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Exercise 9
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SS 06

Exercises on Elementary Particle Physics II

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1. *The MSSM Higgs sector*

   (a) Using the R-parity preserving part of the superpotential, find the part of the scalar potential that contains mass terms for

   $$\text{scalar}(\bar{H}) = \bar{h} = (\bar{h}^+, \bar{h}^0) \quad \text{and} \quad \text{scalar}(H) = h = (h^0, h^-). \tag{1}$$

   Hint: Do not forget to consider the quadratic term $\mu H \bar{H}$.

   (b) Add the D-term contribution from the gauge couplings to the scalar potential:

   $$V_{\text{D-term}} = \frac{1}{2} g_1^2 \left(\sum_{\varphi} \varphi^* Y \varphi \right)^2 + \frac{1}{2} g_2^2 \sum_{a=1}^{3} \left[\sum_{(\varphi^1, \varphi^2)} \left(\varphi^{1*}, \varphi^{2*} \right) T^a \begin{pmatrix} \varphi^1 \\ \varphi^2 \end{pmatrix} \right]^2 \tag{2}$$

   where $\varphi \in \{h^0, h^-, \bar{h}^+, \bar{h}^0\}$, $(\varphi^1, \varphi^2) \in \{h, \bar{h}\}$, Y is the hypercharge and $T^a = \frac{\sigma^a}{2}$ are the generators of SU(2).

   (c) Looking at the full scalar potential for the Higgs fields in unbroken SUSY, is a breaking of the electroweak symmetry possible?

   (d) Include the following soft SUSY breaking terms in the scalar potential

   $$\mathcal{L}_{\text{soft}} = -m^2_{\text{soft},1} |h|^2 - m^2_{\text{soft},2} |\bar{h}|^2 - m_3^2 (\bar{h} h + c.c.)\,, \tag{3}$$

   where $|h|^2 = h^\dagger h = |h^0|^2 + |h^-|^2$ and $\bar{h} h = \bar{h}^a h^b \varepsilon_{ab}$. We want the resulting potential's minimum to break electroweak symmetry. (It is possible to set $\langle \bar{h}^+ \rangle = \langle h^- \rangle = 0$ through a SU(2) gauge transformation. $\langle \bar{h}^0 \rangle$ and $\langle h^0 \rangle$ can be made real and positive by a phase redefinition.)
   Hint: The scalar potential should look like:

   $$V(h, \bar{h}) = m_1^2 |h|^2 + m_2^2 |\bar{h}|^2 + m_3^2 (\bar{h} h + c.c.) + \frac{g_1^2 + g_2^2}{8} \left(|h|^2 - |\bar{h}|^2 \right)^2 \tag{4}$$

   How are m_1^2 and m_2^2 defined?

(e) One requirement for successful electroweak symmetry breaking is a negative $(\text{mass})^2$ term for at least one linear combination of the Higgs fields. What inequality does m_3^2 have to satisfy to achieve this? The second requirement is that the potential should be bounded from below. When is this guaranteed?

(f) Show that $|\mu|^2$, $m^2_{\text{soft},1}$, $m^2_{\text{soft},2}$ and m_3^2 can be related through m_Z^2 if we require agreement with experimental result for the Higgs vev:

$$v_{\text{SM}}^2 = \langle h^0 \rangle^2 + \langle \bar{h}^0 \rangle^2 = \frac{4m_Z^2}{g_1^2 + g_2^2} \approx (246GeV)^2 \tag{5}$$

(Since only the sum of the squares of $\langle h^0 \rangle$ and $\langle \bar{h}^0 \rangle$ is fixed experimentally, the parameter β is introduced to parameterize the remaining choice. One defines $\tan\beta = \bar{v}/v = \langle \bar{h}^0 \rangle / \langle h^0 \rangle$.)

(g) Check that the relations you found satisfy the constraints in (e).

(h) After electroweak symmetry breaking, three of the eight real scalar degrees of freedom of the two Higgs multiplets are swallowed to give mass to the Z^0 and $W^\pm$ bosons. The remaining physical fields are usually named A^0 (a neutral CP-odd pseudoscalar), $H^\pm$ (two charged scalars that are conjugates to each other), H_0 and h_0 (a heavy and a light CP-even scalar filed).

Obtain the mass matrix for H_0 and h_0. Show that m_{h^0} has an upper bound.

Hint: H_0 and h_0 are a mixture of $\text{Re}(h^0) - \langle h^0 \rangle$ and $\text{Re}(\bar{h}^0) - \langle \bar{h}^0 \rangle$. You can use $m_{A^0}^2 = 2m_3^2 / \sin 2\beta$ to simplify the notation.