Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 9
16. Juni 2008
SS 08

Exercises on Elementary Particle Physics II

Prof. Dr. H.-P. Nilles

1. *Supersymmetry breaking*

At experimentally accessible energies no supersymmetry is observed up to now. Thus, SUSY has to be broken at a higher scale (at about 1 TeV) in order to solve the problems of the Standard Model on the one hand and to be compatible with its experimental checks on the other hand. Breaking of (global) supersymmetry is realized as a **spontaneous symmetry breaking**, i.e. the vacuum does not admit the symmetry of the Lagrangian.

First of all let us discuss some general aspects of supersymmetry breaking. Iff $Q_\alpha |0\rangle \neq 0$ SUSY is broken spontaneously. This is equivalent to $\langle X| Q_\alpha |0\rangle \neq 0$ for (at least) one $|X\rangle$ or

$$\langle 0| \left\{Q_\alpha, \hat{X}\right\} |0\rangle = \left\langle \delta_S \hat{X} \right\rangle \neq 0, \tag{1}$$

where $\left\langle \delta_S \hat{X} \right\rangle$ denotes the vacuum expectation value (VEV) of the SUSY variation of the operator $\hat{X}$. Here, $\hat{X}$ is any operator in the theory. In the following we consider only the classical limit, i.e. tree-level, with $\left\langle \delta_S \hat{X} \right\rangle = \delta_S X$ for classical fields X.[1]

(a) What kind of operators $\hat{X}$ are allowed to develop a VEV $\left\langle \hat{X} \right\rangle$ in order to preserve Poincare invariance in the vacuum? Consider the supersymmetry variations of all fields in the chiral multiplet, cf. Ex. 6.1, (j). What does the vanishing of $\delta\psi$ imply for SUSY breaking and for $V = |F|^2$? When is SUSY broken?

(b) Perform the same analysis for the vectormultiplet with SUSY transformations

$$\delta_{(\epsilon,\bar{\epsilon})} V^\mu = -i\bar{\lambda}\bar{\sigma}^\mu \epsilon + i\bar{\epsilon}\bar{\sigma}^\mu \lambda, \qquad \delta_{(\epsilon,\bar{\epsilon})}\lambda = \bar{\sigma}^{\mu\nu}\epsilon F_{\mu\nu} + i\epsilon D,$$
$$\delta_{(\epsilon,\bar{\epsilon})} D = -\epsilon\sigma^\mu D_\mu \bar{\lambda} - D_\mu \lambda\sigma^\mu \bar{\epsilon}, \tag{2}$$

where D_μ denotes the covariant derivative.[2] Note that $V = \frac{1}{2}D^2$ now.

(c) An alternative point of view is the SUSY algebra itself. Express the Hamiltonian H in terms of Q_α, $\bar{Q}_{\dot{\alpha}}$. Infer an inequality for E on the spectrum of any SUSY theory. When is this an equality? Reproduce the results of (a), (b) using $V = |F|^2 + \frac{1}{2}D^2$ for a chiral and a vectormultiplet.

[1] In the full quantum field theory, VEVs of (fundamental) operators are determined by the **effective potential** subject to quantum corrections. However, this equals the classical potential at tree-level.

[2] To check (2) use $\delta_{(\epsilon,\bar{\epsilon})} = \epsilon Q + \bar{Q}\bar{\epsilon}$ acting on V in the normal representation S of Q, $\bar{Q}$.

Thus, we have seen that supersymmetry is broken if and only if auxiliary fields get a nonzero VEV. Therefore, one distinguishes between **F-term** and **D-term breaking** for $\langle F\rangle \neq 0$ and $\langle D\rangle \neq 0$, respectively. However, $\langle V\rangle = 0$ for unbroken SUSY is already a generic feature of the SUSY algebra.

2. *F-term breaking in the O'Raifeartaigh model*

   Consider three left chiral superfields X, Y and Z with component fields denoted by (x, ψ_x, F_x), for example. The **O'Raifeartaigh model** is defined by

   $$\mathcal{L}_D = K(X,Y,Z)|_{\theta^2\bar\theta^2} = (X^\dagger X) + (Y^\dagger Y) + (Z^\dagger Z)\big|_{\theta^2\bar\theta^2}$$

   with the superpotential given by

   $$W = \lambda X(Z^2 - M^2) + gYZ$$

   with λ, g and M real.

