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Example sheet 4
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Elementary Particle Physics II

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1. **Vector superfields and gauge transformations**
   A vector superfield V is defined through $V \equiv V^\dagger$.

   (a) Check that

   $$\begin{aligned} V(x,\theta,\bar\theta) \;=\; & C(x) + i\,\theta\chi(x) - i\,\bar\theta\bar\chi(x) + \frac{1}{2}\,i\,\theta\theta\,[M(x)+iN(x)] \\ & -\frac{1}{2}\,i\,\bar\theta\bar\theta\,[M(x)-iN(x)] + \theta\sigma^\mu\bar\theta\,V_\mu(x) \\ & +i\,\theta\theta\bar\theta\left[\bar\lambda(x) + \frac{i}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right] - i\,\bar\theta\bar\theta\theta\left[\lambda(x) + \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right] \\ & +\frac{1}{2}\,\theta\theta\bar\theta\bar\theta\left[D(x) - \frac{1}{2}\partial_\mu\partial^\mu C(x)\right] \end{aligned} \tag{1}$$

   satisfies this relation.

   (b) Using a chiral superfield Λ, is $i\,(\Lambda - \Lambda^\dagger)$ a vector superfield? Identify the component fields from (a) here.
   The identification for the $V_\mu(x)$ component suggests that the vector superfield transforms under a $U(1)$ gauge transformation as

   $$V \rightarrow V' = V + i\,(\Lambda - \Lambda^\dagger)\,. \tag{2}$$

   (c) One gauge choice for V is the *Wess-Zumino* gauge, where $C = \chi = M = N = 0$. Calculate V_{WZ}, V_{WZ}^2 and V_{WZ}^3.

2. **Gauge invariant couplings**

   (a) Generalizing the $U(1)$ tranformations known from electrodynamics, a chiral superfield transforms as $\Phi \rightarrow \Phi' = e^{-2iq\Lambda}\Phi$. Is $\Phi^\dagger\Phi$ gauge invariant? What about $\Phi^\dagger\, e^{2qV}\,\Phi$?

   (b) Calculate

   $$\left[\Phi^\dagger\, e^{2qV}\,\Phi\right]_{\theta\theta\bar\theta\bar\theta} \tag{3}$$

   in the Wess-Zumino gauge.

3. **Gauge field kinetic terms**

The supersymmetric generalization of the electromagnetic field strength is given by the chiral spinor superfield

$$W_\alpha = \bar{D}\bar{D}D_\alpha V\,, \tag{4}$$

where D and $\bar{D}$ are the covariant derivatives defined on sheet 2.

(a) Show that W_α is a left chiral superfield and that it is invariant under gauge transformations.

(b) Compute the explicit form of the F-term ($\theta\theta$ component) of $W_\alpha W^\alpha$.
(Lengthy calculation! You can use V_{WZ} and a suitable left/right coordinate choice for the derivatives.)
Up to an overall factor, the result should be

$$-\frac{1}{4}V^{\mu\nu}V_{\mu\nu} + i\,\lambda\sigma^\mu\partial_\mu\bar{\lambda} - \frac{1}{4}V^{\mu\nu}({}^*V_{\mu\nu}) + \frac{1}{2}D^2\,. \tag{5}$$