Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 9
December 18^{th}, 2002
WS 2002/2003

Exercises on *Theoretical Elementary Particle Physics*

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1. **Lecture Clean-up and Completion** In the last lecture we were given the Lagrangian density of the (one-generation) Glashow-Salam-Weinberg (GSW) theory:

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{4}\left(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu \;+\; g_W\, \varepsilon^a{}_{bc}\, W^b_\mu\, W^c_\nu\right)\left(\partial^\mu W_a^\nu - \partial^\nu W_a^\mu \;+\; g_W\, \varepsilon_a{}^{de}\, W_d^\mu\, W_e^\nu\right) \\
& -\frac{1}{4}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)\left(\partial^\mu B^\nu - \partial^\nu B^\mu\right) \\
& + i\,\overline{e_R}\,\gamma^\mu\, D_\mu\;\; e_R \;+ i\,\overline{d_R}\,\gamma^\mu\, D_\mu\;\; d_R \;+ i\,\overline{u_R}\,\gamma^\mu\, D_\mu\;\; u_R \\
& + i\,\overline{q}\,\gamma^\mu\, D_\mu\;\; q \;+ i\,\overline{\ell}\,\gamma^\mu\, D_\mu\;\; \ell \\
& + (D_\mu\, \tilde{H})^\dagger\, (D^\mu\, \tilde{H}) \\
& - \mu^2\, \tilde{H}^\dagger \tilde{H} \;+\; \lambda\, (\tilde{H}^\dagger \tilde{H})^2 \quad [\mu^2, \lambda \;\text{ both positive}] \\
& + \left(G^e\, \overline{e_R}\, \tilde{H}^T\, \epsilon\, \ell \;+\; G^d\, \overline{d_R}\, \tilde{H}^T\, \epsilon\, q \;+\; G^u\, \overline{u_R}\, \tilde{H}^\dagger\, q \;+\; c.c.\;\right),
\end{aligned}
$$

with

$$
D_\mu\, q \equiv \left(\partial_\mu + i\, g_Y\; Y_q\; \mathbb{1}\; B_\mu \;+\; i\; g_W\; \sigma_a\; W^a_\mu\right)\, q, \quad \text{etc.}
$$

This Lagrangian density is invariant under local $SU(2)_W \times U(1)_Y$ gauge transformations (but note that the vacuum is not invariant, thus one speaks of a spontaneously *hidden* or *broken* symmetry).

- Why the $+\,c.c.$ in the last line of the Lagrangian density? What about $SU(3)_C$?
- How does the *finite* transformation (Abelian and non-Abelian) for gauge vector fields have to look like?
- Proof that the Yukawa interactions are gauge-invariant.
- How does the Lagrangian density change when we replace $\tilde{H}$ by $\epsilon\, H^*$?
- Recall the steps which we performed in the lecture.
- Why didn't we encounter any goldstone bosons in the lecture?
- Examine $(D_\mu)'$ and $\overline{q'}\,\gamma^\mu\,(D_\mu)'\;\; q'$.

Note that the symmetry breaking scalar potential of the *SM* $\left(-\mu^2\,\tilde{H}^\dagger\tilde{H} + \lambda\,(\tilde{H}^\dagger\tilde{H})^2\right)$ is not able to break $SU(2)_W \times U(1)_Y$ completely but only to $U(1)_Q$. In the minimal supersymmetric standard model (*MSSM*) the scalar potential would in principle be able to break $U(1)_Q$ and also $SU(3)_C$, so one has to demand that the parameters (i.e. coupling constants) are such that this is not the case.

2. **The See-Saw Mechanism** for one generation and real coupling constants (see-saw='Wippe') How does a mass term for neutrinos look like if we introduce righthanded neutrinos? Is it a Dirac or Majorana mass term? Which hypercharge does ν_R have? Which other term in the Lagrangian is thus allowed, what kind of mass term is it? How can the two mass terms be combined? How can this matrix be diagonalized? How do the entries of the diagonal matrix look like, what is the limit if $M \gg m$?

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Homework 4
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1. **Hypercharge Quantization** Internally consistent QFT requires the so-called chiral anomalies of local gauge theories to be absent, which in turn imposes that one has to have

$$\begin{aligned} 2Y_q - Y_{u_R} - Y_{d_R} &= 0, \\ 3Y_q + Y_\ell &= 0, \\ 6Y_q^3 - 3Y_{u_R}^3 - 3Y_{d_R}^3 + 2Y_\ell^3 - Y_{e_R}^3 &= 0; \end{aligned}$$

furthermore, if one believes in quantum gravity,

$$6Y_q - 3Y_{u_R} - 3Y_{d_R} + 2Y_\ell - Y_{e_R} = 0.$$

By solving these four equations, show that there are three solutions (note that demanding that $\overline{e_R}\, H^\dagger\, \ell$ is $U(1)_Y$-invariant fixes Y_H):

$$Y_{e_R} = 2Y_\ell,\quad Y_{u_R} = -\frac{4}{3}Y_\ell,\quad Y_{d_R} = \frac{2}{3}Y_\ell,\quad Y_q = -\frac{1}{3}Y_\ell;$$

$$Y_{e_R} = 2Y_\ell,\quad Y_{u_R} = \frac{2}{3}Y_\ell,\quad Y_{d_R} = -\frac{4}{3}Y_\ell,\quad Y_q = -\frac{1}{3}Y_\ell;$$

$$Y_{e_R} = 0,\quad Y_{u_R} = -Y_{d_R},\quad Y_q = 0,\quad Y_\ell = 0.$$

Which of these solutions is chosen by Nature (this choice is naturally explained in the context of grand unified theories)? Why are the first two solutions so similar?

2. (optional) **Unitarity Gauge** Determine the unitary 2×2 matrix (depending on a, b, c, d, all real) that transforms

$$\begin{pmatrix} a+ib \\ c+id \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ \sqrt{a^2+b^2+c^2+d^2} \end{pmatrix}.$$

3. **Consequences of SSB**

   - Show that by choosing the $SU(2)_W \times U(1)_Y$-gauge such that

     $$H \longrightarrow H' = \begin{pmatrix} 0 \\ \eta + v \end{pmatrix}$$

     one gets, with

     $$v^2 = \frac{\mu^2}{2\lambda}$$

     that

     $$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

     becomes

     $$V(\eta) = -\lambda v^4 + 4\lambda v^2 \eta^2 + 4\lambda v^3 \eta^3 + \lambda \eta^4$$

     .
   - Show that from $(D_\mu' H')^\dagger (D_\mu' H')$ follows the massterm for the $W^\pm$-bosons and the mass matrix for $\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$ given in the lecture. In terms of g_W and g_Y calculate both the orthogonal matrix that diagonalizes the mass matrix and the diagonal matrix in terms of g_W and g_Y. Thus determine $\cos\vartheta_{weinberg}$ and $\sin\vartheta_{weinberg}$.
   - Show that from the kinetic terms of W_a^μ and B^μ after SSB one gets Feynman vertices with $W^+W^-\gamma\gamma$, $W^+W^-Z\gamma$, W^+W^-ZZ, W^+W^-Z, $W^+W^-\gamma$.

4. **Grand Unified Theories** As was explained in the lecture, the GSW theoery does not unify but rather integrate. In order to unify one would need a group that has $SU(3)_C \times SU(2)_W \times U(1)_Y$ as a subgroup (provided that $g_C = g_W = g_Y$, which is certainly not the case at *low* energies). Examinig the generators, show that this is the case for $SU(5)$.

* * * *Merry X-Mas and a HNY* * * *