   (a) Derive the scalar potential using $F_x^* = -\frac{\partial W(x,y,z)}{\partial x}$, e.g., and $V(x,y,z) = \sum_i |F_i|^2$. Show that the vevs of F_x, F_y and F_z cannot vanish simultaneously. Hence SUSY is spontaneously broken.

   (b) Check that the scalar potential $V(x,y,z)$ has a minimum at $z = y = 0$ when

   $$M^2 < \frac{g^2}{2\lambda^2}.$$

   (c) Compute the scalar masses and the masses of the fermions.
   Hint: Make an expansion in terms of fluctuations around the background defined by the VEVs (e.g. $z \to < z > +z$) and look for quadratic terms in the fields. In order to diagonalize the mass of the z field, choose the ansatz $z = \frac{1}{\sqrt{2}}(a + ib)$. Combine ψ_y and ψ_z to a Dirac fermion $\Psi_D = \left(\psi_y, \bar\psi_z\right)^T$ The vev of x is undetermined, so the term $x\Psi_z\Psi_z$ does not contribute to the mass.

   (d) Calculate the **supertrace** of the quadratic mass matrix over spins J defined by

   $$\mathrm{STr}M^2 := \sum_J (-1)^{2J}(2J+1)M_J^2. \tag{3}$$

   The vanishing of $\mathrm{STr}M^2$ is a generic feature of F-term SUSY breaking, cf. Ex. 9.3.

3. *D-term breaking via the Fayet-Iliopoulos mechanism*

   Consider the $\mathcal{N} = 1$ Super Yang-Mills theory of Ex. 8.2 with a gauge group containing at least one U(1)-factor. Then, add the so-called **Fayet-Illiopoulos term** to the action (10) of Ex. 8.2 given by

   $$\mathcal{L}_{\mathrm{FI}} = 2 \sum_{A \equiv \mathrm{U}(1)\text{-factors}} \xi^A \int d^2\theta d^2\bar\theta V^A. \tag{4}$$

Assume a charges q^A for the chiral superfield, i.e.

$$\Phi \mapsto e^{-i2q^A\Lambda^A}\Phi. \tag{5}$$

(a) Why is the term $2\xi^A V^A$ an allowed term? Why is it gauge invariant only for U(1) factors?

(b) What are the equations of motion for the auxiliary fields D^a now? Determine the scalar potential $V(\varphi)$?

(c) Restrict to the case of one U(1) with charge q. Discuss the potential $V(\varphi)$ qualitatively for the two cases: $q\xi < 0$ and $q\xi > 0$. When is SUSY broken? When is the U(1) gauge symmetry broken?

Now turn to a more quantitative analyis.

- Case $q\xi < 0$.
  Illustratively, the shape of the scalar potential $V(\varphi)$ motivates that the radial component of φ gets massive while the angular component remains massless. Verify this by a computation.
  Since φ is charged, the vev of $\varphi\varphi^*$ breaks the U(1) gauge symmetry. Compute the mass of the gauge boson V_μ.
  Is it possible to define a massive Dirac fermion? What is the relation between the Dirac mass and the mass of the gauge boson? Describe the SUSY-Higgs mechanism.
  Hint: The mass of the gauge boson arises from the term $(D_\mu\varphi(x))\,(D^\mu\varphi(x))^*$. The massive Dirac fermion is composed of ψ and the gaugino λ.
- Case $q\xi > 0$.
  Show that all particles except for φ stay massless. Compute the mass of φ.

4. *(Optional!) A Mass formula and the Goldstino*

Here we will collect the necessary pieces of the former exercises to deduce some general features of SUSY breaking mechanisms. Let us focus on the masses of the particles in a supermultiplet which have to equal by supersymmetry in the unbroken case. After SUSY breaking there is no reason for them to equal anymore. These masses depend on the SUSY breaking parameters $\langle F\rangle$ and $\langle D^a\rangle$ in a particular way that we analyze in the following. Therefore, we consider the Super Yang-Mills theory of Ex. 8.2 again.

