^{2\pi iab} q^{a^2/2-1/24} \prod_{n=1}^{\infty} \left[\left(1 + q^{n-1/2+a} e^{2\pi ib}\right)\left(1 + q^{n-1/2-a} e^{-2\pi ib}\right)\right]. \tag{A.33}$$

For $t := i\tau$ the modular transformation property under $\tau \to -1/\tau$ is

$$\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(t^{-1}) = \sqrt{t} e^{2\pi ab} \vartheta\begin{bmatrix} -b \\ a \end{bmatrix}(t) \tag{A.34}$$

$$\eta(t^{-1}) = \sqrt{t}\eta(t) \tag{A.35}$$

for more general characteristics.

## A.2. Divergencies of the type I superstring

To compute the tadpole cancellation conditions for the type I superstring we have to compute the cylinder diagram, the Möbius strip and the Klein bottle [37, 38]. The traces with the appropriate projection operators are:

$$\begin{aligned} \mathcal{C} &= \int_0^\infty \frac{dt}{4t} \mathrm{Tr}_{NS-R}\left[\frac{1+(-1)^F}{2}(-1)^{\mathbf{F}} e^{-2\pi t L_0}\right] \\ &= iV_{10} \frac{1}{2} 2n^2 \int_0^\infty \frac{dt}{4t} \int \frac{d^{10}p}{(2\pi)^{10}} \sum_i \frac{1+(-1)^F}{2}(-1)^{\mathbf{F}} e^{-2\pi t\alpha'(p^2+m_i^2)} \end{aligned} \tag{A.36}$$

$$\begin{aligned} \mathcal{MS} &= \int_0^\infty \frac{dt}{2t} \mathrm{Tr}_{NS-R}\left[\frac{\Omega}{2}\frac{1+(-1)^F}{2}(-1)^{\mathbf{F}} e^{-2\pi t L_0}\right] \\ &= \pm iV_{10} n \int_0^\infty \frac{dt}{2t} \int \frac{d^{10}p}{(2\pi)^{10}} \sum_i \frac{\Omega}{2}\frac{1+(-1)^F}{2}(-1)^{\mathbf{F}} e^{-2\pi t\alpha'(p^2+m_i^2)} \end{aligned} \tag{A.37}$$

$$\begin{aligned} \mathcal{KB} &= \int_0^\infty \frac{dt}{4t} \mathrm{Tr}_{NSNS+RR}\left[\frac{\Omega}{2}\frac{1+(-1)^F}{2} e^{-2\pi t(L_0+\tilde{L}_0)}\right] \\ &= iV_{10} \int_0^\infty \frac{dt}{2t} \int \frac{d^{10}p}{(2\pi)^{10}} \sum_i \frac{\Omega}{2}\frac{1+(-1)^F}{2} e^{-2\pi t\alpha'(p^2/2+m_i^2+\tilde{m}_i^2)} \end{aligned} \tag{A.38}$$

[1] $a$ is defined mod $\mathbb{Z}$, as can be seen from A.4

$\frac{1+(-1)^F}{2}$ is the GSO-projection operator and $(-1)^{\mathbf{F}}$ is the spacetime fermion number operator. The factor of $i$ in front of the integrals comes from the Euclidean signature we use in this section for spacetime and the sum runs over the physical states of the Hilbert space.

## A.2.1. The cylinder

We will carry out the steps for computing the cylinder amplitude in turn.

**Momentum Integration:** The momentum part of the integral is an ordinary Gaussian integral:

$$\int \frac{dt}{4t} \int \frac{d^{10}p}{(2\pi)^{10}} e^{-2\pi t\alpha' p^2} \times \mathrm{Tr}(osc.) = \tag{A.39}$$

$$\int \frac{dt}{4t} \frac{1}{(2\pi)^{10}} \sqrt{\frac{\pi}{2\pi\alpha' t}}^{10} \times \mathrm{Tr}(osc.) = \tag{A.40}$$

$$\int \frac{dt}{4t} \frac{1}{(8\pi^2\alpha' t)^5} \times \mathrm{Tr}(osc.) \tag{A.41}$$

**Traces:** The traces can be split up into a NS and a R part

$$\mathrm{Tr}_{NS-R}(\mathcal{P}_{GSO} e^{-2\pi t N_{NS+R}}) = \mathrm{Tr}_{NS}(osc.) - \mathrm{Tr}_R(osc.). \tag{A.42}$$

Both expressions have a bosonic part which we evaluate first:

$$\begin{aligned} &\mathrm{Tr}\left(e^{-2\pi t \sum_{n>0} \alpha_{-n}\cdot\alpha_n}\right) = \sum_{N_B} \langle N_B | q^{\sum_{n>0} \alpha_{-n}\cdot\alpha_n} | N_B \rangle = \\ &\sum_{N_B} \langle N_B | \prod_{i=1}^{8} \prod_n q^{\alpha^i_{-n}\alpha^i_n} | N_B \rangle = \prod_{i=1}^{8} \prod_n \sum_{N_B} \langle N_B | q^{\alpha^i_{-n}\alpha^i_n} | N_B \rangle = \\ &\prod_{i=1}^{8} \prod_n \sum_{N_B} q^{nN_B} = \prod_{i=1}^{8} \left(\prod_n \frac{1}{1-q^n}\right) = \left(\prod_n \frac{1}{1-q^n}\right)^8 \end{aligned} \tag{A.43}$$

The product over $i = 1, \ldots, 8$ comes from the spacetime dimensions and the light-cone gauge we are using and $N_B$ denotes the bosonic occupation numbers. The fermionic part can be split up in a part with $(-1)^F$ in the trace and one without. The part without the insertion gives:

$$\mathrm{Tr}_{NS}\left(\frac{1}{2} e^{-2\pi t (\sum_{r>0} r\psi_{-r}\cdot\psi_r - \frac{1}{2})}\right) = \frac{1}{2} e^{\pi t} \sum_{N_F=0,1} \langle N_F | q^{\sum_{r>0} r\psi_{-r}\cdot\psi_r} | N_F \rangle =$$

$$\frac{1}{2} e^{\pi t} \sum_{N_F=0,1} \langle N_F | \prod_{i=1}^{8} \prod_{r>0} q^{r\psi^i_{-r}\psi^i_r} | N_F \rangle = \frac{1}{2} e^{\pi t} \prod_{i=1}^{8} \prod_{r>0} (1+q^r) = \tag{A.44}$$

$$\frac{1}{2} q^{-1/2} \prod_{i=1}^{8} \prod_{n=1}^{\infty} (1+q^{n-1/2}) = \frac{1}{2} q^{-1/2} \left[\prod_{n=1}^{\infty} (1+q^{n-1/2})\right]^8 \tag{A.45}$$

For the part with $(-1)^F$, a relative sign between 1 and $q$ arises. As in the NS sector $(-1)^F|\,0\,\rangle_{NS} = -|\,0\,\rangle_{NS}$, we obtain

$$\mathrm{Tr}_{NS}\left(\frac{(-1)^F}{2}e^{-2\pi t(\sum_{r>0} r\psi_{-r}\cdot\psi_r - \frac{1}{2})}\right) = -\frac{1}{2}q^{-1/2}\left[\prod_{n=1}^{\infty}\left(1-q^{n-1/2}\right)\right]^8 . \tag{A.46}$$

The R part of the trace has only contributions without the $(-1)^F$ insertion. With an additional factor of 16 from the degenerate vacua and with the fact in mind, that we have integer moded fermionic oscillators the result is

$$\mathrm{Tr}_{R}\left(\frac{1}{2}e^{-2\pi t(\sum_{r>0} r\psi_{-r}\cdot\psi_r - \frac{1}{2})}\right) = 8\left[\prod_{n=1}^{\infty}(1+q^n)\right]^8 . \tag{A.47}$$

Combining all results we obtain:

$$\mathcal{C} = iV_{10}n^2\int_0^\infty \frac{dt}{8t}\frac{1}{(8\pi^2\alpha' t)^5}\times$$
$$\frac{q^{-1/2}\prod_{n=1}^{\infty}\left(1+q^{n-1/2}\right)^8 - q^{-1/2}\prod_{n=1}^{\infty}\left(1-q^{n-1/2}\right)^8 - 8\prod_{n=1}^{\infty}(1+q^n)^8}{\prod_n (1-q^n)^8}, \tag{A.48}$$

which we can rewrite as

$$\mathcal{C} = iV_{10}n^2\int_0^\infty \frac{dt}{8t}\frac{1}{(8\pi^2\alpha' t)^5}\frac{\vartheta_{00}^4-\vartheta_{01}^4-\vartheta_{10}^4}{\eta^{12}}(t). \tag{A.49}$$

### A.2.2. The Möbius strip

The calculations for the Möbius strip are similar, but we have to include an $\Omega$ insertion into the trace. The action of $\Omega$ on the oscillators is described in section (3.2.1). The momentum integration does not change. The integration over the bosonic part gives

$$\mathrm{Tr}\left(e^{-2\pi t\sum_{n>0}\alpha_{-n}\cdot\alpha_n}\frac{\Omega}{2}\right) = \sum_{N_B}\langle N_B|q^{\sum_{n>0}\alpha_{-n}\cdot\alpha_n}\frac{\Omega}{2}|N_B\rangle =$$
$$\sum_{N_B}\langle N_B|\prod_{i=1}^{8}\prod_n q^{\alpha_{-n}^i\alpha_n^i}\frac{\Omega}{2}|N_B\rangle = \prod_{i=1}^{8}\prod_n\sum_{N_B}\langle N_B|q^{\alpha_{-n}^i\alpha_n^i}\frac{\Omega}{2}|N_B\rangle =$$
$$\prod_{i=1}^{8}\prod_n\sum_{N_B}(-1)^{nN_B}q^{nN_B} = \left(\prod_n\frac{1}{1-q^{2n}}\frac{1}{1+q^{2n+1}}\right)^8 = q^{+8/24}\frac{1}{f_1^8 f_3^8}. \tag{A.50}$$

The NS-part is a bit tricky. With our definitions (see 3.2.1) the action of $\Omega$ on the vacuum gives a $e^{\pi i/2}$. We can work out the trace as follows: We need to compute

$$\mathrm{Tr}_{NS}\left((1+(-1)^F)e^{-2\pi t(\sum_{r>0} r\psi_{-r}\cdot\psi_r-\frac{1}{2})}\Omega\right) =$$

$$\sum_{N_F} e^{\pi t}\langle N_F|\prod_{i=1}^{8}\prod_{r>0} e^{-2\pi t(r\psi_{-r}\cdot\psi_r)}(1+(-1)^F)\Omega|N_F\rangle e^{\pi t} =$$

$$e^{\pi(t-i/2)}\left[\prod_{n=1}^{\infty}\left(1+e^{-2\pi(n-1/2)(t-i/2)}\right)^8 - \prod_{n=1}^{\infty}\left(1-e^{-2\pi(n-1/2)(t-i/2)}\right)^8\right], \tag{A.51}$$

with $q = e^{-\pi(t-i/2)}$ this gives

$$q^{-1}\left[\prod_{n=1}^{\infty}\left(1+q^{2n-1}\right)^8 - \prod_{n=1}^{\infty}\left(1-q^{2n-1}\right)^8\right] = q^{-16/24}(f_3^8(q) - f_4^8(q)). \tag{A.52}$$

With help of the 'abstruse identity' this becomes

$$q^{-16/24}f_2^8(q) = 16\prod_{n=1}^{\infty}\left(1+q^{2n}\right)^8 = \tag{A.53}$$

$$16\prod_{k=1}^{\infty}\left(1+e^{-2\pi(t-i/2)2k}\right)^8\prod_{l=1}^{\infty}\left(1+e^{-2\pi(t-i/2)(2l-1)}\right)^8 = \tag{A.54}$$

$$16\prod_{k=1}^{\infty}\left(1+e^{-2\pi t 2k}\right)^8\prod_{l=1}^{\infty}\left(1-e^{-2\pi t(2l-1)}\right)^8 = \tag{A.55}$$

$$q^{-8/24}f_2^8(e^{-2\pi t})f_4^8(e^{-2\pi t}). \tag{A.56}$$

Therefore the NS part and the bosonic part combine to

$$\mathrm{Tr}_{NS} = \frac{f_1^8(e^{-2\pi t})f_3^8(e^{-2\pi t})}{f_2^8(e^{-2\pi t})f_4^8(e^{-2\pi t})} = \frac{\vartheta_{10}^4\vartheta_{01}^4}{\eta^{12}\vartheta_{00}^4}. \tag{A.57}$$

The Ramond sector is straight forward to compute

$$\mathrm{Tr}_R\left(e^{-2\pi t(\sum_{r>0} r\psi_{-r}\cdot\psi_r-\frac{1}{2})}\Omega\right) = \sum_{N_F} e^{\pi t}\langle N_F|\prod_{i=1}^{8}\prod_{r=1} e^{-2\pi t(r\psi_{-r}\cdot\psi_r)}(1+(-1)^F)\Omega|N_F\rangle =$$

$$16\prod_{r=1}^{\infty}(1+(-1)^r q^r)^8 = 16\prod_{k=1}^{\infty}\left(1+q^{2k}\right)^8\prod_{l=1}^{\infty}\left(1-q^{2l-1}\right)^8 = -q^{-8/24}f_2^8 f_4^8, \tag{A.58}$$

with an extra $(-)$ sign for spacetime fermions. This combines with the bosonic part to

$$\mathrm{Tr}_R = -\frac{f_1^8(e^{-2\pi t})f_3^8(e^{-2\pi t})}{f_2^8(e^{-2\pi t})f_4^8(e^{-2\pi t})} = -\frac{\vartheta_{10}^4\vartheta_{01}^4}{\eta^{12}\vartheta_{00}^4}. \tag{A.59}$$

### A.2.3. The Klein bottle

The computation of the Klein bottle is very similar to the cylinder, because the $\Omega$ insertion projects on states which are left-right symmetric. Thus we obtain the result for the cylinder with doubled weight. The trace we have to compute is

$$\mathcal{KB} = \int_0^\infty \frac{dt}{4t}\mathrm{Tr}_{NSNS+RR}\left[\frac{\Omega}{2}\frac{1+(-1)^F}{2}e^{-2\pi t(L_0+\tilde{L}_0)}\right] \tag{A.60}$$

$$= iV_{10}\int_0^\infty \frac{dt}{2t}\int\frac{d^{10}p}{(2\pi)^{10}}\sum_i \frac{\Omega}{2}\frac{1+(-1)^F}{2}e^{-\pi t\alpha'(p^2/2+m_i^2+\tilde{m}_i^2)}.$$

Taking into account that only left-right symmetric states contribute, the computation is straight forward. The result is

$$\mathcal{KB} = iV_{10}\int_0^\infty \frac{dt}{8t}\frac{1}{(4\pi^2\alpha' t)^5}\frac{f_3^8(e^{-2\pi t})-f_2^8(e^{-2\pi t})}{f_1^8(e^{-2\pi t})} = iV_{10}\int_0^\infty \frac{dt}{8t}\frac{1}{(4\pi^2\alpha' t)^5}\frac{f_4^8(e^{-2\pi t})}{f_1^8(e^{-2\pi t})}. \tag{A.61}$$

For both NS-NS and R-R exchange with a relative minus-sign.

In the next step we have to translate all the amplitudes into the closed string picture and to extract the behaviour for $t \to 0$. To relate the behaviour of the amplitudes we have to rescale them such that the range of the coordinates is the same for all amplitudes. We have actually already imposed the circumference to be $2\pi$ and to rescale the length uniformly we have to exchange $t \to \frac{\pi}{s}, \frac{\pi}{4s}, \frac{\pi}{2s}$ for the cylinder, the Möbius strip and the Klein bottle, respectively. (Thus the $t \to 0$ limit is the same as the $s \to \infty$ limit.) Replacing $t$ by $s$ and then performing a modular transformation $s \to 1/s$ we can extract the divergent parts of the amplitudes. (Each amplitude by itself vanishes by supersymmetry but the parts corresponding to R-R exchange and NS-NS exchange should vanish separately because they signal tadpoles which render the theory inconsistent.)

Extracting the divergent parts from the R-R exchange we obtain:

$$\mathcal{C} = \frac{iV_{10}}{(8\pi^2\alpha')^5 8\pi}n^2\int ds(16+O(e^{-2s})), \tag{A.62}$$

$$\mathcal{MS} = \pm 2n2^5\frac{iV_{10}}{(8\pi^2\alpha')^5 8\pi}\int ds(16+O(e^{-2s})), \tag{A.63}$$

$$\mathcal{KB} = 2^{10}\frac{iV_{10}}{(8\pi^2\alpha')^5 8\pi}\int ds(16+O(e^{-2s})). \tag{A.64}$$

$$\tag{A.65}$$

Thus the total divergence is

$$\mathcal{Z} = -(n\mp 32)^2\frac{iV_{10}}{(8\pi^2\alpha')^5 8\pi}\int ds(16+O(e^{-2s})). \tag{A.66}$$

The signs in the amplitude from the Möbius strip come from the action of $\Omega$ on the Chan-Paton indices with the upper sign for $\gamma_\Omega = \mathbb{1}$. This shows that choosing SO(32) as the gauge group cancels the problematic tadpoles.

