Exercises on 'Elementary Particle Physics'

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1. Some kinematics

Here are some smaller exercises to supplement the lectures.

(a) It was shown that the incident flux in the lab is given by $\vec{F} = (\vec{v}_A - \vec{v}_B) 2E_A 2E_B$. Show that for antiparallel collisions between particles A and B:

$$|\vec{F}| = 4(|\vec{p}_A|E_B + |\vec{p}_B|E_A) = 4((p_A \cdot p_B)^2 - m_A^2 m_B^2)^{1/2}$$
.

Is the flux Lorentz invariant? Comment your answer.

(b) For spinless electron-muon scattering we found in the lecture that

$$|\mathcal{M}|^2 = rac{e^4}{q^4}[(p_A + p_C) \cdot (p_B + p_D)]^2 \ .$$

Assume that $m_i = 0$ and verify that in the CMS

$$\left. \frac{d\sigma}{d\Omega} \right|_{CM} = \frac{\alpha^2}{4s} \left(\frac{3 + \cos \theta}{1 - \cos \theta} \right)^2 ,$$

where $\alpha = \frac{e^2}{4\pi^2}$.

(c) The Mandelstam variables are defined by

$$s = (p_A + p_B)^2,$$

 $t = (p_A - p_C)^2,$
 $u = (p_A - p_D)^2.$

Prove that $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$.

(d) Consider a decay $A \to B_1 + B_2$. Use the results of the lecture to show that

$$\Gamma(A \to B_1 + B_2) = \frac{p_f}{32\pi^2 m_A^2} \int |\mathcal{M}|^2 d\Omega$$
.