(a) Assuming non-vanishing scalar VEVs, determine the mass matrix M_1 of the gauge fields V_μ^a originating from the kinetic term due to the usual Higgs effect. Define

$$D_i^a := \frac{\partial D^a}{\partial \varphi^i} = -g(\varphi^\dagger T^a)_i, \qquad D^{ia} := \frac{\partial D^a}{\partial \varphi_i^\dagger} = -g(T^a\varphi)^i, \qquad D_j^{ai} := -gT_j^{ai} \tag{6}$$

to rewrite the mass matrix. *Hint: Again we work in tree-level, thus $\langle\varphi^n\rangle = \langle\varphi\rangle^n$.*

(b) Read off the mass matrix of the fermions ψ^i and gauginos λ^a from the action (10) of Ex. 8.2. Rewrite this using the determining equation for the scalar VEVs $\langle\varphi^i\rangle$ written as

$$0 = \frac{\partial V}{\partial \varphi^i}(\langle\varphi^i\rangle, \langle\varphi_i^\dagger\rangle) = F^j \frac{\partial^2 W}{\partial\varphi^i\partial\varphi^j} - g^a D^a \varphi_j^\dagger (T^a)_i^j \tag{7}$$

as well as the gauge invariance of W in the form

$$0 = \delta^{(a)}_{\text{gauge}} W = \frac{\partial W}{\partial \varphi^i}\delta^{(a)}_{\text{gauge}}\varphi^i = -F_i^\dagger (T^a)^i_j z^j \tag{8}$$

to obtain

$$M_{\frac{1}{2}} = \begin{pmatrix} \langle F_{ij}\rangle & i\sqrt{2}\langle D_i^b\rangle \\ i\sqrt{2}\langle D_j^a\rangle & 0 \end{pmatrix}. \tag{9}$$

Here, we introduced the abbreviations

$$F^{ij} := \frac{\partial F^i}{\partial \varphi_j^\dagger} = \frac{\partial^2 \bar{W}}{\partial\varphi_j^\dagger\partial\varphi_i^\dagger}, \qquad F_{ij} := \frac{\partial F_i^\dagger}{\partial\varphi^j} = \frac{\partial^2 W}{\partial\varphi^j\partial\varphi^i}. \tag{10}$$

What is the interpretation of the massless fermion called the **Goldstino** associated to the zero eigenvalue in (9)? Calculate $M_{\frac{1}{2}}M_{\frac{1}{2}}^\dagger$.
Hint: There is a factor $\frac{1}{2}$ in front of $M_{\frac{1}{2}}$ in the action!

(c) Show that the mass matrix M_0 of the complex scalars is given by

$$M_0^2 = \begin{pmatrix} \left\langle \frac{\partial^2 V}{\partial\varphi^i\partial\varphi_k^\dagger}\right\rangle & \left\langle \frac{\partial^2 V}{\partial\varphi^i\partial\varphi^l}\right\rangle \\ \left\langle \frac{\partial^2 V}{\partial\varphi_j^\dagger\partial\varphi_k^\dagger}\right\rangle & \left\langle \frac{\partial^2 V}{\partial\varphi_j^\dagger\partial\varphi^l}\right\rangle. \end{pmatrix} \tag{11}$$

Calculate this explicitly using the usual V. *Hint: There is a $\frac{1}{2}$ in front of M_0^2!*

(d) Evaluate the supertrace

$$\text{STr}M^2 = 3\text{tr}M_1^2 - 2\text{tr}M_{\frac{1}{2}}M_{\frac{1}{2}}^\dagger + \text{tr}M_0^2 = -2g\langle D^a\rangle \text{tr}T^a. \tag{12}$$

Thus, the supertrace vanishes for F-term breaking and the magnitude of the non-vanishing D-term $\langle D^a\rangle$ is a parameter for the mass-splitting between bosons and fermions (for $\text{tr}T^a \neq 0$). Calculate $\text{STr}M^2$ for Ex. 9.3.