# B. Brane-Anti-Brane Forces

We will calculate the attractive force between a Dp-brane and a $\overline{\text{Dp}}$-brane. The calculation was first carried out in [9]. We have to focus on the cylinder amplitude, thus we can to a large extend adopt the results from appendix A. The cylinder amplitude for two Dp-branes with a distance $|Y|$ is given by

$$\mathcal{A} = \frac{iV_{p+1}}{2} \int_0^\infty \frac{dt}{t} \frac{1}{(8\pi^2\alpha' t)^{(p+1)/2}} e^{-\frac{|Y|^2 t}{2\pi\alpha'}} \times \\ \frac{-16\prod_{n=1}^\infty (1+q^n)^8 + q^{-1/2}\prod_{n=1}^\infty \left(1+q^{n-1/2}\right)^8 - q^{-1/2}\prod_{n=1}^\infty \left(1-q^{n-1/2}\right)^8}{\prod_{n=1}^\infty (1-q^n)^8}, \tag{B.1}$$

with $q = e^{-2\pi t}$. If we calculate the amplitude between a brane and an anti-brane we have to change the sign in front of the part of the amplitude which belongs to the closed string R-R exchange. Thus we have to interchange the sign in front of the last expression in the numerator. Extracting a factor of $q^{-1/2}$ from the last expression the amplitude can be rewritten

$$\mathcal{A} = \frac{iV_{p+1}}{2} \int_0^\infty \frac{dt}{t} \frac{1}{(8\pi^2\alpha' t)^{(p+1)/2}} e^{-\pi t\left(\frac{|Y|^2}{2\pi^2\alpha'}-1\right)} \\ \frac{1}{2} \frac{-16q^{1/2}\prod_{n=1}^\infty (1+q^n)^8 + \prod_{n=1}^\infty \left(1+q^{n-1/2}\right)^8 + \prod_{n=1}^\infty \left(1-q^{n-1/2}\right)^8}{\prod_{n=1}^\infty (1-q^n)^8}. \tag{B.2}$$

In order to simplify the calculations we rescale $t \to \frac{t}{4\alpha'}$ and focus on the case for D0-branes. The amplitude now is[1]

$$\mathcal{A} = V_1 \int_0^\infty \frac{dt}{t} (2\pi t)^{-1/2} e^{-\frac{t}{4\alpha'}\left(\frac{Y^2}{2\pi^2\alpha'}-1\right)} F(q), \tag{B.3}$$

where $F(q)$ contains the oscillator contributions (without the extracted factor $q^{-1/2}$). $F$ tends to zero for $t \to 0$ and $F(q) \to 1$ for $t \to \infty$. For $Y^2 < 2\pi^2\alpha'$ the integral diverges. We differentiate this expression with respect to $Y$ and obtain with $Z := \frac{1}{4\alpha'}(\frac{Y^2}{2\pi^2\alpha'} - 1)$

$$\mathcal{F} = \frac{Y}{8\pi^3\alpha'^2} \int \frac{dt}{t} (2\pi t)^{+1/2} e^{-tZ} F(q), \tag{B.4}$$

[1] The rescaling of $t$ to $t/4\alpha'$ amounts to the transition to proper time.

which is the force between the branes. Substituting $t = u/Z$ gives

$$\mathcal{F} = \frac{\sqrt{2\pi}}{8\pi^3\alpha'^2}\frac{Y}{\sqrt{Z}}\int\frac{du}{u^{1/2}}e^{-u}F(\frac{u}{Z}). \tag{B.5}$$

For $Z \to 0$ the function $F$ tends to 1 and the integral is proportional to

$$\mathcal{F} \propto \frac{Y}{\sqrt{Z}}\int\frac{du}{u^{1/2}}e^{-u} \propto \frac{1}{\sqrt{Z}}Y. \tag{B.6}$$

From this we see that the attractive force diverges for $Z \to 0$.

# C. T-dualities

We will shortly review some of the T-duality relations used in this thesis. The notion of T-duality was explained in chapter 2 so that we will only show some of the calculations in this chapter, the general strategy is explained in [27].

T-duality on any direction $i$ in the closed string sector acts as follows:

$$
\begin{aligned}
T_i : \tilde{\alpha}_n^i &\to -\tilde{\alpha}_n^i, \\
\tilde{\alpha}_n^a &\to \tilde{\alpha}_n^a \qquad a \neq i, \\
\alpha_n^\mu &\to \alpha_n^\mu \qquad \mu = 0, \dots, 9, \\
\\
\tilde{\psi}_r^i &\to -\tilde{\psi}_r^i, \\
\tilde{\psi}_r^a &\to \tilde{\psi}_r^a, \\
\psi_r^\mu &\to \psi_r^\mu.
\end{aligned}
$$

By performing T-duality in one direction we obtain type IIA from type IIB and vice versa.

## C.1. T-duality and $\Omega$

We will show how different actions of world-sheet parity are related by T-duality. We first introduce the action of $\Omega$ on the different fields. As mentioned in equation (3.15) and equation (3.19), $\Omega$ acts on the massless fields in type IIB theory as

$$
\Omega : \left.\begin{aligned}
g_{\mu\nu} &\to g_{\mu\nu} \\
B_{\mu\nu} &\to -B_{\mu\nu} \\
\phi &\to \phi
\end{aligned}\right\} \text{NS-NS sector} \tag{C.1}
$$

and

$$
\left.\begin{aligned}
A &\to -A \\
A_{\mu\nu} &\to A_{\mu\nu} \\
A^{(+)}_{\mu\nu\rho\sigma} &\to -A^{(+)}_{\mu\nu\rho\sigma}
\end{aligned}\right\} \text{R-R sector} \tag{C.2}
$$

*C. T-dualities*

On the closed string oscillators $\Omega$ acts as follows

$$\Omega : \alpha_n^\mu \to \tilde{\alpha}_n^\mu, \tag{C.3}$$
$$\tilde{\alpha}_n^\mu \to \alpha_n^\mu, \tag{C.4}$$
$$\psi_r^\mu \to \tilde{\psi}_r^\mu, \tag{C.5}$$
$$\tilde{\psi}_r^\mu \to \psi_r^\mu. \tag{C.6}$$

We will further need the action of $R_i$, reflecting the $i$th direction. $R_i$ acts on the oscillators as follows

$$\begin{aligned} R_i : \quad & \alpha_n^i \to -\alpha_n^i, \\ & \alpha_n^a \to \alpha_n^a \qquad a \neq i, \\ & \tilde{\alpha}_n^i \to -\tilde{\alpha}_n^i, \\ & \tilde{\alpha}_n^a \to \tilde{\alpha}_n^a, \\ \\ & \psi_r^i \to -\psi_r^i, \\ & \psi_r^a \to \psi_r^a, \\ & \tilde{\psi}_r^i \to -\tilde{\psi}_r^i, \\ & \tilde{\psi}_r^a \to \tilde{\psi}_r^a. \end{aligned}$$

The claim is, that T-duality in two directions, $i, j$ say, maps $\Omega$ to

$$\Omega \xrightarrow{T_{ij}} \Omega(-1)^{F_L} R_i R_j,$$

where $(-1)^{F_L}$ is the left handed *spacetime fermion number*. It interchanges the signs on the R-R fields and leaves anything else untouched:

$$\begin{aligned} (-1)^{F_L} : \quad g_{\mu\nu} &\to g_{\mu\nu} \\ B_{\mu\nu} &\to B_{\mu\nu} \\ \Phi &\to \Phi \\ \\ A &\to -A \\ A_{\mu\nu} &\to -A_{\mu\nu} \\ A^{(+)}_{\mu\nu\rho\sigma} &\to -A^{(+)}_{\mu\nu\rho\sigma} \end{aligned}$$

Thus we have to show that $\Omega = T_{ij}\Omega R_i R_j (-1)^{F_L} T_{ij}^{-1}$. Let us exemplify this on a T-duality transformation in the directions 8 and 9. On the NS-NS oscillators this gives:

$$\psi^a_{-1/2} \xrightarrow{T_{89}} \psi^a_{-1/2} \xrightarrow{\Omega} \tilde{\psi}^a_{-1/2} \xrightarrow{R_{89}} \tilde{\psi}^a_{-1/2} \xrightarrow{(-1)^{F_L}} \tilde{\psi}^a_{-1/2} \xrightarrow{T_{89}^{-1}} \tilde{\psi}^a_{-1/2} \; = \Omega \psi^a_{-1/2}$$

$$\tilde{\psi}^a_{-1/2} \xrightarrow{T_{89}} \tilde{\psi}^a_{-1/2} \xrightarrow{\Omega} \psi^a_{-1/2} \xrightarrow{R_{89}} \psi^a_{-1/2} \xrightarrow{(-1)^{F_L}} \psi^a_{-1/2} \xrightarrow{T_{89}^{-1}} \psi^a_{-1/2} \; = \Omega \tilde{\psi}^a_{-1/2}$$

$$\psi^8_{-1/2} \xrightarrow{T_{89}} \psi^8_{-1/2} \xrightarrow{\Omega} \tilde{\psi}^8_{-1/2} \xrightarrow{R_{89}} -\tilde{\psi}^8_{-1/2} \xrightarrow{(-1)^{F_L}} -\tilde{\psi}^8_{-1/2} \xrightarrow{T_{89}^{-1}} \tilde{\psi}^8_{-1/2} \; = \Omega \psi^8_{-1/2}$$

$$\tilde{\psi}^8_{-1/2} \xrightarrow{T_{89}} -\tilde{\psi}^8_{-1/2} \xrightarrow{\Omega} -\psi^8_{-1/2} \xrightarrow{R_{89}} \psi^8_{-1/2} \xrightarrow{(-1)^{F_L}} \psi^8_{-1/2} \xrightarrow{T_{89}^{-1}} \psi^8_{-1/2} \; = \Omega \tilde{\psi}^8_{-1/2}$$

where $a \neq 8, 9$. For the R-R fields this calculation is as follows:

$$A \xrightarrow{T_{89}} A_{89} \xrightarrow{\Omega} A_{89} \xrightarrow{R_{89}} A_{89} \xrightarrow{(-1)^{F_L}} -A_{89} \xrightarrow{T_{89}^{-1}} -A \; = \Omega A$$

$$A_{ab} \xrightarrow{T_{89}} A_{ab89} \xrightarrow{\Omega} -A_{ab89} \xrightarrow{R_{89}} -A_{ab89} \xrightarrow{(-1)^{F_L}} A_{ab89} \xrightarrow{T_{89}^{-1}} A_{ab} \; = \Omega A_{ab}$$

$$A_{8a} \xrightarrow{T_{89}} A_{a9} \xrightarrow{\Omega} A_{a9} \xrightarrow{R_{89}} -A_{a9} \xrightarrow{(-1)^{F_L}} A_{a9} \xrightarrow{T_{89}^{-1}} A_{8a} \; = \Omega A_{8a}$$

$$A_{89} \xrightarrow{T_{89}} A \xrightarrow{\Omega} -A \xrightarrow{R_{89}} -A \xrightarrow{(-1)^{F_L}} A \xrightarrow{T_{89}^{-1}} A_{89} \; = \Omega A_{89}$$

$$A_{abcd} \xrightarrow{T_{89}} A_{abcd89} \xrightarrow{\star} A_{ef} \xrightarrow{\Omega} A_{ef} \xrightarrow{R_{89}} A_{ef} \xrightarrow{(-1)^{F_L}} -A_{ef} \xrightarrow{T_{89}^{-1}} -A_{ef89} \xrightarrow{\star} -A_{abcd} \; = \Omega A_{abcd}$$

$$A_{ab89} \xrightarrow{T_{89}} A_{ab} \xrightarrow{\Omega} A_{ab} \xrightarrow{R_{89}} A_{ab} \xrightarrow{(-1)^{F_L}} -A_{ab} \xrightarrow{T_{89}^{-1}} -A_{ab89} \; = \Omega A_{ab89}$$

$$A_{abc9} \xrightarrow{T_{89}} A_{abc8} \xrightarrow{\Omega} -A_{abc8} \xrightarrow{R_{89}} A_{abc8} \xrightarrow{(-1)^{F_L}} -A_{abc8} \xrightarrow{T_{89}^{-1}} -A_{abc9} \; = \Omega A_{abc9}$$

and similarly for the other cases.

The same line of argument applies to T-duality in four directions which relates $\Omega$ and $\Omega R_{ijkl}$. This can be proven by an analogous calculation.

# D. The $\mathbb{Z}_3$ model

In this section the construction and spectrum of the $\mathbb{Z}_3$ orientifold are explained and derived. The considered model is a type IIB string theory, compactified on a Torus $T^4$ (here seen as $T^2 \times T^2$) and moded out by a $\mathbb{Z}_3\Omega$ projection. Therefore, we have to derive the spectrum of type IIB on $T^4/\mathbb{Z}_3\Omega$. The spacetime twist acts on the two tori via the twist vector $v = \frac{1}{3}(1, -1)$. Instead of an ordinary $\Omega$ projection we use the world-sheet parity projection combined with reflections $R_3, R_4$ of the two complex planes. I.e. we use $\Omega' \equiv \Omega R_3 R_4$. This model needs D5-branes to satisfy the tadpole cancelation conditions and is T-dual to a model that exclusively needs D9-branes. As the models are T-dual their spectra are identical but in what is to follow the description using D5-branes is more intuitive. This model was first discussed in [12, 20] and in the non-supersymmetric setting it was introduced in [5]. We start with the derivation of the closed string spectrum.

## D.1. Closed string spectrum

The closed string spectrum consists of an untwisted sector and two (equivalent) twisted sectors. We will derive them in turn.

### D.1.1. Untwisted sector

In the closed string sector for the left movers the energy is

$$T_0 = \frac{1}{2}p^2 + N - \frac{1}{2} = \frac{\alpha'}{4}m^2. \tag{D.1}$$

The formula for the right-movers is given by

$$\tilde{T}_0 = \frac{1}{2}\tilde{p}^2 + \tilde{N} - \frac{1}{2} = \frac{\alpha'}{4}m^2, \tag{D.2}$$

where $p, \tilde{p}$ are the center-of-mass momenta, $N, \tilde{N}$ contain the oscillator contributions to the energy and the factor $-\frac{1}{2}$ accounts for the zero point energy of the superstring. The NS-NS ground state $|\,0\,\rangle_{NS}$ has $(\text{mass})^2 = -\frac{1}{2}$ and is therefore tachyonic. By the usual assignment it has $(-1)^F$ eigenvalue $-1$ and therefore suffers the GSO projection. The Ramond sector ground states $|\,\mathbf{s}\,\rangle$ with $\mathbf{s} = (\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2})$ are massless. We choose the GSO projection such that it projects on states with an odd number of minus signs. In type IIB left and right movers have identical GSO-projections and thus the following analysis is valid for both sectors.

### NS sector

We start with the derivation of the massless spectrum of the NS sector. In chapter 3 we explained in general the action of the orbifold twist $\theta$ on the NS sector. The action is given by

$$\theta\psi^i_{-1/2} = e^{2\pi i v_i}\psi^i_{-1/2} \qquad i = 3, 4, \tag{D.3}$$

with the twist vector $v = \frac{1}{3}(1, -1)$. The index $i = 3, 4$ labels the two complex planes, 3 denotes the 6-7 plane and 4 denotes the 8-9 plane. $\theta$ acts trivially in the uncompactified directions, which will be denoted by $\mu$. We consider the linear combinations

$$\psi^3_{-1/2} := \psi^6_{-1/2} + i\psi^7_{-1/2} \quad \text{and } \psi^4_{-1/2} := \psi^8_{-1/2} + i\psi^9_{-1/2}, \tag{D.4}$$

$$\psi^{\bar{3}}_{-1/2} := \psi^6_{-1/2} - i\psi^7_{-1/2} \quad \text{and } \psi^{\bar{4}}_{-1/2} := \psi^8_{-1/2} - i\psi^9_{-1/2}. \tag{D.5}$$

The eigenvalues of the oscillators under the $\theta$ twist are given by ($\alpha = e^{2\pi i \frac{1}{3}}$):

| oscillator | eigenvalue |
|---|---|
| $\psi^\mu_{-1/2}$ | $\alpha^0$ |
| $\psi^3_{-1/2}$ | $\alpha^1$ |
| $\psi^4_{-1/2}$ | $\alpha^2$ |
| $\psi^{\bar{3}}_{-1/2}$ | $\alpha^2$ |
| $\psi^{\bar{4}}_{-1/2}$ | $\alpha^1$ |

The massless NS-$\widetilde{\text{NS}}$ states, which are invariant under the orbifold action, are

$$\begin{array}{ll}
\alpha^0\cdot\alpha^0 & \psi^\mu_{-1/2}\tilde{\psi}^\nu_{-1/2}|\,0\,\rangle_{NS}, \\
 & \\
\alpha^1\cdot\alpha^2 & \psi^3_{-1/2}\tilde{\psi}^4_{-1/2}|\,0\,\rangle_{NS}, \\
 & \psi^3_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2}|\,0\,\rangle_{NS}, \\
 & \psi^{\bar{4}}_{-1/2}\tilde{\psi}^4_{-1/2}|\,0\,\rangle_{NS}, \\
 & \psi^{\bar{4}}_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2}|\,0\,\rangle_{NS}, \\
 & \\
\alpha^2\cdot\alpha^1 & \psi^{\bar{3}}_{-1/2}\tilde{\psi}^{\bar{4}}_{-1/2}|\,0\,\rangle_{NS}, \\
 & \psi^{\bar{3}}_{-1/2}\tilde{\psi}^3_{-1/2}|\,0\,\rangle_{NS}, \\
 & \psi^4_{-1/2}\tilde{\psi}^{\bar{4}}_{-1/2}|\,0\,\rangle_{NS}, \\
 & \psi^4_{-1/2}\tilde{\psi}^3_{-1/2}|\,0\,\rangle_{NS}.
\end{array}$$

These states are invariant under the orbifold projection but to render them invariant under the complete orientifold group, we have to symmetrise over the left and right movers.

Therefore we obtain the following $\Omega'$ invariant states

$$\psi^{(\mu}_{-1/2}\tilde{\psi}^{\nu)}_{-1/2}|\,0\,\rangle_{NS}, \tag{D.6}$$
$$\psi^{3}_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2}+\psi^{\bar{3}}_{-1/2}\tilde{\psi}^{3}_{-1/2}|\,0\,\rangle_{NS}, \tag{D.7}$$
$$\psi^{3}_{-1/2}\tilde{\psi}^{4}_{-1/2}+\psi^{4}_{-1/2}\tilde{\psi}^{3}_{-1/2}|\,0\,\rangle_{NS}, \tag{D.8}$$
$$\psi^{\bar{4}}_{-1/2}\tilde{\psi}^{4}_{-1/2}+\psi^{4}_{-1/2}\tilde{\psi}^{\bar{4}}_{-1/2}|\,0\,\rangle_{NS}, \tag{D.9}$$
$$\psi^{\bar{4}}_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2}+\psi^{\bar{3}}_{-1/2}\tilde{\psi}^{\bar{4}}_{-1/2}|\,0\,\rangle_{NS}. \tag{D.10}$$

The states in the uncompactified directions correspond to the graviton $G^{\mu\nu}$ and the dilaton $\Phi$, while the states in the complex directions correspond to four scalar degrees of freedom. The anti-symmetric tensor $B_{\mu\nu}$ is projected out by the $\Omega'$ projection.

#### The R sector

In the Ramond sector we choose the GSO projection such that the operators $(-1)^F$ and $(-1)^{\tilde{F}}$ have eigenvalue $-1$ on the R-R states. Therefore, states with an odd number of $(-\frac{1}{2})$'s survive the GSO projection. These states and their eigenvalues are:

| $\lvert s\, s_2\, s_3\, s_4\rangle$ | $\mathbf{s}\cdot v$ | eigenvalue |
|---|---|---|
| $\lvert +-++\rangle$ | 0 | $\alpha^0$ |
| $\lvert -+++\rangle$ | 0 | $\alpha^0$ |
| $\lvert +---\rangle$ | 0 | $\alpha^0$ |
| $\lvert -+--\rangle$ | 0 | $\alpha^0$ |
| $\lvert +++-\rangle$ | 2/3 | $\alpha^2$ |
| $\lvert ++-+\rangle$ | -2/3 | $\alpha^1$ |
| $\lvert --+-\rangle$ | 2/3 | $\alpha^2$ |
| $\lvert ---+\rangle$ | -2/3 | $\alpha^1$ |

We can group these states according to their transformation property under the spacetime SO(4)$\widehat{=}$SU(2) $\times$ SU(2). The spinors with eigenvalue $\alpha^0$ transform as two copies of the $(\mathbf{1},\mathbf{2})$[1]. The spinors with any of the other two eigenvalues under $\theta$ transform in the $(\mathbf{2},\mathbf{1})$ representation of SU(2) $\times$ SU(2).

