Physikalisches Institut
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Theoretische Physik

Exercise 10
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SS 09

Exercises on Theoretical Astroparticle Physics

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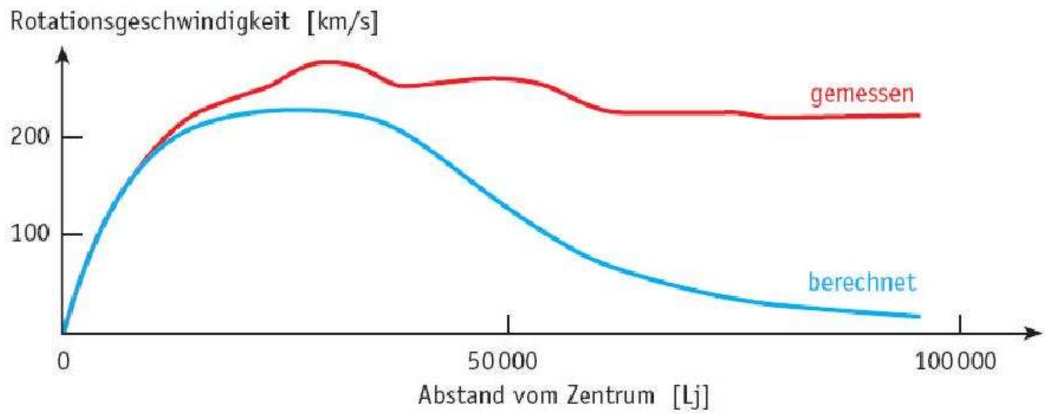

Figure 1: rotation curve of a galaxy

1. **Dark Matter in Galaxies**
   The observation of the galactical rotation curves (see fig. 1) yields a deficit of mass in the galaxy. Under the assumption of spherical symmetry of a rotating galaxy one can calculate the mass inside a sphere of a given radius from the circular velocity of the stars at its surface and compare it to an estimation from the visible stars.

   (a) Give a formula which expresses the circular velocity in terms of the enclosed mass and the distance to the galactic center. Verify the virial theorem for gravitationally bound systems $< T >= - < V > /2$.

   (b) Assume the simplest case of a constant mass density ρ_0 inside a radius r_0. How does the rotation curve look like?

   (c) A more realistic distribution is of the form

   $$\rho(r) = \frac{\rho_0 r_0^2}{r^2 \left(1 + r/r_0\right)^{\alpha}} .$$

   Derive the rotation curve $v(r)$. Which value of α gives a flat rotation curve at $r \gg r_0$ as shown in the measurements?

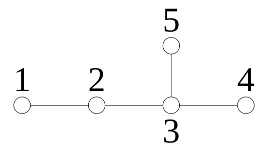

Figure 2: Dynkin diagramm of D5

(d) At $r = 100000ly$ the measurement yields $v_{calc} = 15km/s$ and $v_{meas} = 225km/s$. Calculate the visible as well as the true galaxy mass. What is the percentage of dark matter in the galaxy? How high is the average dark matter mass density? *Hint:* $G = 6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$

2. **More GUT breaking**

In fig. 2 you can see the Dynkin diagramm of $D_5 = \mathfrak{so}(10)$, which is also a desirable Lie group for unification. It can be broken to $\mathfrak{su}(5)$ by removing the simple root α_5.

(a) White down the Cartan matrix $A_{ij} = \frac{2<\alpha_i,\alpha_j>}{<\alpha_i\alpha_i>}$.

(b) Express the $U(1)$-generator which is orthogonal to $\mathfrak{su}(5)$ in terms of the Cartan algebra basis $\{H_i\}$ which satisfies $\alpha_i(H_j) = A_{ij}$.

(c) Construct the **16** by starting with the Dynkin label of the highest weight $\Lambda_i = \frac{2<\alpha_i,\mu>}{<\alpha_i\alpha_i>} = (0,0,0,1,0)$.

(d) Find the decomposition of the **16** of $\mathfrak{so}(10)$ as follows: $\mathbf{16} \rightarrow \mathbf{10} \oplus \bar{\mathbf{5}} \oplus \mathbf{1}$.

We immideately see the advantage of a $\mathfrak{so}(10)$-GUT. All matter of one generation fits into the spinor representation, and there is a singlett left which e.g. can act as a righthanded neutrino. Furthermore a $\mathfrak{so}(2N)$ gauge theory with chiral matter in any representation is always free of gauge anomalies.