Universität Bonn
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Theoretische Physik

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Example sheet 1
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General Relativity

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1. ***Lorentz transformations***

   Let us write a linear transformation of the space-time coordinates as a matrix Λ which acts on the coordinate 4-vectors as $x'^{\mu} = \Lambda^{\mu}{}_{\nu}\, x^{\nu}$. The metric η is defined as $-\eta_{\mu\nu} \equiv \mathrm{diag}(1, -1, -1, -1)$.

   (a) Requiring the length element $ds^2 = -\eta_{\mu\nu} dx^{\mu} dx^{\nu}$ to be constant leads to a constraint equation for Λ, which defines Λ as a Lorentz transformation. What does this constraint look like?

   (b) What form does Λ take for purely spatial rotations? Check that this Λ satisfies the condition in (a).

   (c) Now consider a *boost* along the z-axis (*i.e.* the origin of the x'^{μ}-system moves along the z-axis with constant speed v). Use condition (a) to derive Λ for this situation.
   (Hint: the only interesting components are $\Lambda^0{}_0$*,* $\Lambda^0{}_3$*,* $\Lambda^3{}_0$ *and* $\Lambda^3{}_3$*.*
   Also, $\cosh x = \frac{1}{\sqrt{1-\tanh^2 x}}$, $\sinh x = \frac{\tanh x}{\sqrt{1-\tanh^2 x}}$ *and* $\cosh^2 x - \sinh^2 = 1$*)*

2. ***Time and space dilations***

   Have a look at figure 1: it shows the following situation from the viewpoint of an observer R at rest: At event A, two spaceships S and T are launched with a velocity such that $\gamma = 100$. At event B, the ships encounter a planet, where one ship (T) remains, while the other ship (S) continues on its old course. At event C, ship T begins its return journey to Earth, with the same velocity as before. Its arrival back on Earth is event D.

   (a) Calculate the velocity of the ships from $\gamma = 100$ as a percentage of c.

   (b) Calculate the distance d from Earth to the planet, in the frame of R. How far is it from the spaceships' point of view?

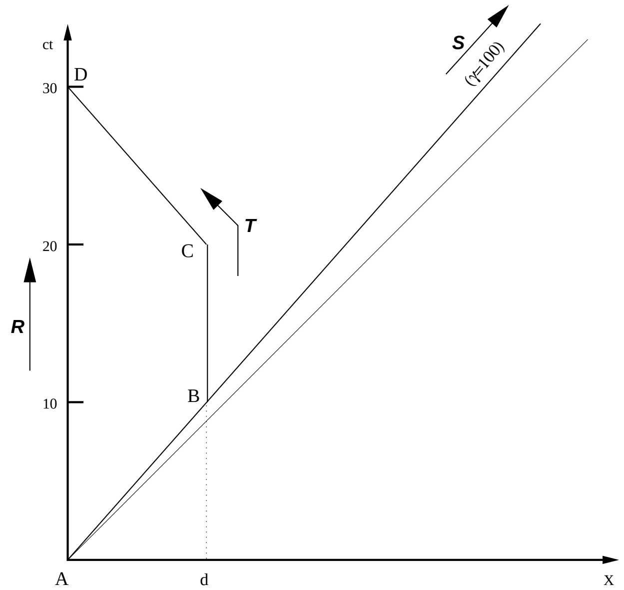

Figure 1: Space-time diagram for Question 2.

(c) Complete the following table:

	A	B	C	D
$(t,x)_R$	(0,0)	(10,)	(20,)	(30,0)
$(t,x)_S$	(0,0)	(,0)	(,)	(,)
$(t,x)_T$	(0,0)	(,0)	(,0)	(,0)

(d) In the frame of R, calculate $\tau_R(A \to D)$, $\tau_T(A \to D)$ and $\tau_S(t_R = 30)$.

(e) Sketch a space-time diagram in which S is at rest. Use the table to mark the events A–D.

(f) In the frame of S, calculate $\tau_R(A \to D)$, $\tau_T(A \to D)$ and $t_S(D)$.

3. *Accelerated objects*

(a) Sketch the rest-frame space-time diagram of an object O moving along the path

$$x(t) = \sin t \qquad (1)$$

from $t = 0$ until O returns to the starting point.
Let's call the event at the origin A, the one at the return point B.

(b) How much time elapses between A and B for an observer R at rest?

(c) Integrate over $d\tau$ to calculate the time between A and B for the co-moving observer O. ($\beta \equiv v/c$)