Unlike the NS sector, we have to antisymmetrise the states in the R sector under the $\Omega'$ projection. We need to antisymmetrise $2\cdot 2\cdot(\mathbf{1},\mathbf{2})\otimes(\mathbf{1},\mathbf{2}) = 4\cdot[(\mathbf{1},\mathbf{3})+(\mathbf{1},\mathbf{1})]$, such that the states are left-right antisymmetric. We obtain states which transform under $(\mathbf{1},\mathbf{3})+3\cdot(\mathbf{1},\mathbf{1})$ of SU(2) $\times$ SU(2).

Because these states have the $\theta$ eigenvalue $\alpha^0$, they belong into the same SUSY multiplets as $G^{\mu\nu}$ and $\Phi$. Together with their superpartners from the NS-R and R-NS sector, which we derive below, they form the supergravity (SUGRA) multiplet and the dilaton (i.e. tensor) multiplet.

[1] In labeling these spacetime transformations we follow the conventions of [37]. The spinors in the representation $(\mathbf{2},\mathbf{1})$ are sometimes called "chiral" spinors, whereas the spinors in the representation $(\mathbf{1},\mathbf{2})$ are called "anti-chiral".

The spinors with $\theta$ eigenvalue $\alpha^1$ and $\alpha^2$ transform under $2 \cdot (\mathbf{2}, \mathbf{1})$ in the left-moving as well as in the right-moving sector. From anti-symmetrising we obtain states which transform as $4 \cdot (\mathbf{1}, \mathbf{1})$.

### Untwisted NS-R + R-NS sector

We can use the values we derived above and combine them into $\mathbb{Z}_3\Omega$ invariant states in the R-NS sector. For $\alpha^0$ this gives

$$(\mathbf{2}, \mathbf{2}) \otimes (\mathbf{1}, \mathbf{2}) \cdot 2 = 2 \cdot [(\mathbf{2}, \mathbf{3}) + (\mathbf{2}, \mathbf{1})]. \tag{D.11}$$

For $\alpha^1 \cdot \alpha^2$ we obtain

$$2 \cdot [(\mathbf{1}, \mathbf{1}) \otimes (\mathbf{2}, \mathbf{1})] = 2 \cdot (\mathbf{2}, \mathbf{1}), \tag{D.12}$$

and similarly for $\alpha^2 \cdot \alpha^1$. The $\Omega$ projection interchanges the NS-R and R-NS sector. Therefore, taking all states into account we obtain two gravitinos, transforming under the spacetime representation $(\mathbf{2},\mathbf{3})$ and four fermions, transforming in the $(\mathbf{2},\mathbf{1})$. Together with their bosonic superpartners from the NS-NS and R-R sector these states fill the SUGRA-multiplet, the dilaton tensor multiplet and two hypermultiplets of D = 6 $\mathcal{N}$=1 supersymmetry.

## D.1.2. Twisted sector

### Twisted NS sector

As we explained in section 3, the zero point energy in the twisted sector is given by

$$E^{(1)} = \frac{1}{2} \sum_i |v_i|(1 - |v_i|). \tag{D.13}$$

For the first twisted sector the zero point energy is $E^{(1)} = \frac{2}{9}$. With the usual factor of $-\frac{1}{2}$ and the zero mode momentum for the twisted ground state, this gives a tachyonic NS ground state with $(\text{mass})^2 = -1/6$ which is projected out by the GSO projection. The lowest oscillators in the twisted sector have the following modings

$$\psi^{\mu}_{-1/2}, \quad \psi^{3}_{-1/6}, \quad \psi^{\bar{3}}_{-5/6}, \quad \psi^{4}_{-5/6}, \quad \psi^{\bar{4}}_{-1/6}. \tag{D.14}$$

Therefore two massless NS states, namely

$$\psi^{3}_{-1/6} | \, 0 \, \rangle^{(1)}_{NS}, \tag{D.15}$$

$$\psi^{\bar{4}}_{-1/6} | \, 0 \, \rangle^{(1)}_{NS} \tag{D.16}$$

survive the GSO projection.

#### Twisted R sector

The zero point energy in the R-sector added to the zero mode momenta result in a massless ground state. The lowest oscillators in the twisted sector have the moding

$$\psi^{\mu}_{0}, \quad \psi^{3}_{-2/3}, \quad \psi^{\bar{3}}_{-1/3}, \quad \psi^{4}_{-1/3}, \quad \psi^{\bar{4}}_{-2/3}. \tag{D.17}$$

The ground states compatible with the GSO projection are

$$|+\frac{1}{2},+\frac{1}{2}\rangle, \tag{D.18}$$

$$|-\frac{1}{2},-\frac{1}{2}\rangle. \tag{D.19}$$

The second twisted sector is identical to the first twisted sector. The only difference is that we have to interchange the modings between complex conjugated oscillators.

#### The complete closed spectrum

The twisted states have to be made $\Omega'$ invariant. $\Omega'$ maps the twisted sectors onto each other, therefore we have to symmetrise (anti-symmetrise) in the NS-NS (R-R) sector among the states from the different twisted sectors. From the NS sector we obtain four singlets under the spacetime SO(4), namely

$$\psi^{3}_{-1/6}|\,0\,\rangle^{(1)}_{NS}\otimes\tilde{\psi}^{\bar{3}}_{-1/6}|\,0\,\rangle^{(2)}_{NS}+\psi^{\bar{3}}_{-1/6}|\,0\,\rangle^{(2)}_{NS}\otimes\tilde{\psi}^{3}_{-1/6}|\,0\,\rangle^{(1)}_{NS}, \tag{D.20}$$

$$\psi^{\bar{4}}_{-1/6}|\,0\,\rangle^{(1)}_{NS}\otimes\tilde{\psi}^{4}_{-1/6}|\,0\,\rangle^{(2)}_{NS}+\psi^{4}_{-1/6}|\,0\,\rangle^{(2)}_{NS}\otimes\tilde{\psi}^{\bar{4}}_{-1/6}|\,0\,\rangle^{(1)}_{NS}, \tag{D.21}$$

$$\psi^{3}_{-1/6}|\,0\,\rangle^{(1)}_{NS}\otimes\tilde{\psi}^{4}_{-1/6}|\,0\,\rangle^{(2)}_{NS}+\psi^{\bar{4}}_{-1/6}|\,0\,\rangle^{(1)}_{NS}\otimes\tilde{\psi}^{\bar{3}}_{-1/6}|\,0\,\rangle^{(2)}_{NS}, \tag{D.22}$$

$$\psi^{3}_{-1/6}|\,0\,\rangle^{(1)}_{NS}\otimes\tilde{\psi}^{\bar{4}}_{-1/6}|\,0\,\rangle^{(1)}_{NS}+\psi^{4}_{-1/6}|\,0\,\rangle^{(2)}_{NS}\otimes\tilde{\psi}^{\bar{3}}_{-1/6}|\,0\,\rangle^{(2)}_{NS}. \tag{D.23}$$

The states in the R sector transform under the $(\mathbf{2},\mathbf{1})$ of the spacetime SO(4). By tensoring the left and right movers we obtain states in the representations $(\mathbf{3},\mathbf{1})$ + $(\mathbf{1},\mathbf{1})$. Therefore we obtain one tensor and one hypermultiplet of D=6 $\mathcal{N}$=1 SUSY from each of the nine fixed points. The complete closed string sector is summarised in table D.1.

| | |
|---|---|
| Untwisted sector | $\mathcal{N}$=1 SUGRA multiplet + 2 $\mathbf{T}$ + 2 $\mathbf{H}$ |
| Twisted sector | 9 $\mathbf{T}$ + 9 $\mathbf{H}$ |

Table D.1.: The closed string spectrum of the $\mathbb{Z}_3$ orientifold

## D.2. Open string spectrum

In this section the open spectrum of the $\mathbb{Z}_3$ orientifold is derived. The general technique is explained in section 3.

### D.2.1. The supersymmetric setting

In this model we use the $\Omega R_3 R_4$ projection, therefore only D5-branes which are extended in the non-compact space are needed to satisfy the tadpole cancellation conditions (see 3.1). The model is T-dual to a model with $\Omega$ projection where only D9-branes are required. The spectrum of this model was derived in [20]. To satisfy the tadpole cancellation conditions the following conditions have to be fulfilled

$$\begin{aligned} n_5 &= 32, && \text{(D.24)} \\ \mathrm{Tr}(\gamma_{1,5}) &= 8, && \text{(D.25)} \\ \mathrm{Tr}(\gamma_{2,5}) &= 8, && \text{(D.26)} \end{aligned}$$

where $n_5$ is the number of D5-branes in the model and $\gamma_{i,5}$ denotes the CP-matrix corresponding to the twist $\theta^i$. The CP factors have to satisfy

$$\begin{aligned} \lambda^{(0)} &= \gamma_{k,5}\lambda^{(0)}\gamma_{k,5}^{-1}, && \text{(D.27)} \\ \lambda^{(0)} &= -\gamma_{\Omega',5}\lambda^{(0)T}\gamma_{\Omega',5}^{-1}, && \text{(D.28)} \\ \lambda^{(a)} &= e^{2\pi i v_a}\gamma_{k,5}\lambda^{(a)}\gamma_{k,5}^{-1}, && \text{(D.29)} \\ \lambda^{(a)} &= -\gamma_{\Omega',5}\lambda^{(a)T}\gamma_{\Omega',5}^{-1}, && \text{(D.30)} \end{aligned}$$

where $a = 3, 4$ labels the complex planes and 0 labels the CP-factors for the uncompactified directions. We choose our CP matrices as:

$$\gamma_{1,5} = \mathrm{diag}(e^{2\pi i\frac{1}{3}}\mathbb{1}_8, e^{-2\pi i\frac{1}{3}}\mathbb{1}_8, \mathbb{1}_{16}), \tag{D.31}$$

$$\gamma_{\Omega,5} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \otimes \mathbb{1}_8. \tag{D.32}$$

With these choices all consistency conditions are atisfied, as can be checked easily. Furthermore, $\gamma_{\Omega',5} = \gamma_{\Omega',5}^T$ and $\gamma_{\Omega',5}$ interchanges $\gamma_{1,5}$ and $\gamma_{2,5}$. After imposing the projections, the CP factors have the following structure:

$$\lambda^{(0)} = \begin{pmatrix} M_{8\times8} & 0 & \\ 0 & -M^T_{8\times8} & 0 \\ & 0 & A^{as}_{16\times16} \end{pmatrix}, \tag{D.33}$$

$$\lambda^{(+)} = \begin{pmatrix} 0 & A^{(+)}_{8\times8} & 0 & 0 \\ 0 & 0 & B^{(3)}_{8\times8} & B^{(4)}_{8\times8} \\ -B^{(1)T}_{8\times8} & 0 & 0 & 0 \\ -B^{(2)T}_{8\times8} & 0 & 0 & 0 \end{pmatrix}, \tag{D.34}$$

$$\lambda^{(-)} = \begin{pmatrix} 0 & 0 & B^{(1)}_{8\times8} & B^{(2)}_{8\times8} \\ A^{(-)}_{8\times8} & 0 & 0 & \\ 0 & -B^{(3)T}_{8\times8} & 0 & 0 \\ 0 & -B^{(4)T}_{8\times8} & 0 & 0 \end{pmatrix}, \tag{D.35}$$

where $A^{as}_{16\times16}$ is an anti-symmetric $16\times16$ matrix and all other matrices are unitary matrices of the indicated rank. The $\Omega$ projection interchanges the two matrices $\lambda^{(+)}$ and $\lambda^{(-)}$ as they belong to conjugate eigenvalues. These CP factors therefore give rise to hypermultiplets transforming under the (**28**, **1**) and (**8**,**16**) of U(8)×SO(16). The supersymmetric spectrum is summarised in table (D.2).

| | | |
|---|---|---|
| closed spectrum | untwisted sector | $\mathcal{N}$=1 SUGRA multiplet + 2 **T** + 2 **H** |
| | twisted sector | 9 **T** + 9 **H** |
| open spectrum | gauge sector | vector multiplet of U(8) × SO(16) |
| | matter | hypermultiplets in (**28**,**1**) and (**8**,**16**) |

Table D.2.: The complete spectrum of the supersymmetric $\mathbb{Z}_3$ orientifold.

## D.2.2. The non-supersymmetric setting

In this section we discuss the spectrum of a non-supersymmetric extension with D5-branes and $\overline{\text{D5}}$-branes. In order to obtain a consistent model, the branes and anti-branes have either to be separated from each other such that the tachyonic modes become haevy or the tachyonic mode between branes and anti-branes has to be projected out. As the origin of supersymmetry breaking is purely in the open string sector, the closed string sector is not affected and therefore does not change. This setting was first analysed in [5]. To satisfy the tadpole cancellation conditions, the formulae (D.24)-(D.26) have to be modified in the following way

$$n_5 - n_{\bar{5}} = 32, \tag{D.36}$$
$$\mathrm{Tr}(\gamma_{1,5,0}) - \mathrm{Tr}(\gamma_{1,\bar{5},0}) = 8, \tag{D.37}$$
$$\mathrm{Tr}(\gamma_{2,5,0}) - \mathrm{Tr}(\gamma_{2,\bar{5},0}) = 8, \tag{D.38}$$
$$\mathrm{Tr}(\gamma_{k,5,M}) = \mathrm{Tr}(\gamma_{k,\bar{5},M}) = 0, \tag{D.39}$$

where the third index on the CP matrices labels the fixed points, and $M$ denotes $\mathbb{Z}_3$ fixed points other than the origin.

We place 4 D5-branes and 8 $\overline{\mathrm{D5}}$-branes at the origin. They fill the non-compact space and as they are placed at the same fixed point in the internal space, the tachyonic sector has to be projected out to obtain a stable model. As we have to satisfy equation (D.36), we place 18 D5-branes on one $\mathbb{Z}_3$ fixed point away from the origin and another 18 D5-branes on its image point under $\Omega'$. To satisfy the equations (D.37-D.39), the CP matrices are chosen as follows

$$\gamma_{1,5,0} = \mathbb{1}_4, \tag{D.40}$$
$$\gamma_{1,\bar{5},0} = \mathrm{diag}(e^{2\pi i\frac{1}{3}}\mathbb{1}_4, e^{2\pi i\frac{2}{3}}\mathbb{1}_4), \tag{D.41}$$
$$\gamma_{1,5,M} = \mathrm{diag}(\mathbb{1}_6, e^{2\pi i\frac{1}{3}}\mathbb{1}_6, e^{2\pi i\frac{2}{3}}\mathbb{1}_6), \tag{D.42}$$
$$\gamma_{\Omega',5,0} = \mathbb{1}_4, \tag{D.43}$$
$$\gamma_{\Omega',\bar{5},0} = \begin{pmatrix} 0 & \mathbb{1}_4 \\ \mathbb{1}_4 & 0 \end{pmatrix}. \tag{D.44}$$

As $\Omega'$ interchanges the $\mathbb{Z}_3$ fixed points labeled by $M$, there are no CP matrices for $\Omega'$-action on these fixed points. The CP factors in the NS sector have to satisfy the same conditions as they had to satisfy in the supersymmetric case. The projections act identically on the branes and on the anti-branes. The projections on the CP-factors of the R sector differ between branes and anti-branes. On the D5-branes they are the same as in the supersymmetric case, while on the $\overline{\mathrm{D5}}$-branes the $\Omega'$ projection acts with an additional minus sign (we explain this in section (2.8)). Therefore, the conditions on the CP-factors for the R sector are

$$\lambda^{(0)} = \gamma_{k,\bar{5}}\lambda^{(0)}\gamma_{k,\bar{5}}^{-1}, \tag{D.45}$$
$$\lambda^{(0)} = +\gamma_{\Omega',\bar{5}}\lambda^{(0)T}\gamma_{\Omega',\bar{5}}^{-1}, \tag{D.46}$$
$$\lambda^{(a)} = e^{2\pi i v_a}\gamma_{k,\bar{5}}\lambda^{(a)}\gamma_{k,\bar{5}}^{-1}, \tag{D.47}$$
$$\lambda^{(a)} = +\gamma_{\Omega',\bar{5}}\lambda^{(a)T}\gamma_{\Omega',\bar{5}}^{-1}. \tag{D.48}$$

In order to obtain the spectrum of the open sector we have to impose the orientifold projections on the CP-factors. This gives the gauge group $\mathrm{SO}(4)_{55} \times \mathrm{U}(4)_{\bar{5}\bar{5}}$ at the origin. The corresponding fermions are those in the $(\mathbf{1},\mathbf{2})$ representation of the spacetime SO(4). Matter living on the D5-branes is projected out by the action of $\theta$ on the CP-factors:

$$\lambda^{(a)} = e^{2\pi i\frac{1}{3}}\gamma_{i,5,0}\lambda^{(a)}\gamma_{i,5,0}^{-1} \Rightarrow \lambda^{(a)} = 0 \tag{D.49}$$

If there were a matter sector on the D5-branes, the matter multiplets had the same number of bosons and fermions. On the $\overline{\text{D5}}$-branes the matter sector is non-supersymmetric. As we explained above, the $\Omega'$ projection acts with an additional $(-)$ sign on the R sector. Imposing this modified condition on the CP factors gives complex scalars in the $(\mathbf{1},\mathbf{6})+(\mathbf{1},\bar{\mathbf{6}})$ of U(4) and fermions in the $(\mathbf{1},\mathbf{10})$ of U(4).

In the D5-$\overline{\text{D5}}$ sector the GSO projection is opposite to the usual one. It projects on the tachyonic ground state in the NS sector and on an even number of $-\frac{1}{2}$'s in the R sector. With the chosen CP-matrices the tachyon is projected out by the $\theta$ projection.

$$\theta : |0, ab\rangle_{NS}\lambda_{ba} \to e^{\pm 2\pi i/3}|0, ab\rangle_{NS}\lambda_{ba}. \tag{D.50}$$

In the R sector the fermions with negative chirality survive the projection. They transform in the bifundamental representation of the gauge groups.

