Physikalisches Institut
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Theoretische Physik

Exercise 2
21.10.02
WS 02/03

# Exercises on 'Elementary Particle Physics'

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1. *Massless fermions*

   We now want to find the solutions of the *Dirac* equation for massless particles.

   (a) Proceed analogously to exercise 1 of the last sheet. What's the *Dirac* equation for massless particles? Which anticommutation relations do you get? And which hermiticity properties?

   Unlike for the massive case the lowest dimensionality matrices satisfying the relations of (1) are now $2 \times 2$.

   (b) Which matrices could that be?

   (c) Show that the massless *Dirac* equation divides into two decoupled equations for two-component spinors $\chi(\vec{p})$ and $\phi(\vec{p})$:

   $$E\chi = -\vec{\sigma} \cdot \vec{p}\chi \ , \quad E\phi = +\vec{\sigma} \cdot \vec{p}\phi \ .$$

   (d) What can you say about the energy eigenvalues of these two equations? Classify the solutions in terms of the properties particle/antiparticle and helicity.

2. *Chirality and helicity*

   (a) Define the chirality ($\equiv$ handedness) operator $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ and show that

   $$\gamma^{5\dagger} = \gamma^5 \ , \quad (\gamma^5)^2 = \mathbb{1} \ , \quad \{\gamma^5, \gamma^\mu\} = 0 \ .$$

   (b) Give $\gamma^5$ in terms of the $\alpha$'s and $\beta$ and compute the commutator $[H, \gamma^5]$ for the massive case. What's about the massless case?

   (c) Compute $\gamma^5$ in the *Pauli-Dirac* representation and show that chirality and helicity coincide for

      (i) $m = 0$ (massless particles).

      (ii) $E \gg m$ (ultrarelativistic limit).

   (d) Discuss when which quantities are 'good' quantum numbers.