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Exercise 10
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SS 08

Exercises on Elementary Particle Physics II

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1. *The Minimal Supersymmetric Standard Model*

   The Minimal Supersymmetric Standard Model (MSSM) is the easiest extension of the standard model taking supersymmetry into account. The gauge sector consists of a Super Yang-Mills theory V with the standard model gauge group $G_{\mathrm{SM}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ that is coupled to a matter sector of chiral multiplets. These multiplets contain the SM-matter and the **two** Higgses transforming in the following representations of G_{SM}:

quarks	$\mathbb{U}_i = \left(\mathbf{3}, \mathbf{2}, \frac{1}{6}\right)$	$\bar{U}_i = \left(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}\right)$	$\bar{D}_i = \left(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}\right)$
leptons	$\mathbb{L}_i = \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right)$	$\bar{E}_i = (\mathbf{1}, \mathbf{1}, 1)$	
higgs	$H = \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right)$	$\bar{H} = \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}\right)$	

   The kinetic terms of the MSSM's chiral multiplets are just the canonical ones, $K = \sum_i \Phi_i^\dagger e^V \Phi_i$. The superpotential W has to contain terms allowing for the right Yukawa couplings and scalar potential of the Higgses. Furthermore, SUSY is broken by soft SUSY-breaking terms what completes the full Lagrangian of the MSSM.

   First, let us analyze the matter sector, namely the superpotential of the MSSM.

   (a) How do the component fields (of e.g. $\mathbb{L}_i$) transform under gauge transformations? Why is the term $\mathbb{L}H\bar{E}$ gauge invariant?

   (b) What are the general constraints on a superpotential W of a renormalizable theory? Write down the most general, gauge invariant cubic superpotential for the matter superfields. Add also a Higgs mass, sometimes called the μ**-term**, and determine the Yukawa couplings and scalar potential.
   Hint: Remember that the superpotential is ***holomorphic*** *in the chiral superfields by SUSY. Thus, no complex conjugates of fields are allowed. Denote the superpartners of the SM by e.g. $\tilde{H}$ for the Higgs or $\tilde{e_L}$ for the left-handed selectron, i.e. by the "tilded", but same letter.*

   (c) Why do we need a second Higgs $\bar{H}$ in the MSSM? There are various reasons!

(d) Identify the terms that conserve baryon and lepton number and those that do not. Introduce the discrete symmetry of **R-parity** given by $R_p = (-1)^{3B+L+2s}$ as another defining property of the MSSM. Show that it forbids exactly those terms that violate baryon or lepton number. Why are superpartners always produced in pairs? This is a symmetry that does not commute with SUSY!

Now, we turn to an explicit analysis of the Higgs sector in the MSSM and discuss electro-weak symmetry breaking.

(e) Using the R-parity preserving part of the superpotential, find the part of the scalar potential that contains mass terms for

$$\text{scalar}(\bar{H}) = \bar{h} = (\bar{h}^+, \bar{h}^0) \quad \text{and} \quad \text{scalar}(H) = h = (h^0, h^-). \tag{1}$$

(f) Add the D-term contribution from the gauge couplings to the scalar potential:

$$V_{\text{D-term}} = \frac{1}{2} g_1^2 \left(\sum_{\varphi} \varphi^* Y \varphi \right)^2 + \frac{1}{2} g_2^2 \sum_{a=1}^{3} \left[\sum_{(\varphi^1, \varphi^2)} \left(\varphi^{1*}, \varphi^{2*} \right) T^a \begin{pmatrix} \varphi^1 \\ \varphi^2 \end{pmatrix} \right]^2 \tag{2}$$

where $\varphi \in \{h^0, h^-, \bar{h}^+, \bar{h}^0\}$, $(\varphi^1, \varphi^2) \in \{h, \bar{h}\}$, Y is the hypercharge and $T^a = \frac{\sigma^a}{2}$ are the generators of SU(2). Deduce this form for the potential and expand the sums.

Considering the full scalar potential for the Higgs fields in unbroken SUSY, is a breaking of the electroweak symmetry possible? State the conditions on the parameters.

(g) Include the following **soft SUSY breaking** terms in the scalar potential

$$\mathcal{L}_{\text{soft}} = -m_{\text{soft},1}^2 |h|^2 - m_{\text{soft},2}^2 |\bar{h}|^2 - m_3^2 (\bar{h} h + c.c.)\,, \tag{3}$$

where $|h|^2 = h^\dagger h = |h^0|^2 + |h^-|^2$ and $\bar{h} h = \bar{h}^a h^b \varepsilon_{ab}$. The resulting potential's minimum should break the electroweak symmetry.[1]

Show that the scalar potential can be written as

$$V(h, \bar{h}) = m_1^2 |h|^2 + m_2^2 |\bar{h}|^2 + m_3^2 (\bar{h} h + c.c.) + \frac{g_1^2 + g_2^2}{8} \left(|h|^2 - |\bar{h}|^2 \right)^2. \tag{4}$$

How are m_1^2 and m_2^2 defined?

(h) One requirement for successful electroweak symmetry breaking is a negative $(\text{mass})^2$ term for at least one linear combination of the Higgs fields. Derive an inequality for m_3^2 to achieve this? The second requirement is that the potential should be bounded from below. When is this guaranteed?

[1] It is possible to set $\langle \bar{h}^+ \rangle = \langle h^- \rangle = 0$ through a SU(2) gauge transformation. $\langle \bar{h}^0 \rangle$ and $\langle h^0 \rangle$ can be made real and positive by a phase redefinition.

(i) Show that $|\mu|^2$, $m^2_{\text{soft},1}$, $m^2_{\text{soft},2}$ and m_3^2 can be related through m_Z^2 if we require agreement with experimental result for the Higgs vev:

$$v_{\text{SM}}^2 = \langle h^0\rangle^2 + \langle \bar{h}^0\rangle^2 = \frac{4m_Z^2}{g_1^2 + g_2^2} \approx (246GeV)^2 \tag{5}$$

Since only the sum of the squares of $\langle h^0\rangle$ and $\langle \bar{h}^0\rangle$ is fixed experimentally, the parameter β is introduced to parameterize the remaining freedom. One defines $\tan\beta = \bar{v}/v = \langle \bar{h}^0\rangle / \langle h^0\rangle$.

(j) Check that the relations you found satisfy the constraints in (e).

(k) After electroweak symmetry breaking, three of the eight real scalar degrees of freedom of the two Higgs multiplets are swallowed to give mass to the Z^0 and $W^\pm$ bosons. The remaining physical fields are usually named A^0 (a neutral CP-odd pseudoscalar), $H^\pm$ (two charged scalars that are conjugates to each other), H_0 and h_0 (a heavy and a light CP-even scalar filed).

Obtain the mass matrix for H_0 and h_0. Show that m_{h^0} has an upper bound.

Hint: H_0 and h_0 are a mixture of $Re(h^0) - \langle h^0\rangle$ and $Re(\bar{h}^0) - \langle \bar{h}^0\rangle$. You can use $m^2_{A^0} = 2m_3^2/\sin 2\beta$ to simplify the notation.