On the D5-branes at the orbifold fixed points $M$ the gauge group is $\mathrm{U}(6)^3$. Applying the usual projections we find matter hypermultiplets in the $(\bar{\mathbf{6}},\mathbf{6},\mathbf{1})+(\mathbf{1},\bar{\mathbf{6}},\mathbf{6})+(\mathbf{6},\mathbf{1},\bar{\mathbf{6}})$. In table (D.4) we list the complete open spectrum.

| Open spectrum | | | | |
|---|---|---|---|---|
| | Gauge sector | Fermions in $(\mathbf{1},\mathbf{2})$ | complex scalars | Fermions in $(\mathbf{2},\mathbf{1})$ |
| 55 (origin) | U(4) | $(\mathbf{Adj},1)$ | – | – |
| $\bar{5}\bar{5}$ | SO(4) | $(\mathbf{1},\,\mathbf{Adj})$ | $(\mathbf{1},\substack{\square\\\square})+(\mathbf{1},\overline{\substack{\square\\\square}})$ | $(\mathbf{1},\square\!\square)$ |
| $5\bar{5}$ | – | $(\square,\square)$ | – | – |
| 55 ($\mathbb{Z}_3$ FP) | $\mathrm{U}(6)^3$ | $(\mathbf{Adj},1)$ | $(\overline{\square},\square,\mathbf{1})+(\mathbf{1},\overline{\square},\square)$ $+(\square,\mathbf{1},\overline{\square})$ | $(\overline{\square},\square,\mathbf{1})+(\mathbf{1},\overline{\square},\square)$ $+(\square,\mathbf{1},\overline{\square})$ |

Table D.4.: The complete spectrum of the non-supersymmetric $\mathbb{Z}_3$ orientifold

## D.2.3. All branes at the origin

As we have discussed in section (4.1), the D5-branes at the orbifold fixed points can move into the bulk, since they are not stuck at the fixed points. They are attracted by the $\overline{\text{D5}}$-branes at the origin and therefore will possibly move towards the origin. When the D5-branes reach the origin, the model becomes unstable and tends to annihilate into the supersymmetric setting. We derive the spectrum with all branes placed at the origin to understand, where the instailities arise. The remarks on the CP projections apply as in the cases we considered before. The CP matrices are as follows

$$\gamma_{1,5,0} = \mathrm{diag}(e^{2\pi i\frac{1}{3}}\mathbb{1}_{12}, e^{2\pi i\frac{2}{3}}\mathbb{1}_{12}, \mathbb{1}_{16}), \tag{D.51}$$

$$\gamma_{1,\bar{5},0} = \mathrm{diag}(e^{2\pi i\frac{1}{3}}\mathbb{1}_{4}, e^{2\pi i\frac{2}{3}}\mathbb{1}_{4}), \tag{D.52}$$

$$\gamma_{\Omega',5,0} = \begin{pmatrix} 0 & \mathbb{1}_{12} & 0 \\ \mathbb{1}_{12} & 0 & 0 \\ 0 & 0 & \mathbb{1}_{16} \end{pmatrix}, \tag{D.53}$$

$$\gamma_{\Omega',\bar{5},0} = \begin{pmatrix} 0 & \mathbb{1}_{4} \\ \mathbb{1}_{4} & 0 \end{pmatrix}. \tag{D.54}$$

This yields the gauge group $U(12)_{55} \times SO(16)_{55} \times U(4)_{\bar{5}\bar{5}}$. With these CP matrices the tachyon is not longer projected out, signaling the instability of the system. The complete open spectrum is

| Open spectrum | | | | |
|---|---|---|---|---|
| | Gauge sector | Fermions in $(\mathbf{1},\mathbf{2})$ | complex scalars | Fermions in $(\mathbf{2},\mathbf{1})$ |
| 55 | $U(12) \times SO(16)$ | $(\mathbf{Adj},\mathbf{1},\mathbf{1})$<br>$(\mathbf{1},\mathbf{Adj},\mathbf{1})$ | $(\begin{array}{c}\square\\[-4pt]\square\end{array},\mathbf{1},\mathbf{1})$<br>$+(\square,\square,\mathbf{1})$ | $(\begin{array}{c}\square\\[-4pt]\square\end{array},\mathbf{1},\mathbf{1})$<br>$+(\square,\square,\mathbf{1})$ |
| $\bar{5}\bar{5}$ | $SO(4)$ | $(\mathbf{1},\mathbf{1}, \mathbf{Adj})$ | $(\mathbf{1},\mathbf{1}, \begin{array}{c}\square\\[-4pt]\square\end{array}) + (\mathbf{1},\mathbf{1}, \overline{\begin{array}{c}\square\\[-4pt]\square\end{array}})$ | $(\mathbf{1},\mathbf{1}, \square\!\square)$ |
| $5\bar{5}$ | | $(\square, \square, \square)$ | $(\square, \mathbf{1}, \square)$ (tachyons) | $(\square,\mathbf{1},\square)$ |

Table D.6.: The complete open spectrum with all branes at the origin

# E. The $\mathbb{Z}_6'$ model

In this section we will derive the spectrum of the IIB on $T^6/\Omega\mathbb{Z}_6'$ model. We will start with the closed string spectrum. The twist vector for this model is $v = \frac{1}{6}(1, -3, 2)$. This model is equivalent to a $\mathbb{Z}_3 \times \mathbb{Z}_2$ orientifold model, considered in [59] and was discussed in [1] in the supersymmetric context and in [5] in the non-supersymmetric case.

## E.1. Closed string spectrum

The closed string sector consists of untwisted states and twisted states. We derive these sectors in turn, paying particularly attention to the multiplicities of the twisted sector states. We start with the untwisted spectrum.

### E.1.1. Untwisted sector

The main strategy is as in the $\mathbb{Z}_3$ model which we considered in the previous chapter. Therefore, we will pay in particular attention to the new features of the $\mathbb{Z}_6'$ orientifold and our discussion of the points common to both models will be less exhaustive.

#### NS sector

We start with the oscillators of the untwisted NS sector and their eigenvalues. The index $\mu$ refers to the uncompactified directions. For the compactified directions we use a complexified description with $\psi^i = \psi^{2i} + i\psi^{2i+1}$ and $\psi^{\bar{\imath}} = \psi^{2i} - i\psi^{2i+1}$. For convenience we set $\alpha = e^{\pi i/6}$. The eigenvalues of the oscillators under the twist are:

| oscillator | eigenvalue |
|---|---|
| $\psi^{\mu}_{-1/2}$ | $\alpha^0$ |
| $\psi^{2}_{-1/2}$ | $\alpha^2$ |
| $\psi^{3}_{-1/2}$ | $\alpha^6$ |
| $\psi^{4}_{-1/2}$ | $\alpha^4$ |
| $\psi^{\bar{2}}_{-1/2}$ | $\bar{\alpha}^2$ |
| $\psi^{\bar{3}}_{-1/2}$ | $\bar{\alpha}^6$ |
| $\psi^{\bar{4}}_{-1/2}$ | $\bar{\alpha}^4$ |

*E. The $\mathbb{Z}_6'$ model*

In order to derive the untwisted spectrum, we have to consider the states which are invariant under the $\mathbb{Z}_6'$ as well as under the orientifold projection. Proceeding as in the $\mathbb{Z}_3$ model, we find the following states invariant under all projections

$$\psi^{(\mu}_{-1/2}\tilde{\psi}^{\nu)}_{-1/2}|\,0\,\rangle_{NS} \widehat{=} G_{\mu\nu} + \Phi, \tag{E.1}$$

$$\psi^{2}_{-1/2}\tilde{\psi}^{\bar{2}}_{-1/2} + \psi^{\bar{2}}_{-1/2}\tilde{\psi}^{2}_{-1/2}|\,0\,\rangle_{NS}, \tag{E.2}$$

$$\psi^{3}_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2} + \psi^{\bar{3}}_{-1/2}\tilde{\psi}^{3}_{-1/2}|\,0\,\rangle_{NS}, \tag{E.3}$$

$$\psi^{3}_{-1/2}\tilde{\psi}^{3}_{-1/2}|\,0\,\rangle_{NS}, \tag{E.4}$$

$$\psi^{\bar{3}}_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2}|\,0\,\rangle_{NS}, \tag{E.5}$$

$$\psi^{4}_{-1/2}\tilde{\psi}^{\bar{4}}_{-1/2} + \psi^{\bar{4}}_{-1/2}\tilde{\psi}^{4}_{-1/2}|\,0\,\rangle_{NS}. \tag{E.6}$$

These states correspond to the metric and the dilaton in the uncompactified dimensions, and in addition we obtain five scalar degrees of freedom from the compactified directions.

### The R sector

In the R sector, we choose the GSO projection again such that we have an odd number of $-\frac{1}{2}$'s. We list the eigenvalues of the R sector states grouped after their chirality.

| $\|s\, s_2\, s_3\, s_4\rangle$ | $\mathbf{s}\cdot v$ | eigenvalue$=e^{2\pi i v\cdot\mathbf{s}}$ |
|---|---|---|
| $\|+---\rangle$ | $0$ | $\alpha^0$ |
| $\|+-++\rangle$ | $-1/6$ | $\bar{\alpha}^2$ |
| $\|++-+\rangle$ | $1/2$ | $\alpha^6$ |
| $\|+++-\rangle$ | $-1/3$ | $\bar{\alpha}^4$ |
| $\|-+++\rangle$ | $0$ | $\alpha^0$ |
| $\|-+--\rangle$ | $1/6$ | $\alpha^2$ |
| $\|--+-\rangle$ | $-1/2$ | $\alpha^6$ |
| $\|---+\rangle$ | $1/3$ | $\alpha^4$ |

The following closed string states are invariant under the complete orientifold group.

$$|+---\rangle_L \otimes |-+++\rangle_R - |-+++\rangle_L \otimes |+---\rangle_R, \tag{E.7}$$

$$|+-++\rangle_L \otimes |-+--\rangle_R - |-+--\rangle_L \otimes |+-++\rangle_R, \tag{E.8}$$

$$|++-+\rangle_L \otimes |--+-\rangle_R - |--+-\rangle_L \otimes |++-+\rangle_R, \tag{E.9}$$

$$|+++-\rangle_L \otimes |---+\rangle_R - |---+\rangle_L \otimes |+++-\rangle_R. \tag{E.10}$$

The states with eigenvalue $\alpha^0$ correspond to the SUGRA multiplet of D=4 $\mathcal{N}$=1 supersymmetry together with one chiral multiplet, the dilaton multiplet. From the oscillators in the compact dimensions we obtain four chiral multiplets. The NS-R and R-NS sector is as straightforward as in the $\mathbb{Z}_3$ case therefore we will not repeat the derivation at this point. The states fill up the supersymmetric multiplets.

## E.1.2. Twisted sector

### Twisted NS sector

In this section we consider the twisted states from the NS sector.

*First twisted sector*

The first twisted sector has zero-point energy of $z.p.e = -\frac{7}{36}$. The zero mode momenta and the zero point energy add up to a massless ground state. The oscillators leading to the lowest excited levels are:

$$\psi^{\mu}_{-1/2}, \quad \psi^{2}_{-5/6}, \quad \psi^{3}_{0}, \quad \psi^{4}_{-1/6}, \quad \psi^{\bar{2}}_{-1/6}, \quad \psi^{\bar{3}}_{0}, \quad \psi^{\bar{4}}_{-5/6}. \tag{E.11}$$

In the 6-7 plane the orbifold twist acts as a $\mathbb{Z}_2$ twist, i.e. as a $\pi$ rotation. Therefore the oscillators in this plane are shifted such that they are integer moded and a degeneracy arises. From the two possible ground states one is projected out by the GSO projection such that we are left with one massless ground state in the first twisted sector $|\,0\,\rangle^{(1)}_{NS} = |0, -\frac{1}{2}\rangle$.

*Second twisted sector*

The second twisted sector has the zero point energy $z.p.e = -\frac{5}{18}$. The $z.p.e$ adds up with the zero mode momenta to a $(\text{mass})^2$ of $-1/6$ and therefore gives rise to a tachyonic ground state. The oscillators which act as raising operators for the first excited level are

$$\psi^{\mu}_{-1/2}, \quad \psi^{2}_{-1/6}, \quad \psi^{3}_{-1/2}, \quad \psi^{4}_{-5/6}, \quad \psi^{\bar{2}}_{-1/6}, \quad \psi^{\bar{3}}_{0}, \quad \psi^{\bar{4}}_{-1/6}. \tag{E.12}$$

where the oscillator in the 6-7-plane is not shifted, because $\theta$ acts as an order two twist in this plane. There are two massless ground states, compatible with the GSO projection, namely

$$\psi^{2}_{-1/6}|\,0\,\rangle^{(2)}_{NS}, \tag{E.13}$$
$$\psi^{\bar{4}}_{-1/6}|\,0\,\rangle^{(2)}_{NS}. \tag{E.14}$$

*Third twisted sector*

In the third twisted sector the ground states are massless and there are two ground states compatible with the GSO projection, namely

$$|0, +\tfrac{1}{2}, +\tfrac{1}{2}, 0\rangle, \tag{E.15}$$
$$|0, -\tfrac{1}{2}, -\tfrac{1}{2}, 0\rangle. \tag{E.16}$$

The oscillators for the first excited level are

$$\psi^{\mu}_{-1/2}, \quad \psi^{2}_{0}, \quad \psi^{3}_{0}, \quad \psi^{4}_{-1/2}, \quad \psi^{\bar{2}}_{0}, \quad \psi^{\bar{3}}_{0}, \quad \psi^{\bar{4}}_{-1/2}. \tag{E.17}$$

The fourth and fifth twisted sectors correspond to the second and first twisted sector respectively, with the moding of the oscillators interchanged between $\psi^{i}$ and $\psi^{\bar{\imath}}$.

**Twisted R sector**

*First twisted sector*

Similarly to the NS sector there is one massless ground state in the R sector. The oscillators have the following moding

$$\psi_0^\mu, \quad \psi^2_{-5/6}, \quad \psi^3_{-1/2}, \quad \psi^4_{-2/3}, \quad \psi^{\bar{2}}_{-1/6}, \quad \psi^{\bar{3}}_{-1/2}, \quad \psi^{\bar{4}}_{-1/3}. \tag{E.18}$$

After imposing the GSO projection one ground state survives,

$$|0\rangle_R^{(1)} = |+\tfrac{1}{2}, 0\rangle. \tag{E.19}$$

*Second twisted sector*

We will from now on only list the oscillator modings and the massless ground states. In the second twisted sector the lowest oscillators are

$$\psi_0^\mu, \quad \psi^2_{-2/3}, \quad \psi^3_0, \quad \psi^4_{-1/3}, \quad \psi^{\bar{2}}_{-1/3}, \quad \psi^{\bar{3}}_0, \quad \psi^{\bar{4}}_{-2/3} \tag{E.20}$$

and the ground states are

$$|0\rangle_{R,1}^{(2)} = |+\tfrac{1}{2}, +\tfrac{1}{2}\rangle, \tag{E.21}$$

$$|0\rangle_{R,2}^{(2)} = |-\tfrac{1}{2}, -\tfrac{1}{2}\rangle. \tag{E.22}$$

*Third twisted sector*

The oscillators are

$$\psi_0^\mu, \quad \psi^2_{-1/2}, \quad \psi^3_{-1/2}, \quad \psi^4_0, \quad \psi^{\bar{2}}_{-1/2}, \quad \psi^{\bar{3}}_{-1/2}, \quad \psi^{\bar{4}}_0, \tag{E.23}$$

and the massless ground states are:

$$|0\rangle_{R,1}^{(3)} = |+\tfrac{1}{2}, 0, 0 - \tfrac{1}{2}\rangle, \tag{E.24}$$

$$|0\rangle_{R,2}^{(2)} = |-\tfrac{1}{2}, 0, 0 + \tfrac{1}{2}\rangle. \tag{E.25}$$

### E.1.3. Construction of invariant states

Before we are going to construct the twisted states, which are invariant under the orientifold group, we consider the fixed points of the various powers of the orbifold twist. Each torus can be taken as an SU(3) lattice. In the first torus the twist $\theta$ acts as a $\mathbb{Z}_6$ twist, therefore only the origin is invariant under $\theta^1$. In the second complex plane $\theta^1$ acts as a $\mathbb{Z}_2$ twist, thus four points are invariant under the twist action. In the third complex plane $\theta$ acts as a $\mathbb{Z}_3$ twist, hence three points are fixed in the third complex plane.

$\theta^2$ acts in the first complex plane as a $\mathbb{Z}_3$ twist, thus has three fixed points. In the second complex plane it acts as the identity, giving no fixed points and in the third complex plane it acts as an $\mathbb{Z}_3$ twist.

$\theta^3$ acts as a $\mathbb{Z}_2$ twist in the first and second complex plane, thus giving four fixed points in each of these planes. In the third plane it acts as the identity. For higher twists the analysis is identical. In figure (E.1) the fixed points of the complex planes are shown and labeled. In table (E.1) the sets of invariant fixed points are summarised.

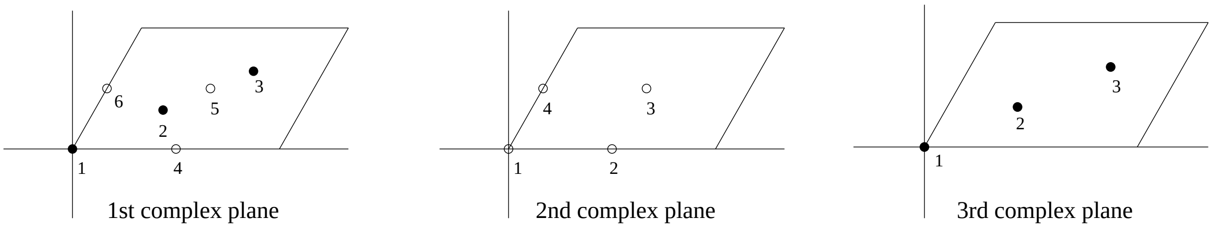


Figure E.1.: The complex planes and their fixed points. In the first plane the points 2 and 3 are invariant under $\theta^2$ and 4,5 and 6 are invariant under $\theta^3$.

| | 1st complex plane | 2nd complex plane | 3rd complex plane | fixed points |
|---|---|---|---|---|
| $\theta^1$ | $1$ | $\{1,2,3,4\}$ | $\{1,2,3\}$ | 12 |
| $\theta^2$ | $\{1,2,3\}$ | – | $\{1,2,3\}$ | 9 |
| $\theta^3$ | $\{1,4,5,6\}$ | $\{1,2,3,4\}$ | – | 16 |

Table E.1.: Fixed points

Under $\theta^1$ the fixed points 4, 5 and 6 in the first complex plane are mapped onto each other and the points 2 and 3 are interchanged. In order to render states at fixed points invariant under $\theta$ we have to take combinations of twisted states sitting at different fixed points. These combinations are shown in table (E.2). We are now able to construct all

| | 1st complex plane | 2nd complex plane | 3rd complex plane | multiplicity |
|---|---|---|---|---|
| $\theta^1$ | $1$ | $\{1,2,3,4\}$ | $\{1,2,3\}$ | 12 |
| $\theta^2$ | $\{1,2+3\}$ | – | $\{1,2,3\}$ | 6 |
| $\theta^3$ | $\{1,4+5+6\}$ | $\{1,2,3,4\}$ | – | 8 |

Table E.2.: $\theta$ Invariant fixed points

states which are invariant under the orientifold group.

### NS sector

The $\Omega$ projection maps states from the first twisted sector onto the fifth twisted sector and vice-versa, furthermore states from the second twisted sector are mapped onto the fourth twisted sector. To obtain invariant states we have to symmetrise with respect to the twisted sectors as follows:

| twisted sector | $\Omega\theta$-invariant states |
|---|---|
| 1 + 5 | $\vert 0\rangle_L^{(1)}\otimes\vert 0\rangle_R^{(1)}+\vert 0\rangle_L^{(5)}\otimes\vert 0\rangle_R^{(5)}$<br>$\vert 0\rangle_L^{(1)}\otimes\vert 0\rangle_R^{(5)}+\vert 0\rangle_L^{(5)}\otimes\vert 0\rangle_R^{(1)}$ |
| 2 + 4 | $\psi^2_{-1/6}\tilde\psi^{\bar 2}_{-1/6}\vert 0\rangle_{NS}+\psi^{\bar 2}_{-1/6}\tilde\psi^{2}_{-1/6}\vert 0\rangle_{NS}$<br>$\psi^{\bar 4}_{-1/6}\tilde\psi^{4}_{-1/6}\vert 0\rangle_{NS}+\psi^{4}_{-1/6}\tilde\psi^{\bar 4}_{-1/6}\vert 0\rangle_{NS}$ |
| 3 | $\vert 0,\frac12,\frac12,0\rangle_L\otimes\vert 0,-\frac12,-\frac12,0\rangle_R+\vert 0,-\frac12,-\frac12,0\rangle_L\otimes\vert 0,\frac12,\frac12,0\rangle_R$ |

Except of these states there are another two states in the twisted sector which are:

$$\left(\psi^4_{-1/2}\tilde\psi^4_{-1/2}\vert 0\rangle_{NS}\right)_4+e^{2\pi i/3}\left(\psi^4_{-1/2}\tilde\psi^4_{-1/2}\vert 0\rangle_{NS}\right)_5+e^{4\pi i/3}\left(\psi^4_{-1/2}\tilde\psi^4_{-1/2}\vert 0\rangle_{NS}\right)_6$$

and

$$\left(\psi^{\bar 4}_{-1/2}\tilde\psi^{\bar 4}_{-1/2}\vert 0\rangle_{NS}\right)_4+e^{-2\pi i/3}\left(\psi^{\bar 4}_{-1/2}\tilde\psi^{\bar 4}_{-1/2}\vert 0\rangle_{NS}\right)_5+e^{-4\pi i/3}\left(\psi^{\bar 4}_{-1/2}\tilde\psi^{\bar 4}_{-1/2}\vert 0\rangle_{NS}\right)_6 .$$

The indices on the brackets denote, on which fixed point in the first plane the state resides. Under $\theta$ the fixed points 4,5 and 6 are mapped onto each other and the states pick up a phase. However, as one can check, the above combinations are invariant under $\theta$ as well as under $\Omega$.

We want to determine the multiplicity of the states from the various twisted sectors. States from the first twisted sector can only sit at fixed points under $\theta$, therefore these states appear with a multiplicity of 12. States from the second twisted sector sit at fixed points under $\theta^2$, therefore these states appear with a multiplicity of 9 and the states from the third twisted sector appear with a multiplicity of 16. The two additional states have a multiplicity of 4. (Half the multiplicity of $\theta^3$.) Therefore a counting of the states gives:

$$2\cdot 12+2\cdot 6+1\cdot 8+2\cdot 4=52$$

From which we obtain 52 scalar degrees of freedom from the twisted NS sector.

### R sector

From the R sector we obtain the following invariant states:

| twisted sector | $\Omega\theta$-invariant states |
|---|---|
| 1 + 5 | $\vert 0\rangle_L^{(1)}\otimes\vert 0\rangle_R^{(5)}-\vert 0\rangle_L^{(5)}\otimes\vert 0\rangle_R^{(1)}$ |
| 2 + 4 | $\vert +\frac12,+\frac12\rangle_L^{(2)}\otimes\vert -\frac12,-\frac12\rangle_R^{(4)}-\vert -\frac12,-\frac12\rangle_L^{(2)}\otimes\vert \frac12,\frac12\rangle_R^{(4)}$ |
| 3 | $\vert -\frac12,0,0,\frac12\rangle_L\otimes\vert +\frac12,0,0,-\frac12\rangle_R-\vert +\frac12,0,0,-\frac12\rangle_L\otimes\vert -\frac12,0,0,\frac12\rangle_R$ |

And the same procedure of "distributing states" at different fixed points works for the R sector, but here the states are from the second twisted sector, namely

$$\begin{aligned}&\Big(|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_L^{(2)}\otimes|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_R^{(2)}\Big)_2-\Big(|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_L^{(4)}\otimes|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_R^{(4)}\Big)_2\\-&\Big(|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_L^{(2)}\otimes|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_R^{(2)}\Big)_3+\Big(|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_L^{(4)}\otimes|+\tfrac{1}{2},+\tfrac{1}{2}\rangle_R^{(4)}\Big)_3\end{aligned}$$

and similarly

$$\begin{aligned}&\Big(|-\tfrac{1}{2},-\tfrac{1}{2}\rangle_L^{(2)}\otimes|-\tfrac{1}{2},-\tfrac{1}{2}\rangle_R^{(2)}\Big)_2-\Big(|-\tfrac{1}{2},-\tfrac{1}{2}\rangle_L^{(4)}\otimes|-\tfrac{1}{2},-\tfrac{1}{2}\rangle_R^{(4)}\Big)_2\\-&\Big(|-\tfrac{1}{2},-\tfrac{1}{2}\rangle_L^{(2)}\otimes|-\tfrac{1}{2},-\tfrac{1}{2}\rangle_R^{(2)}\Big)_3+\Big(|-\tfrac{1}{2},-\tfrac{1}{2}\rangle_L^{(4)}\otimes|-\tfrac{1}{2},-\tfrac{1}{2}\rangle_R^{(4)}\Big)_3.\end{aligned}$$

Counting the degrees of freedom in the R sector we obtain

$$1\cdot 12+1\cdot 6+1\cdot 8+2\cdot 3=32.$$

From both sectors there are 84 scalar degrees of freedom which can be grouped into 42 chiral multiplets if we take the states from the twisted R-NS and NS-R sector into account.

## E.2. Open string spectrum

We will now construct the open string spectrum. In order to satisfy the tadpole cancellation conditions for this model, given in [1], we have to use D9-branes and D5-branes. The D5-branes are needed, because the orbifold group has an order two twist, inverting the first and second complex plane. Therefore we need $D5_3$-branes in this model to satisfy the tadpole conditions. The $D5_3$-branes fill spacetime and the the third complex plane. Tadpole cancellation further implies

$$n_9=32, \tag{E.26}$$

$$n_5-n_{\bar 5}=32, \tag{E.27}$$

$$\mathrm{Tr}\gamma_{k,9}-2\mathrm{Tr}\gamma_{k,5,L}+2\mathrm{Tr}\gamma_{k,\bar 5,L}=0;\qquad k=1,5, \tag{E.28}$$

$$\mathrm{Tr}\gamma_{2,9}=-8, \tag{E.29}$$

$$\mathrm{Tr}\gamma_{4,9}=8, \tag{E.30}$$

$$\sum_M\Big(\left(\mathrm{Tr}\gamma_{2,5,M}\right)^2-\left(\mathrm{Tr}\gamma_{2,\bar 5,M}\right)^2\Big)+16\sum_L\left(\mathrm{Tr}\gamma_{2,5,L}-\mathrm{Tr}\gamma_{2,\bar 5,L}\right)=-64, \tag{E.31}$$

$$\sum_M\Big(\left(\mathrm{Tr}\gamma_{4,5,M}\right)^2-\left(\mathrm{Tr}\gamma_{4,\bar 5,M}\right)^2\Big)-16\sum_L\left(\mathrm{Tr}\gamma_{4,5,L}-\mathrm{Tr}\gamma_{4,\bar 5,L}\right)=-64, \tag{E.32}$$

here $L=0,\dots,3$ refers to the $\theta$ fixed points in the first two tori, while $M=0,1,2$ refers to the $\theta^2$ fixed points. In order to satisfy these conditions, we will use 32 D9-branes which are extended in all dimensions. We choose the CP matrices for the D9-branes as

$$\gamma_{\theta,9}=\mathrm{diag}(\alpha\mathbb{1}_4,\alpha^3\mathbb{1}_8,\alpha^5\mathbb{1}_4,\alpha^7\mathbb{1}_4,\alpha^9\mathbb{1}_8,\alpha^{11}\mathbb{1}_4). \tag{E.33}$$

Furthermore we place some D$5_3$-branes at the origin of the second complex plane with CP-matrices

$$\gamma_{\theta,5,0} = \mathrm{diag}(\alpha \mathbb{1}_4, \alpha^5 \mathbb{1}_4, \alpha^7 \mathbb{1}_4, \alpha^{11} \mathbb{1}_4). \tag{E.34}$$

We place $2M$ $\overline{\text{D5}}$-branes at some other $\theta$ fixed point, labeled '1' with CP-matrices

$$\gamma_{\theta,\bar{5},1} = \mathrm{diag}(\alpha^3 \mathbb{1}_M, \alpha^9 \mathbb{1}_M), \tag{E.35}$$

and $16 + 2M$ D$5_3$-branes at another fixed point labeled '2', with CP-matrices

$$\gamma_{\theta,5,2} = \mathrm{diag}(\alpha^3 \mathbb{1}_{8+M}, \alpha^9 \mathbb{1}_{8+M}). \tag{E.36}$$

The CP-matrices for the $\Omega$ projection are chosen as follows

$$\gamma_{\Omega,9} = \begin{pmatrix} 0 & \mathbb{1}_{16} \\ \mathbb{1}_{16} & 0 \end{pmatrix}; \qquad \gamma_{\Omega,5,P} = \begin{pmatrix} 0 & -i\mathbb{1}_{N_5} \\ i\mathbb{1}_{N_5} & 0 \end{pmatrix}. \tag{E.37}$$

These CP-matrices fulfill the tadpole cancellation conditions.

## E.2.1. D9-branes

In the 99-sector we obtain the gauge group $\mathrm{U}(4) \times \mathrm{U}(8) \times \mathrm{U}(4)$. Matter from the 4-5 plane has CP-factors of the following structure:

$$\lambda^{(2)} = \begin{pmatrix} 0 & A^{12}_{4\times4} & A^{13}_{4\times4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A^{24}_{4\times4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A^{34}_{4\times4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A^{45}_{4\times4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A^{56}_{4\times4} & A^{57}_{4\times4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A^{68}_{4\times4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A^{78}_{4\times4} \\ A^{81}_{4\times4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{E.38}$$

$$\lambda^{(\bar{2})} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & A^{18}_{4\times4} \\ A^{21}_{4\times4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ A^{31}_{4\times4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A^{42}_{4\times4} & A^{43}_{4\times4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A^{54}_{4\times4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A^{65}_{4\times4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A^{75}_{4\times4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A^{86}_{4\times4} & A^{87}_{4\times4} & 0 \end{pmatrix}, \tag{E.39}$$

where the $\Omega$ projection implies $\lambda^{(\bar{a})} = -\lambda^{(a)T}$. Similarly for the other compact directions we obtain

$$\lambda^{(3)} = \begin{pmatrix} 0 & 0 & 0 & 0 & A^{15}_{4\times4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A^{26}_{4\times4} & A^{27}_{4\times4} & 0 \\ 0 & 0 & 0 & 0 & 0 & A^{36}_{4\times4} & A^{37}_{4\times4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A^{48}_{4\times4} \\ A^{51}_{4\times4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A^{62}_{4\times4} & A^{63}_{4\times4} & 0 & 0 & 0 & 0 & 0 \\ 0 & A^{72}_{4\times4} & A^{73}_{4\times4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A^{15}_{4\times4} & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{E.40}$$

$$\lambda^{(4)} = \begin{pmatrix} 0 & 0 & 0 & A^{14}_{4\times4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A^{25}_{4\times4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A^{35}_{4\times4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A^{46}_{4\times4} & A^{47}_{4\times4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A^{58}_{4\times4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{E.41}$$

The CP-factors for the complex conjugate oscillators can be deduced by an analogous calculation. In order to derive under which representations of the gauge group these matrices transform, we proceed in the following manner: The gauge bosons have CP-factors

$$\begin{array}{cccccccc} \overline{4}_1 & \overline{8} & \overline{8} & \overline{4}_2 & 4_1 & 8 & 8 & 4_2 \end{array} \tag{E.42}$$

$$\lambda^{(0)} = \begin{array}{c} \begin{pmatrix} A^{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A^{22} & A^{23} & 0 & 0 & 0 & 0 & 0 \\ 0 & A^{32} & A^{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A^{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -A^{11^T} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -A^{22^T} & -A^{32^T} & 0 \\ 0 & 0 & 0 & 0 & 0 & -A^{23^T} & -A^{33^T} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -A^{44^T} \end{pmatrix} \end{array} \begin{array}{c} 4_1 \\ 8 \\ 8 \\ 4_2 \\ \overline{4}_1 \\ \overline{8} \\ \overline{8} \\ \overline{4}_2 \end{array}, \tag{E.43}$$

where each $A^{ij}$ denotes a four by four unitary matrix. The numbers aside the matrix label the representation to which this part of the matrix corresponds. Note that we decomposed for technical reasons the matrix corresponding to the **8** of U(8) into four by four blocks but it should be thought of as one eight by eight matrix. The form of the matrix is determined by the projections of the CP-matrices.

After determining the representations for the gauge bosons we can determine the representations of the matter multiplets. In order to obtain the correct matter multiplets, we have to take the action of $\Omega$ into account. It is for an arbitrary CP-matrix:

$$\Omega : \begin{pmatrix} A^{11}_{16\times16} & A^{12}_{16\times16} \\ A^{21}_{16\times16} & A^{22}_{16\times16} \end{pmatrix} \to \begin{pmatrix} -A^{22^T}_{16\times16} & -A^{12^T}_{16\times16} \\ -A^{21^T}_{16\times16} & A^{11^T}_{16\times16} \end{pmatrix} \tag{E.44}$$

i.e. $\Omega$ maps the upper left diagonal block to the lower right diagonal block and transposes the off-diagonal blocks. Therefore, we have to consider one diagonal block and the two off-diagonal blocks to obtain the matter multiplets. Furthermore it is enough for $\lambda^{(2)}$ and $\lambda^{(4)}$ to consider only one CP-factor, as the other one belongs to the CPT conjugate particles. For $\lambda^{(3)}$ the CP-factors are mapped onto itself, therefore we obtain anti-symmetric representations in this sector. From $\lambda^{(2)}$ we obtain matter transforming in the $(\mathbf{4},\overline{\mathbf{8}},\mathbf{1})$, $(\mathbf{1},\mathbf{8},\overline{\mathbf{4}})$, $(\mathbf{4},\mathbf{1},\mathbf{4})$ and $(\overline{\mathbf{4}},\mathbf{1},\overline{\mathbf{4}})$. $\lambda^{(4)}$ gives matter multiplets in the $(\mathbf{4},\mathbf{1},\overline{\mathbf{4}})$, $(\mathbf{4},\mathbf{8},\mathbf{1})$ and $(\mathbf{1},\mathbf{8},\mathbf{4})$. And from $\lambda^{(3)}$ we obtain multiplets in the $(\mathbf{6},\mathbf{1},\mathbf{1})$, $(\mathbf{1},\mathbf{28},\mathbf{1})$, $(\mathbf{1},\mathbf{1},\mathbf{6})$, $(\overline{\mathbf{6}},\mathbf{1},\mathbf{1})$, $(\mathbf{1},\overline{\mathbf{28}},\mathbf{1})$ and $(\mathbf{1},\mathbf{1},\overline{\mathbf{6}})$.

### E.2.2. D5-branes

*D5-branes at the origin*

From the CP-factors of the $D5_3$-branes at the origin, we obtain the gauge group $U(4)\times U(4)$. By the same method which we explained for the D9-branes, we obtain matter in the representations $(\mathbf{4},\mathbf{4})$ from $\lambda^{(2)}$, $(\mathbf{4},\overline{\mathbf{4}})+(\overline{\mathbf{4}},\mathbf{4})$ from $\lambda^{(\mathbf{4})}$. The CP-factor $\lambda^{(3)}$ contributes matter in the antisymmetric representations $(\mathbf{1},\mathbf{6})+(\mathbf{6},\mathbf{1})$. There exist further matter multiplets from strings stretching between the $D5_3$-branes and the D9-branes. Because these strings have N boundary conditions on one end and D boundary conditions on the other end their oscillators have a half-integer moding. In this model the D5-branes fill the third complex plane therefore we have to take strings with $|s_1,s_2;ba\rangle\lambda^{59}_{ab}$ and the GSO projection implies $s_1=s_2$. We obtain $\mathcal{N}$=1 chiral multiplets in the representations

$$(\mathbf{4},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{4})+(\mathbf{1},\mathbf{4};\mathbf{1},\overline{\mathbf{8}},\mathbf{1})+(\overline{\mathbf{4}},\mathbf{1};\mathbf{1},\mathbf{8},\mathbf{1})+(\mathbf{1},\overline{\mathbf{4}};\overline{\mathbf{4}},\mathbf{1},\mathbf{1}). \tag{E.45}$$

*D5-branes at the fixed point '2'*

The branes at this fixed point transform under the gauge group $U(8+M)$. Matter in the first and third complex plane is projected out by the orbifold projection and only matter multiplets from states in the second complex plane survive. These multiplets transform in the $(\begin{smallmatrix}\square\\\square\end{smallmatrix}+\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}})$ representation of $U(8+M)$. As for the branes at the origin the $D5_3$-branes under consideration have a 59 matter sector with particles in chiral multiplets transforming under $((\square;\,\overline{\mathbf{4}},\mathbf{1},\mathbf{1})+(\overline{\square};\,\mathbf{1},\mathbf{1},\mathbf{4}))$.

*$\overline{\text{D5}}$-branes at fixed point '1'*

The gauge bosons in this sector transform under the adjoint representations of $U(M)$. As was explained in section (2.8) we expect that the $\Omega$ projection picks out different representations for bosons and fermions. The matter bosons transform in the antisymmetric representation $(\begin{smallmatrix}\square\\\square\end{smallmatrix}+\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}})$, as it is the case on the $D5_3$-branes at fixed point '2'. $\Omega$ acts with an additional $-1$ in the R sector, therefore it projects on the symmetric representations. Thus the fermions transform under the $(\square\square+\overline{\square\square})$. In the $\overline{5}9$-sector the GSO projection acts with an additional $-1$ and states from the NS sector transform under the $(M;\mathbf{4},\mathbf{1},\mathbf{1})+(\overline{M};\mathbf{1},\mathbf{1},\overline{\mathbf{4}})$. The states from the R sector transform under $(M;\overline{\mathbf{4}},\mathbf{1},\mathbf{1})+(\overline{M};\mathbf{1},\mathbf{1},\mathbf{4})$. The complete spectrum of the model is summarised in table (E.4).

| Open spectrum | | |
|---|---|---|
| 99 | gauge sector<br>matter | $\mathrm{U}(4)\times\mathrm{U}(8)\times\mathrm{U}(4)$<br>$(\mathbf{4},\overline{\mathbf{8}},\mathbf{1})+(\mathbf{1},\mathbf{8},\overline{\mathbf{4}})+(\mathbf{4},\mathbf{1},\mathbf{4})+(\overline{\mathbf{4}},\mathbf{1},\overline{\mathbf{4}})+(\mathbf{4},\mathbf{1},\overline{\mathbf{4}})+$<br>$(\mathbf{4},\mathbf{8},\mathbf{1})+(\mathbf{1},\mathbf{8},\mathbf{4})+(\mathbf{6},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{28},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{6})+$<br>$(\overline{\mathbf{6}},\mathbf{1},\mathbf{1})+(\mathbf{1},\overline{\mathbf{28}},\mathbf{1})+(\mathbf{1},\mathbf{1},\overline{\mathbf{6}})$ |
| 55 (origin) | gauge sector<br>matter | $\mathrm{U}(\mathbf{4})\times\mathrm{U}(4)$<br>$(\mathbf{4},\mathbf{4})+(\mathbf{4},\overline{\mathbf{4}})+(\overline{\mathbf{4}},\mathbf{4})+(\mathbf{1},\mathbf{6})+(\mathbf{6},\mathbf{1})$ |
| 55 at '2' | gauge sector<br>matter | $\mathrm{U}(8+M)$<br>$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$ |
| $\overline{55}$ | gauge bosons<br>fermions$_-$<br>complex scalars<br>fermions$_+$ | $\mathrm{U}(M)$<br>**Adj**<br>$\begin{smallmatrix}\Box\\\Box\end{smallmatrix},\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$<br>$\Box\Box+\overline{\Box\Box}$ |
| $95_0$ | matter | $(\mathbf{4},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{4})+(\mathbf{1},\mathbf{4};\mathbf{1},\overline{\mathbf{8}},\mathbf{1})+(\overline{\mathbf{4}},\mathbf{1};\mathbf{1},\mathbf{8},\mathbf{1})+(\mathbf{1},\overline{\mathbf{4}};\overline{\mathbf{4}},\mathbf{1},\mathbf{1})$ |
| $95_2$ | matter | $(\Box;\ \overline{\mathbf{4}},\mathbf{1},\mathbf{1})+(\overline{\Box};\ \mathbf{1},\mathbf{1},\mathbf{4})$ |
| $9\overline{5}_1$ | complex scalars<br>fermions$_-$ | $(M;\mathbf{4},\mathbf{1},\mathbf{1})+(\overline{M};\mathbf{1},\mathbf{1},\overline{\mathbf{4}})$<br>$(M;\overline{\mathbf{4}},\mathbf{1},\mathbf{1})+(\overline{M};\mathbf{1},\mathbf{1},\mathbf{4})$ |

Table E.4.: The complete open spectrum

### *E. The $\mathbb{Z}_6'$ model*

# F. The $T^4/\widetilde{G}$ model

We will derive the spectra of the different phases of the $T^4/\widetilde{G}$ orientifold. This model was first proposed in [40].

The important idea about this model is that it is constructed such that it does not contain untwisted R-R tadpoles but twisted R-R tadpoles. To construct models of this kind, recall as mentioned in section (3), that elements $\Omega h$ of $\widetilde{G}$ give rise to tadpoles in the $h^2$ twisted sector. Thus, to obtain models without untwisted tadpoles, $\widetilde{G}$ has to contain only elements of the form $\Omega h$ such that $h^2 \neq 1$. Thus $G_2$ must not contain elements of order 2.

In this section we consider Type IIB compactified on $(T^2 \times T^2)/\widetilde{G}$ with

$$\widetilde{G} = \{1, \Omega' \hat{\alpha}, \hat{\alpha}^2 =: \theta, \Omega' \hat{\alpha}^3\}, \tag{F.1}$$

where $\hat{\alpha}$ generates a $\mathbb{Z}_4$, $\hat{\alpha}$ acts as $\hat{\alpha} : (z_1, z_2) \to (e^{2\pi i \frac{1}{4}} z_1, e^{-2\pi i \frac{1}{4}} z_2)$ and $\Omega' = \Omega R$ where $R$ reflects all compact directions[1]. From the general remarks in section (3.2) this implies that the model will need D5-branes for tadpole cancellation.

There is another new feature in this model. In the order two twisted sector we will take left-right symmetric states in the R-R sector and left-right antisymmetric states in the NS-NS sector. There is a freedom of choice in this sector which modifies the tadpole cancellation conditions and with this choice we find the more interesting model. (The model with left-right symmetric (anti-symmetric) states in the NS-NS (R-R) sector does not need an open string sector for consistency [40].)

With the action of $\widetilde{G}$ as described and with rectangular lattices on both tori with lattice vectors $e_1 = R_i$ and $e_2 = iR_i$ with $i = 1, 2$ for the two complex planes, the fixed point structure of this model is as follows: There are 16 points fixed under $\theta$, namely the points $z_i \in \{0, 1R_i, iR_i, \frac{1+i}{2}R_i\}$. The points $(z_1, z_2) = (0,0), (0, \frac{1+i}{2}R_2), (\frac{1+i}{2}R_1, 0), (\frac{1+i}{2}R_1, \frac{1+i}{2}R_2)$ are invariant under $\hat{\alpha}$ as well, as is shown in fig. (F.1).

## F.1. Closed string spectrum

First we construct the closed string spectrum. The twist vector corresponding to $\hat{\alpha}$ is $\tilde{v} = \frac{1}{4}(1, -1)$ and the twist vector corresponding to $\theta$ is $v = \frac{1}{2}(1, -1)$.

[1] Choosing $\Omega$ without the reflection would lead to a T-dual model.

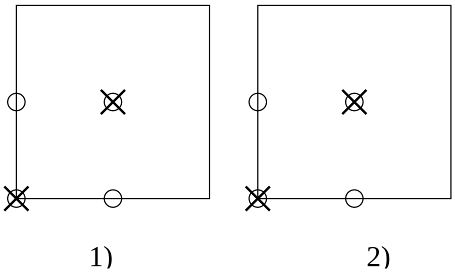


Figure F.1.: The complex planes and their fixed points. The points indicated by $\times$ are invariant under $\hat{\alpha}$, the points indicated by circles are invariant under $\theta$

## F.1.1. The untwisted sector

### NS sector

The oscillators in the NS sector have the following eigenvalues under $\Omega\hat{\alpha}$ and $\theta$ ($\alpha := e^{2\pi i \frac{1}{4}}$):

| osc. | $\theta$ | $\Omega'\hat{\alpha}$ |
|---|---|---|
| $\psi^{\mu}_{-\frac{1}{2}}$ | $\alpha^0$ | $\tilde{\psi}^{\mu}_{-\frac{1}{2}}$ |
| $\tilde{\psi}^{\mu}_{-\frac{1}{2}}$ | $\alpha^0$ | $\psi^{\mu}_{-\frac{1}{2}}$ |
| $\psi^{3}_{-\frac{1}{2}}$ | $\alpha^2$ | $-\alpha\,\tilde{\psi}^{3}_{-\frac{1}{2}}$ |
| $\psi^{\bar{3}}_{-\frac{1}{2}}$ | $\overline{\alpha}^2$ | $-\overline{\alpha}\,\tilde{\psi}^{\bar{3}}_{-\frac{1}{2}}$ |
| $\psi^{4}_{-\frac{1}{2}}$ | $\overline{\alpha}^2$ | $-\overline{\alpha}\,\tilde{\psi}^{4}_{-\frac{1}{2}}$ |
| $\psi^{\bar{4}}_{-\frac{1}{2}}$ | $\alpha^2$ | $-\alpha\,\tilde{\psi}^{\bar{4}}_{-\frac{1}{2}}$ |

They form the following invariant combinations:

$$
\begin{aligned}
&\psi^{(\mu}_{-1/2}\tilde{\psi}^{\nu)}_{-1/2}|\,0\,\rangle_{NS} \widehat{=} G_{\mu\nu} + \Phi,\\
&\psi^{3}_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2} + \psi^{\bar{3}}_{-1/2}\tilde{\psi}^{3}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{4}_{-1/2}\tilde{\psi}^{3}_{-1/2} + \psi^{3}_{-1/2}\tilde{\psi}^{4}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{3}_{-1/2}\tilde{\psi}^{\bar{4}}_{-1/2} + \psi^{\bar{4}}_{-1/2}\tilde{\psi}^{3}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{\bar{3}}_{-1/2}\tilde{\psi}^{4}_{-1/2} - \psi^{4}_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{\bar{3}}_{-1/2}\tilde{\psi}^{\bar{4}}_{-1/2} + \psi^{\bar{4}}_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{4}_{-1/2}\tilde{\psi}^{\bar{4}}_{-1/2} + \psi^{\bar{4}}_{-1/2}\tilde{\psi}^{4}_{-1/2}|\,0\,\rangle_{NS}.
\end{aligned}
$$

### The R sector

In the R sector we obtain with the usual GSO projection:

| oscillators | $\theta$ | $\Omega'\hat{\alpha}$ |
|---|---|---|
| $\vert +---\rangle_{L/R}$ | $\alpha^0$ | $-\vert +---\rangle_{R/L}$ |
| $\vert +++-\rangle_{L/R}$ | $\alpha^2$ | $\alpha\vert +++-\rangle_{R/L}$ |
| $\vert ++-+\rangle_{L/R}$ | $\overline{\alpha}^2$ | $\overline{\alpha}\vert +++-\rangle_{R/L}$ |
| $\vert +-++\rangle_{L/R}$ | $\alpha^0$ | $-\vert +-++\rangle_{R/L}$ |
| $\vert -+++\rangle_{L/R}$ | $\alpha^0$ | $-\vert -+++\rangle_{R/L}$ |
| $\vert -+--\rangle_{L/R}$ | $\alpha^0$ | $-\vert -+--\rangle_{R/L}$ |
| $\vert --+-\rangle_{L/R}$ | $\alpha^2$ | $\alpha\vert --+-\rangle_{R/L}$ |
| $\vert ---+\rangle_{L/R}$ | $\overline{\alpha}^2$ | $\overline{\alpha}\vert ---+\rangle_{R/L}$ |

For definiteness we have incorporated the reflection as $e^{2\pi i\frac{1}{2}(s_3+s_4)}$. These states form the following $\Omega'\hat{\alpha}$ invariant combinations. The states corresponding to the eigenvalue $\alpha^0$ give the invariant states:

$$\begin{aligned}
&|+-++\rangle_L\otimes|+---\rangle_R-|+---\rangle_L\otimes|+-++\rangle_R,\\
&|+-++\rangle_L\otimes|-+++\rangle_R-|-+++\rangle_L\otimes|+-++\rangle_R,\\
&|+-++\rangle_L\otimes|-+--\rangle_R-|-+--\rangle_L\otimes|+-++\rangle_R,\\
&|+---\rangle_L\otimes|-+++\rangle_R-|-+++\rangle_L\otimes|+---\rangle_R,\\
&|+---\rangle_L\otimes|-+--\rangle_R-|-+--\rangle_L\otimes|+---\rangle_R,\\
&|-+++\rangle_L\otimes|-+--\rangle_R-|-+--\rangle_L\otimes|-+++\rangle_R,
\end{aligned}$$

and the states with the eigenvalues $\alpha^{\pm 2}$ give

$$\begin{aligned}
&|+++-\rangle_L\otimes|+++-\rangle_R,\\
&|--+-\rangle_L\otimes|--+-\rangle_R,\\
&|++-+\rangle_L\otimes|++-+\rangle_R,\\
&|---+\rangle_L\otimes|---+\rangle_R,\\
&|+++-\rangle_L\otimes|++-+\rangle_R-|++-+\rangle_L\otimes|+++-\rangle_R,\\
&|--+-\rangle_L\otimes|---+\rangle_R-|---+\rangle_L\otimes|--+-\rangle_R,\\
&|+++-\rangle_L\otimes|--+-\rangle_R+|--+-\rangle_L\otimes|+++-\rangle_R,\\
&|++-+\rangle_L\otimes|---+\rangle_R+|---+\rangle_L\otimes|++-+\rangle_R,\\
&|+++-\rangle_L\otimes|---+\rangle_R-|---+\rangle_L\otimes|+++-\rangle_R,\\
&|++-+\rangle_L\otimes|--+-\rangle_R-|--+-\rangle_L\otimes|++-+\rangle_R.
\end{aligned}$$

The NS-NS and R-R states form the bosonic particle content of the SUGRA, 3 tensor and two hypermultiplets of $D=6$, $\mathcal{N}$=1 SUSY.

### F.1.2. The twisted sector

As the $\Omega$ projection introduces no twisted sectors, the only twisted sector comes from the $\theta$-twist.

### Twisted NS sector

The zero-point energy is $z.p.e = -\frac{1}{4}$ and the moding of the oscillators is integer in the compact directions. After the GSO projection we have two degenerate ground states, namely

$$\begin{aligned}|\,1\,\rangle_{NS} &= |0, 0, \tfrac{1}{2}, \tfrac{1}{2}\rangle,\\ |\,2\,\rangle_{NS} &= |0, 0, -\tfrac{1}{2}, -\tfrac{1}{2}\rangle.\end{aligned}$$

The lowest lying creation operators are

$$\psi^{\mu}_{-1/2}, \quad \psi^{3}_{0}, \quad \psi^{4}_{0}, \quad \psi^{\bar{3}}_{0}, \quad \psi^{\bar{4}}_{0}. \tag{F.2}$$

After anti-symmetrising we obtain one state transforming as a singlet under the spacetime little group SO(4).

### Twisted R sector

In the twisted R sector the oscillators in the compact directions have half-integer moding. Thus, we again have two degenerate ground states after the GSO-projection. They are given by:

$$\begin{aligned}|\,1\,\rangle_{R} &= |\tfrac{1}{2}, \tfrac{1}{2}, 0, 0\rangle,\\ |\,2\,\rangle_{R} &= |-\tfrac{1}{2}, -\tfrac{1}{2}, 0, 0\rangle.\end{aligned}$$

The lowest lying creation operators are

$$\psi^{\mu}_{0}, \quad \psi^{3}_{-1/2}, \quad \psi^{4}_{-1/2}, \quad \psi^{\bar{3}}_{-1/2}, \quad \psi^{\bar{4}}_{-1/2}. \tag{F.3}$$

As we have to symmetrise the states to obtain $\Omega$ invariant combinations, we obtain states which transform under the $(\mathbf{3}, \mathbf{1})$ representation of the spacetime SO(4)$\widehat{=}$SU(2) $\times$ SU(2). Taking NS-NS and R-R twisted states together we obtain one tensor multiplet for each $\hat{\alpha}$ fixed point and one tensor plus one hyper multiplet for each pair of fixed points which are mapped onto each other under $\hat{\alpha}$. Thus with the fixed point structure mentioned in the beginning of this section, we obtain 6 hypermultiplets and 6 + 4 = 10 tensor multiplets from the twisted sector.

## F.2. Open string spectrum

Calculating the tadpole cancellation conditions gives the following condition for the cancellation of twisted tadpoles:

$$4\left(\mathrm{Tr}(\gamma_{\theta,5,P}) - \mathrm{Tr}(\gamma_{\theta,\bar{5},P})\right) - 32 = 0, \tag{F.4}$$

where $P$ denotes the fixed points and $\gamma_\theta$ denotes the projection matrix for the $\theta$ twist. There are different ways to satisfy this condition. As they are connected with each other by nucleating brane-anti-brane pairs into the bulk and distributing them differently among the fixed points, we will only analyse one model that satisfies the tadpole cancellation conditions. We distribute sets of 8 D5-branes at the $\hat{\alpha}$ fixed points $(z_1, z_2) = (0,0)$ and $(z_1, z_2) = (\frac{1+i}{2}R_1, \frac{1+i}{2}R_2)$ and 8 $\overline{\mathrm{D5}}$-branes at the fixed points $(z_1, z_2) = (\frac{1+i}{2}R_1, 0)$ and $(z_1, z_2) = (0, \frac{1+i}{2}R_2)$. The CP-matrices are

$$\gamma_{\alpha^2,5} = \mathbb{1}_8, \qquad \gamma_{\alpha^2,\bar{5}} = -\mathbb{1}_8. \tag{F.5}$$

The other $\theta$ fixed points are not fixed under $\hat{\alpha}$ and thus generate no tadpoles. The CP-matrices for the $\Omega$ projection are:

$$\gamma_{\Omega'\alpha,5} = \mathbb{1}_8, \qquad \gamma_{\Omega'\alpha,\bar{5}} = \begin{pmatrix} 0 & \mathbb{1}_4 \\ -\mathbb{1}_4 & 0 \end{pmatrix}. \tag{F.6}$$

With this projection matrices we obtain on the D5-branes

$$\lambda_{55} = -\lambda_{55}^T, \tag{F.7}$$

Which gives rise to the gauge group SO(8) on the D5-branes. On the $\overline{\mathrm{D5}}$-branes we obtain

$$-\lambda_{\bar{5}\bar{5}} = -\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} 0 & \mathbb{1}_4 \\ -\mathbb{1}_4 & 0 \end{pmatrix} \cdot \begin{pmatrix} A_{11}^T & A_{21}^T \\ A_{12}^T & A_{22}^T \end{pmatrix} \cdot \begin{pmatrix} 0 & -\mathbb{1}_4 \\ \mathbb{1}_4 & 0 \end{pmatrix} = \begin{pmatrix} A_{22}^T & A_{12}^T \\ A_{21}^T & A_{11}^T \end{pmatrix}. \tag{F.8}$$

This defines the gauge group USp(8). On the R sector of the $\overline{\mathrm{Dp}}$-brane the $\Omega$ projection acts with an additional minus sign. Therefore we obtain fermions in the $(\mathbf{27{+}1})$ of USp(8) on each stack of $\overline{\mathrm{D5}}$-branes. On all branes matter multiplets are projected out by the $\theta$ projection. The complete spectrum is listed in table (F.1).

| | | |
|---|---|---|
| Closed | Untwisted | $\mathcal{N}$=1 SUGRA + 3 **T** + 2 **H** |
| | Twisted | 10 **T** + 6 **H** |
| Open | 55 | $\mathcal{N}$=1 Vector of SO(8)$^2$ |
| | $\overline{55}$ | Gauge bosons of USp(8)$^2$ |
| | | Fermions in $(\mathbf{27}+\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{27}+\mathbf{1})$ |

Table F.1.: The complete spectrum of the $T^4/\widetilde{G}$ orientifold.

For large radii of the compact directions this model is stable. However, as we have explained in section (4.3) this model undergoes phase transitions as the radii of the compact tori are varied.

### F.2.1. The spectrum at $R_1 = 1$

For $R_2$ large and[2] $R_1 = 1$ the D5- and $\overline{\mathrm{D5}}$-branes can combine to non-BPS D6-branes (denoted $\widehat{\mathrm{D6}}$-branes) wrapped on a non-supersymmetric cycle [40, 45]. We are going to construct the spectrum of this phase of the model.

[2] In this section we work in units where $\alpha' = 1$

We need two stacks of $\widehat{\mathrm{D6}}$-branes, one stack denoted by $\widehat{\mathrm{D6}}_1$ sitting at $z_2 = 0$ and passing through $z_1 = (0, 0)$ and $(\frac{1+i}{2}R_1, 0)$ and another one denoted by $\widehat{\mathrm{D6}}_2$ sitting at $z_2 = \frac{1+i}{2}R_2$ and passing through $z_1 = (0, \frac{1+i}{2}R_2)$ and $(\frac{1+i}{2}R_1, \frac{1+i}{2}R_2)$. Furthermore we have to introduce the images under $\Omega'\alpha$ of these branes. The complete configuration is sketched in figure (F.2).

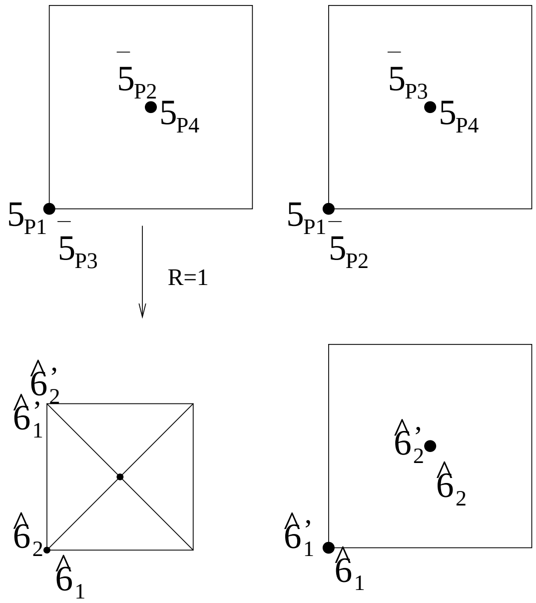


Figure F.2.: The brane configurations for $R_1 = 1$

In order for this brane configuration to cancel all tadpoles, we have to choose

$$\gamma_{\theta,\hat{6}_1} = \gamma_{\theta,\hat{6}_2} = \mathbb{1}_4. \tag{F.9}$$

The contribution of the $\widehat{\mathrm{D6}}$-branes and their $\Omega'\alpha$ images add up at each fixed point and thus cancel the twisted tadpoles. The spectrum is derived as follows: As the branes can be viewed as a system of branes and anti-branes [45], we have to consider both GSO-projections. Thus in the NS sector we have before the orientifold projection the massless fields $\psi^a_{-1/2}|\,0\,\rangle_{NS}$ and the tachyonic ground state $|\,0\,\rangle_{NS}$. In the R sector we have fermions with an odd and an even number of minus signs. The $\Omega'\alpha$ projection maps $\widehat{\mathrm{D6}}$- to $\widehat{\mathrm{D6}}$'-branes and thus imposes no projections on the CP-factors. On the branes only the $\theta$ projection acts. With our choice of CP-matrices the CP-factors for the gauge bosons are projected onto themselves, thus the gauge group is U(4) on each stack of branes. The matter scalars pick up a minus sign from the action of $\theta$ on the oscillators and are thus projected out. The fermions, corresponding to the gauge bosons, are of the form $|s_1, s_2, s_3, s_4\rangle$ with $s_3 = s_4$ and thus $s_1 = -s_2$. They transform in the adjoint representation of U(4), as well. In the sector with the opposite GSO-projection we would obtain the tachyon and additional fermions. As the tachyon is odd under the $\mathbb{Z}_2$ action [45], it is projected out but another fermion remains in this sector. $\Omega'\hat{\alpha}$ maps the spectrum on the $\widehat{\text{D6}}$-branes to the $\widehat{\text{D6}}$'-branes and thus it is sufficient to derive the spectrum on the $\widehat{\text{D6}}$-branes and to perform no projection.

The next sector we have to consider is the spectrum of strings stretching between $\widehat{\text{D6}}$-branes and $\widehat{\text{D6}}$'-branes at the intersection points. The states in the NS sector are all projected out (the tachyon, which is present in this sector is projected out since it is odd under the $\mathbb{Z}_2$-twist) but there remain states from the R sector: At each intersection point one fermion in the $\hat{6}$-$\hat{6}'$ and one in the $\hat{6}'$-$\hat{6}$ sector remains. Both sectors are mapped to themselves under $\Omega$ and therefore the fermions transform in the antisymmetric representation. Thus we have two fermions per intersection and two intersections and thus a total of four additional fermions. The complete spectrum is listed in table F.2.

| | |
|---|---|
| $\widehat{6},\widehat{6}\quad\widehat{6}',\widehat{6}'$ | Gauge bosons of $\mathrm{U}(4)^2$ |
| | Fermions: $2{\times}[(\mathbf{Adj},\,\mathbf{1})+(\mathbf{1},\,\mathbf{Adj})]$ |
| $\widehat{6},\widehat{6}'$ | Fermions: $2\times[(\mathbf{6},\mathbf{1})+(\mathbf{1},\mathbf{6})]$ |
| $\widehat{6}',\widehat{6}$ | Fermions: $2\times[(\mathbf{6},\mathbf{1})+(\mathbf{1},\mathbf{6})]$ |

Table F.2.: Spectrum of $\widehat{\text{D6}}$-$\widehat{\text{D6}}$'-branes

As we have explained in section (4.3) the model with $\widehat{\text{D6}}$-branes is only marginally stable as there exist tachyonic modes for $R_1 \neq 1$. If we shrink $R_1$ further to $R_1 < 1$ another phase transition appears and the $\widehat{\text{D6}}$-branes transform into a system of D7-$\overline{\text{D7}}$-branes.

### F.2.2. The spectrum for $R_1 < 1$

The spectrum is constructed as follows: We obtain two sets of D7-$\overline{\text{D7}}$-branes wrapped on the first torus and sitting at the fixed points $z_2 = 0$ and $z_2 = \frac{1+i}{2}R_2$ in the second torus. The configuration is shown in figure F.3. To obtain the correct twisted charges the projection matrices have to be

$$\gamma_{\theta,7} = e^{\pi i \frac{1}{2}}\mathbb{1}_8, \qquad \gamma_{\theta,\bar{7}} = e^{\pi i \frac{3}{2}}\mathbb{1}_8, \tag{F.10}$$

as was discussed in section 4.3. Because the seven-branes wrap the first torus, they contribute to the twisted charges at the $\theta$ fixed points which are not fixed under $\Omega'\hat{\alpha}$. To cancel these contributions we have to introduce Wilson lines on the first torus. We will use

$$\gamma_{W_1,7} = \gamma_{W_1,\bar{7}} = \mathrm{diag}(\mathbb{1}_4, -\mathbb{1}_4). \tag{F.11}$$

The spectrum is derived as follows. The 77-sector is mapped to the $\bar{7}\bar{7}$-sector under $\Omega'\hat{\alpha}$ [58], thus we have to compute only one of these sectors. On the D7-branes, the $\theta$-projection would give rise to a U(8) gauge group, which is broken down by the Wilson lines to $\mathrm{U}(4)\times\mathrm{U}(4)$. Thus we obtain the particles of a $\mathcal{N}$=1 vector multiplet transforming in the adjoint of $\mathrm{U}(4)^2$. The D7-$\overline{\text{D7}}$ sector is mapped onto itself under the orbifold and

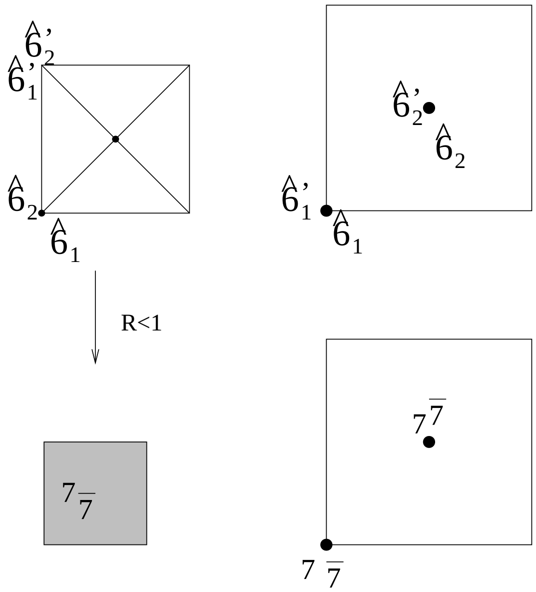


Figure F.3.: The brane configurations for $R_1 < 1$

orientifold projection. The tachyonic NS ground state survives the opposite GSO in this sector, but it is odd under the $\theta$-projection and is thus projected out. Analogously to the $\widehat{\mathrm{D6}}$-brane system we obtain fermions in the anti-symmetric representation of the gauge group. As there are two stacks of branes and anti-branes the whole spectrum is doubled. The complete spectrum is listed in table F.3.

| $77 + \bar{7}\,\bar{7}$ | Gauge bosons of $\mathrm{U}(4)^4$ |
|---|---|
| | Fermion $(\mathbf{Adj},\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{Adj},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{Adj},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{Adj})$ |
| $7\bar{7}$ | Fermion $(\mathbf{6},\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{6},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{6},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{6})$ |
| $\bar{7}7$ | Fermion $(\mathbf{6},\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{6},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{6},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{6})$ |

Table F.3.: Spectrum of D7-$\overline{\mathrm{D7}}$-branes

If we keep $R_1$ small and start to decrease the radius of the second torus we obtain models with $\widehat{\mathrm{D8}}$-branes at $R_2$=1 and a model with D9-$\overline{\mathrm{D9}}$-branes for $R_1, R_2 < 1$. But these configurations are T-dual to the spectra we just derived. The model with $\widehat{\mathrm{D8}}$-branes is T-dual to the model with $\widehat{\mathrm{D6}}$-branes and the model with D9-$\overline{\mathrm{D9}}$ branes is T-dual to the model with D5-$\overline{\mathrm{D5}}$-branes we started with. Thus, we will not repeat the analysis. The details can be found in [40].

# G. The $T^6/\widetilde{G}$ model

In this chapter we will derive the spectrum of the IIB on $T^2 \times T^2 \times T^2/\widetilde{G}$ orientifold. The orientifold group is, as in appendix F,

$$\widetilde{G} = \{1, \Omega'\alpha, \theta = \alpha^2, \Omega'\alpha^3\}.$$

But this time the orientifold group acts on six compactified dimensions. The twist vectors associated to $\hat{\alpha}$ and $\theta$ are $\tilde{v} = \frac{1}{4}(1, 1, -2)$ and $v = 2\tilde{v} = \frac{1}{2}(1, 1, -2)$. $\hat{\alpha}$ has two fixed points in the first and second torus and four fixed points in the third torus. $\theta$ has four fixed points in the first two tori and acts as the identity on the third torus. $\Omega'$ is taken to act as $\Omega' = \Omega R_1 R_2 R_3 (-1)^{F_L}$ where the $R_i$ are reflections $z_i \to -z_i$ in all complex planes and $(-1)^{F_L}$, is the left handed spacetime fermion number. This model was first discussed in [40]. The fixed points are sketched in figure G.1.

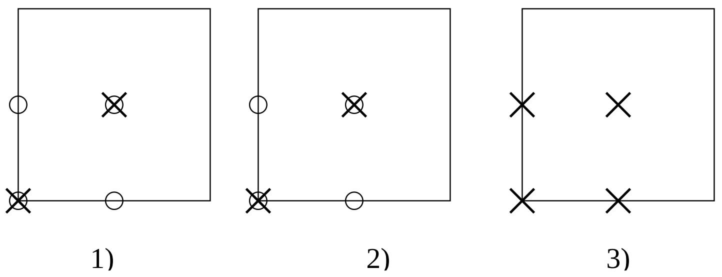


Figure G.1.: Fixed points in the three complex planes

## G.1. Closed string spectrum

We will, again, first focus on the closed string sector. Unlike the previous model we will symmetrise in the twisted NS-NS sector under $\Omega'\hat{\alpha}$ and antisymmetrise in the R-R sector.

### G.1.1. Untwisted sector

#### NS sector

The oscillators in the untwisted NS sector have the following eigenvalues under the elements of the orientifold group ($\alpha = e^{2\pi i \frac{1}{4}}$)

*G. The $T^6/\widetilde{G}$ model*

| osc. | $\theta$ | $\Omega'\hat{\alpha}$ |
|---|---|---|
| $\psi^{\mu}_{-\frac{1}{2}}$ | $\alpha^0$ | $\tilde{\psi}^{\mu}_{-\frac{1}{2}}$ |
| $\tilde{\psi}^{\mu}_{-\frac{1}{2}}$ | $\alpha^0$ | $\psi^{\mu}_{-\frac{1}{2}}$ |
| $\psi^{2}_{-\frac{1}{2}}$ | $\alpha^2$ | $-\alpha\,\tilde{\psi}^{2}_{-\frac{1}{2}}$ |
| $\psi^{\bar{2}}_{-\frac{1}{2}}$ | $\overline{\alpha}^2$ | $-\overline{\alpha}\,\tilde{\psi}^{\bar{2}}_{-\frac{1}{2}}$ |
| $\psi^{3}_{-\frac{1}{2}}$ | $\alpha^2$ | $-\alpha\,\tilde{\psi}^{3}_{-\frac{1}{2}}$ |
| $\psi^{\bar{3}}_{-\frac{1}{2}}$ | $\overline{\alpha}^2$ | $-\overline{\alpha}\,\tilde{\psi}^{\bar{3}}_{-\frac{1}{2}}$ |
| $\psi^{4}_{-\frac{1}{2}}$ | $\alpha^0$ | $\tilde{\psi}^{4}_{-\frac{1}{2}}$ |
| $\psi^{\bar{4}}_{-\frac{1}{2}}$ | $\alpha^0$ | $\tilde{\psi}^{\bar{4}}_{-\frac{1}{2}}$, |

with which we can form the following invariant states:

$$
\begin{aligned}
&\psi^{(\mu}_{-1/2}\tilde{\psi}^{\nu)}_{-1/2}|\,0\,\rangle_{NS}\widehat{=}G_{\mu\nu}+\Phi,\\
&\psi^{\mu}_{-1/2}\tilde{\psi}^{4}_{-1/2}+\psi^{4}_{-1/2}\tilde{\psi}^{\mu}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{\mu}_{-1/2}\tilde{\psi}^{\bar{4}}_{-1/2}+\psi^{\bar{4}}_{-1/2}\tilde{\psi}^{\mu}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{2}_{-1/2}\tilde{\psi}^{\bar{2}}_{-1/2}+\psi^{\bar{2}}_{-1/2}\tilde{\psi}^{2}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{3}_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2}+\psi^{\bar{3}}_{-1/2}\tilde{\psi}^{3}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{4}_{-1/2}\tilde{\psi}^{\bar{4}}_{-1/2}+\psi^{\bar{4}}_{-1/2}\tilde{\psi}^{4}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{2}_{-1/2}\tilde{\psi}^{3}_{-1/2}-\psi^{3}_{-1/2}\tilde{\psi}^{2}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{\bar{2}}_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2}-\psi^{\bar{3}}_{-1/2}\tilde{\psi}^{\bar{2}}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{2}_{-1/2}\tilde{\psi}^{\bar{3}}_{-1/2}+\psi^{\bar{3}}_{-1/2}\tilde{\psi}^{2}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{\bar{2}}_{-1/2}\tilde{\psi}^{3}_{-1/2}+\psi^{3}_{-1/2}\tilde{\psi}^{\bar{2}}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{4}_{-1/2}\tilde{\psi}^{4}_{-1/2}|\,0\,\rangle_{NS},\\
&\psi^{\bar{4}}_{-1/2}\tilde{\psi}^{\bar{4}}_{-1/2}|\,0\,\rangle_{NS}.
\end{aligned}
$$

**The R sector**

With the usual GSO projection (odd number of "$-$" signs) we obtain the following states

| oscillators | $\theta$ | $\Omega'\hat{\alpha}$ |
|---|---|---|
| $\vert+---\rangle_{L/R}$ | $1$ | $\alpha\vert+---\rangle_{R/L}$ |
| $\vert+++-\rangle_{L/R}$ | $1$ | $\overline{\alpha}\vert+++-\rangle_{R/L}$ |
| $\vert++-+\rangle_{L/R}$ | $-1$ | $\vert++-+\rangle_{R/L}$ |
| $\vert+-++\rangle_{L/R}$ | $-1$ | $\vert+-++\rangle_{R/L}$ |
| $\vert-+++\rangle_{L/R}$ | $1$ | $\overline{\alpha}\vert-+++\rangle_{R/L}$ |
| $\vert-+--\rangle_{L/R}$ | $-1$ | $\vert-+--\rangle_{R/L}$ |
| $\vert--+-\rangle_{L/R}$ | $-1$ | $\vert--+-\rangle_{R/L}$ |
| $\vert---+\rangle_{L/R}$ | $1$ | $\alpha\vert---+\rangle_{R/L}$. |

The invariant states are:

$$
\begin{array}{lr}
|+++-\rangle\otimes|+---\rangle-|+---\rangle\otimes|+++-\rangle & h=1,\\
|+++-\rangle\otimes|---+\rangle-|---+\rangle\otimes|+++-\rangle, & \\
|+++-\rangle\otimes|-+++\rangle+|-+++\rangle\otimes|+++-\rangle, & \\
|+---\rangle\otimes|---+\rangle+|---+\rangle\otimes|+---\rangle, & \\
|+---\rangle\otimes|-+++\rangle-|-+++\rangle\otimes|+---\rangle, & \\
|---+\rangle\otimes|-+++\rangle-|-+++\rangle\otimes|---+\rangle & h=-1,\\
|++-+\rangle\otimes|+-++\rangle-|+-++\rangle\otimes|++-+\rangle & h=1,\\
|++-+\rangle\otimes|--+-\rangle-|--+-\rangle\otimes|++-+\rangle, & \\
|++-+\rangle\otimes|-+--\rangle-|-+--\rangle\otimes|++-+\rangle, & \\
|+-++\rangle\otimes|--+-\rangle-|--+-\rangle\otimes|+-++\rangle, & \\
|+-++\rangle\otimes|-+--\rangle-|-+--\rangle\otimes|+-++\rangle, & \\
|--+-\rangle\otimes|-+--\rangle-|-+--\rangle\otimes|--+-\rangle & h=-1,\\
|---+\rangle\otimes|---+\rangle & h=-1,\\
|-+++\rangle\otimes|-+++\rangle & h=-1,\\
|+---\rangle\otimes|+---\rangle & h=1,\\
|+++-\rangle\otimes|+++-\rangle & h=1,
\end{array}
$$

where $h$ denotes the helicity of the states. All states where we have omitted $h$, transform as scalars. The bosonic states from the NS-NS and R-R states and their fermionic partners from the R-NS and NS-R sector we obtain the $\mathcal{N}$=1, D=4 SUGRA multiplet, 6 vector multiplets, the dilaton multiplet and 8 additional chiral multiplets.

### G.1.2. Twisted sector

The $\theta$-twisted sector gives the following:

#### NS sector

There are two twisted ground states in the NS sector, namely

$$|1\rangle^{NS}=|0,+\tfrac{1}{2},-\tfrac{1}{2},0\rangle, \tag{G.1}$$

$$|2\rangle^{NS}=|0,-\tfrac{1}{2},+\tfrac{1}{2},0\rangle, \tag{G.2}$$

from which we can form the invariant combinations

$$|1\rangle_L^{NS}\otimes|1\rangle_R^{NS}, \tag{G.3}$$

$$|1\rangle_L^{NS}\otimes|2\rangle_R^{NS}+|2\rangle_L^{NS}|\otimes|1\rangle_R^{NS}, \tag{G.4}$$

$$|2\rangle_L^{NS}\otimes|2\rangle_R^{NS}. \tag{G.5}$$

### R sector

There are two twisted ground states in the R sector, as well, namely

$$|1\rangle^R = |+\tfrac{1}{2}, 0, 0, -\tfrac{1}{2}\rangle, \tag{G.6}$$
$$|2\rangle^R = |-\tfrac{1}{2}, 0, 0, +\tfrac{1}{2}\rangle, \tag{G.7}$$

from which we can form only one invariant anti-symmetric combination:

$$|1\rangle^R_L \otimes |2\rangle^R_R - |2\rangle^R_L \otimes |1\rangle^R_R. \tag{G.8}$$

From the four bosonic states from the twisted NS-NS and R-R sector we obtain two chiral multiplets per $\Omega'\hat{\alpha}$ fixed point, which gives a total of 8 chiral multiplets. For the other $\theta$ fixed points not fixed under $\Omega'\hat{\alpha}$ we obtain

$$(|1\rangle^{NS}_L \otimes |1\rangle^{NS}_R)_p + (|1\rangle^{NS}_L \otimes |1\rangle^{NS}_R)_{(\Omega'\alpha)p}, \tag{G.9}$$
$$(|1\rangle^{NS}_L \otimes |2\rangle^{NS}_R)_p + (|2\rangle^{NS}_L \otimes |1\rangle^{NS}_R)_{(\Omega'\alpha)p}, \tag{G.10}$$
$$(|2\rangle^{NS}_L \otimes |1\rangle^{NS}_R)_p + (|1\rangle^{NS}_L \otimes |2\rangle^{NS}_R)_{(\Omega'\alpha)p}, \tag{G.11}$$
$$(|2\rangle^{NS}_L \otimes |2\rangle^{NS}_R)_p + (|2\rangle^{NS}_L \otimes |2\rangle^{NS}_R)_{(\Omega'\alpha)p}, \tag{G.12}$$
$$(|1\rangle^R_L \otimes |2\rangle^R_R)_p - (|2\rangle^R_L \otimes |1\rangle^R_R)_{(\Omega'\alpha)p}, \tag{G.13}$$
$$(|2\rangle^R_L \otimes |1\rangle^R_R)_p - (|1\rangle^R_L \otimes |2\rangle^R_R)_{(\Omega'\alpha)p}, \tag{G.14}$$
$$(|1\rangle^R_L \otimes |1\rangle^R_R)_p - (|1\rangle^R_L \otimes |1\rangle^R_R)_{(\Omega'\alpha)p} \tag{G.15}$$
$$(|2\rangle^R_L \otimes |2\rangle^R_R)_p - (|2\rangle^R_L \otimes |2\rangle^R_R)_{(\Omega'\alpha)p}. \tag{G.16}$$

As there are 12 $\theta$-fixed points not fixed under $\Omega'\hat{\alpha}$ we obtain another $12/2 \times 6 = 36$ scalars and another $12/2 \times 1 = 6$ vectors. Thus, there are another 18 chiral multiplets and another 6 vector multiplets in the spectrum for a total of 26 chiral and 6 vector multiplets in the twisted sector.

## G.2. Open string spectrum

We have to use D7-branes to cancel the twisted tadpoles, as can be seen from the volume dependence of the tadpoles, derived in [40]. Let us consider the case where all radii of the compact dimensions are large. From the structure of the fixed points, mentioned in the beginning of this chapter, we see that there are 16 $\Omega'\hat{\alpha}$ fixed points, namely the points $(z_1, z_2, z_3)$ with $z_1, z_2 = \{0, \frac{1+i}{2}R_{1,2}\}$ and $z_3 = \{R_3, \frac{1}{2}R_3, \frac{i}{2}R_3, \frac{1+i}{2}R_3\}$. In addition there are 16 tori fixed under $\theta$ of which four with coordinates $z_1, z_2 = \{0, \frac{1+i}{2}\}$ are fixed under $\Omega'\hat{\alpha}$ and twelve are not. The tadpole cancellation conditions for the $\Omega'\hat{\alpha}$ fixed points are

$$[\mathrm{Tr}\gamma_{\theta,7,P} - \mathrm{Tr}\gamma_{\theta,\bar{7},P}] - 16 = 0. \tag{G.17}$$

The tori fixed under $\theta$ but not fixed under $\Omega'\hat{\alpha}$ do not generate twisted tadpoles thus the twisted charge of the D7- and $\overline{\mathrm{D7}}$-branes should be zero there

$$\mathrm{Tr}\gamma_{\theta,7,Q} - \mathrm{Tr}\gamma_{\theta,\bar{7},Q} = 0. \tag{G.18}$$

We solve this equations as follows: We locate 16 $D7_1$-branes[1] at the origin of the first complex plane, and another 16 $\overline{D7}_1$-branes at the point $z_1 = \frac{1+i}{2}$. With the CP-matrices

$$\gamma_{\theta,7_1,0} = \mathbb{1}_{16}, \qquad \gamma_{\theta,\bar{7}_1,(1+i)/2} = -\mathbb{1}_{16}. \tag{G.19}$$

which satisfy the tadpole cancellation conditions of equation (G.17) at the points $P$ fixed under $\Omega'\hat{\alpha}$. But as the branes wrap the second complex plane completely, they introduce twisted charges at the points $Q$, which should be absent. To avoid these twisted charges we introduce Wilson lines along the second complex plane

$$\gamma_{W_2,7_1} = \mathrm{diag}(\mathbb{1}_8, -\mathbb{1}_8), \qquad \gamma_{W_2,\bar{7}_1} = \mathrm{diag}(\mathbb{1}_8, -\mathbb{1}_8). \tag{G.20}$$

As explained in [40], this model belongs to a class of orientifold models which is termed 'with vector structure' (which means $(\gamma_\theta)^N = +\mathbb{1}$ in this context) and for these models we have to take

$$\gamma_{\Omega,7_1} = \begin{pmatrix} 0 & \mathbb{1}_4 & 0 & 0 \\ -\mathbb{1}_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbb{1}_4 \\ 0 & 0 & -\mathbb{1}_4 & 0 \end{pmatrix} \qquad \gamma_{\Omega,\bar{7}_1} = -\mathbb{1}_{16}. \tag{G.21}$$

The unusual form of $\gamma_{\Omega,7_1}$ comes from the fact that it should project on USp but should not mix sectors with different Wilson line factors.

The Wilson lines break the original U(16) gauge group down to U(8) × U(8) on either branes and anti-branes. On the D7-branes the $\Omega'\hat{\alpha}$ projection leaves the $\mathrm{USp}(8)^2$ subgroup invariant and on the $\overline{D7}$-branes the $\mathrm{SO}(8)^2$ subgroup survives the projections in the gauge sector. On the branes as well as on the anti-branes the transverse scalars in the first two complex planes are projected out, but the scalar and fermionic degrees of freedom in the third complex plane survive. On the D7-branes they form chiral multiplets in the $(\mathbf{36},\mathbf{1})+(\mathbf{1},\mathbf{36})$ as fermions and bosons are projected equally on branes. On the $\overline{D7}$ branes the $\Omega'\hat{\alpha}$ projection acts with an additional factor of $-1$ on the R sector states (as explained in section 3.2.2). From this it follows that for the CP-factors of the R states the projection condition is

$$\lambda = +\gamma_{\Omega,\bar{7}_1}\lambda^T\gamma^{-1}_{\Omega,\bar{7}_1} \Rightarrow \lambda = \lambda^T. \tag{G.22}$$

Therefore the fermions transform under the symmetric representations of SO(8), whereas the NS states transform under the anti-symmetric representation. The complete spectrum of this model is listed in table G.1.

### G.2.1. The spectrum at $R_1 = 1$

The analysis in this section will be rather similar to the considerations of section F. We will vary the radius of the first, torus keeping the other radii large during the analysis. If we shrink the first torus to $R_1 = 1$, the strings stretching between the D7-branes and the $\overline{D7}$-branes survive the projections and become massless. This indicates the possibility

[1] A $D7_i$-brane denotes a brane which is transverse to the $i$th plane

| | | |
|---|---|---|
| Closed | Untwisted | $\mathcal{N}$=1 SUGRA + 6 **V** + 9 **Ch** |
| | Twisted | 6 **V** + 26 **Ch** |
| Open | $7_1 7_1$ | $\mathcal{N}$=1 Vector of USp$(8)^2$ |
| | | Chiral in (**36**,**1**) + (**1**,**36**) |
| | $\overline{7}_1\overline{7}_1$ | Gauge bosons of SO$(8)^2$ |
| | | Fermion$_+$ in (**35** + **1**, **1**) + (**1**, **35** + **1**) |
| | | Complex scalars (**28**, **1**) + (**1**, **28**) |
| | | Fermion$_-$ in (**35** + **1**, **1**) + (**1**, **35** + **1**) |

Table G.1.: The complete spectrum of the $T^6/\widetilde{G}$ orientifold.

for the system to decay into a system of non-BPS D8-branes [45], denoted by $\widehat{\text{D8}}$. This $\widehat{\text{D8}}$-brane stretches on a non-supersymmetric cycle between the points $z_1 = 0$ and $z_1 = \frac{1+i}{2}$, completely wrapping the other two complex planes. This is sketched in figure G.2. The $\widehat{\text{D8}}$-

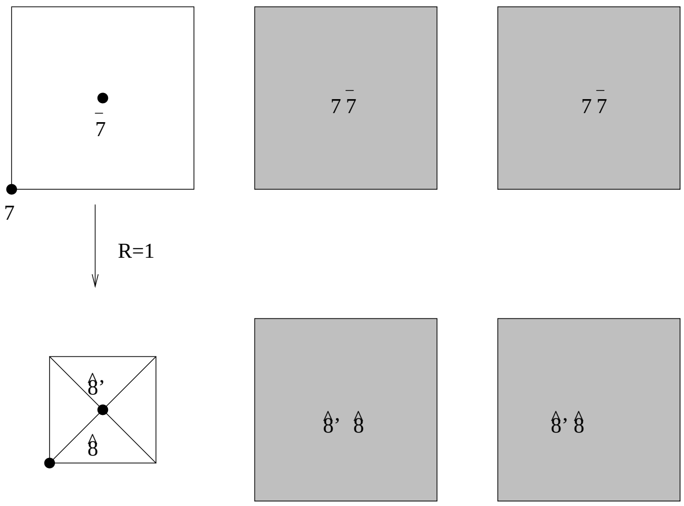


Figure G.2.: The transition at $R_1 = 1$

branes are not fixed under $\Omega'\hat{\alpha}$ thus we have to introduce their image under this projection, denoted by $\widehat{\text{D8}}$'. To satisfy the tadpole cancellation conditions we introduce the CP-matrices

$$\gamma_{\theta,\hat{8}} = \mathbb{1}_8. \tag{G.23}$$

In addition we have to satisfy the tadpole cancellation conditions for $\theta$-fixed points not fixed under $\Omega'\hat{\alpha}$. To meet this requirement, we introduce Wilson lines along the second complex plane of the form

$$\gamma_{W_2,\hat{8}} = \gamma_{W_2,\hat{8}'} = \mathrm{diag}(\mathbb{1}_4, -\mathbb{1}_4). \tag{G.24}$$

In order to derive the spectrum, it is sufficient to derive it on the $\widehat{\mathrm{D8}}$-branes and to perform no $\Omega'\hat{\alpha}$ projection. Additionally we have to take into account the $\widehat{\mathrm{D8}}$-$\widehat{\mathrm{D8}}$' sector, as it is projected onto itself and therefore survives the $\Omega'\hat{\alpha}$ projection. On the branes we have a sector with the ordinary GSO and one sector with the reversed GSO projection.

### The NS sector

We start with the NS sector with the ordinary GSO projection. As the $\theta$ projection acts as the identity, the gauge group is U(8), but it is broken to U(4) $\times$ U(4) by the Wilson lines. Thus, we obtain the gauge bosons of U(4) $\times$ U(4) plus complex scalars from the third complex plane, transforming in the adjoint, as well.

From the sector with the opposite GSO-projection we would obtain a tachyon but it is projected out by $\theta$ since it is odd under the $\mathbb{Z}_2$ twist [45]. The $\widehat{\mathrm{D8}}$-$\widehat{\mathrm{D8}}$' strings give no contribution in the NS sector because all states are projected out.

### The R sector

From the R sector with the usual GSO-projection we obtain the would-be supersymmetric partners to the states from the NS sector with usual GSO projection, namely four fermionic states, combining into two Majorana fermions transforming under the adjoints of U(4) $\times$ U(4).

From the sector with the opposite GSO projection we obtain another two fermions transforming in the adjoint representation.

In the $\widehat{\mathrm{D8}}$-$\widehat{\mathrm{D8}}$' sector we obtain two fermions, suffering the orientifold projection and thus transforming in the antisymmetric representation. The same holds for the $\widehat{\mathrm{D8}}$'-$\widehat{\mathrm{D8}}$ sector. As the branes intersect twice we obtain two copies of these. The complete spectrum is written down in table G.2.

| Open | $\widehat{\mathrm{D8}}\,\widehat{\mathrm{D8}}$, $\widehat{\mathrm{D8}}$'$\widehat{\mathrm{D8}}$' | $\mathcal{N}$=1 Vector of U(4)$^2$ |
|---|---|---|
| | | Chiral in (**Adj**,**1**) + (**1**,**Adj**) |
| | | 2 Fermions in (**Adj**, **1**) + (**1**, **Adj**) |
| | $\widehat{\mathrm{D8}}\widehat{\mathrm{D8}}$'+$\widehat{\mathrm{D8}}$'$\widehat{\mathrm{D8}}$ | 8 Fermions in (**6**, **1**) + (**1**, **6**) |

Table G.2.: The complete spectrum of the $T^6/\widetilde{G}$ orientifold at $R_1 = 1$.

## G.2.2. The spectrum at $R_1 < 1$

If we vary the radius away from $R_1 = 1$, tachyonic modes arise. For $R_1 > 1$ we obtain a tachyonic momentum mode along the world volume of the brane, whereas for $R_1 < 1$ a stretching mode becomes tachyonic. In the region $R_1 < 1$ the system transforms into a system of D9-$\overline{\mathrm{D9}}$-branes. The complete transition is sketched in figure G.3.

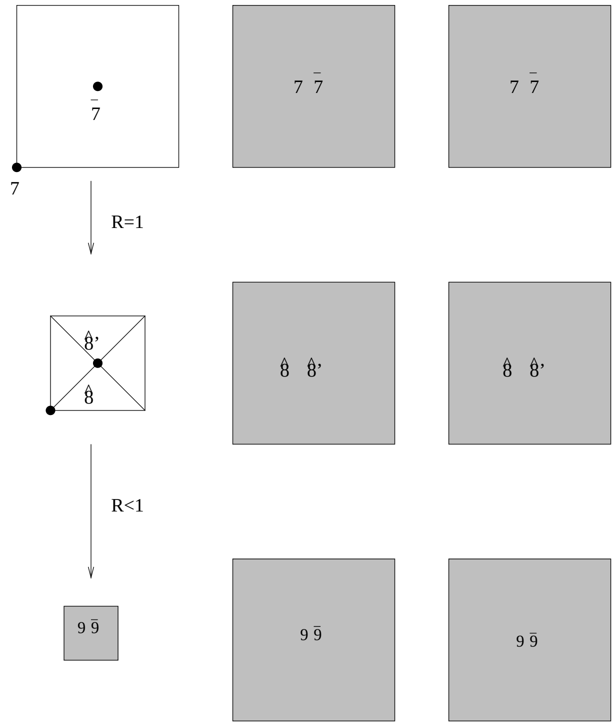


Figure G.3.: The transitions for varying $R_1$

In the region $R_1 = 1$ the D9-$\overline{\text{D9}}$ system is stable, as well as the system of the D7-$\overline{\text{D7}}$-branes. From analysing the $\theta$ action on 7-9 strings we see that $\theta$ has to act on the D9-$\overline{\text{D9}}$ strings with a CP-matrix of the form

$$\gamma_{\theta,9} = e^{i\frac{\pi}{2}}\mathbb{1}_{16}, \qquad \gamma_{\theta,\bar{9}} = e^{i\frac{3\pi}{2}}\mathbb{1}_{16}. \tag{G.25}$$

The argument is as follows: Consider a string stretching form a D7-brane to a D9-brane. This string has ND boundary conditions in one complex plane. Thus it has a half-integer moding in this plane and the $\theta$ action on the oscillator gives a factor of $i$. As these strings should be invariant under the projection, the factor from the oscillators should be canceled by the CP-matrices. This gives an additional factor of $i$ on the CP-matrices. Thus we obtain a model "without vector structure" $((\gamma_\theta)^N = -1)$. $\Omega'\hat{\alpha}$ interchanges D9- and $\overline{\text{D9}}$-branes so it is enough to consider the $\theta$ projection on one of them and perform no $\Omega$ projection.

In order to cancel the twisted charges at the points not fixed under $\Omega'\hat{\alpha}$ we have to introduce Wilson lines. We choose them as follows:

$$\gamma_{W_1,9} = \text{diag}(\mathbb{1}_8, -\mathbb{1}_8), \quad \gamma_{W_1,\bar{9}} = \text{diag}(-\mathbb{1}_8, \mathbb{1}_8), \tag{G.26}$$

$$\gamma_{W_2,9} = \text{diag}(\mathbb{1}_8, -\mathbb{1}_8), \quad \gamma_{W_2,\bar{9}} = \text{diag}(\mathbb{1}_8, -\mathbb{1}_8). \tag{G.27}$$

The gauge group we obtain is $\text{U}(8) \times \text{U}(8)$, the subgroup of $\text{U}(16)$ which is unbroken by the Wilson lines. From the sector with ordinary GSO-projection we obtain a vector multiplet plus an additional chiral multiplet. The D9-$\overline{\text{D9}}$ sector is mapped onto itself. The tachyon survives the GSO projection but is projected out by the $\theta$ twist. We obtain two fermions in the antisymmetric representation of the group. The $\overline{\text{D9}}$-D9 sector is analogous. The results are summarised in table G.3

| | | |
|---|---|---|
| Open | D9-D9, $\overline{\text{D9}}$-$\overline{\text{D9}}$ | $\mathcal{N}$=1 Vector of $U(8)^2$ |
| | | Chiral in (**Adj**,**1**) + (**1**,**Adj**) |
| | D9-$\overline{\text{D9}}$+$\overline{\text{D9}}$-D9 | 4 Fermions in (**28**, **1**) + (**1**, **28**) |

Table G.3.: The complete spectrum of the $T^6/\widetilde{G}$ orientifold at $R_1 < 1$.

*G. The* $T^6/\widetilde{G}$ *model*

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