Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercises 5
17 May 2010
SS 2010

Exercises on Theoretical Particle Physics II

Dr. S.Förste

Due 05/31/2010

As you might know by now, SUSY has many appealing features. For example, it leads to a unification of the gauge couplings and it alleviates fine-tuning by protecting the Higgs mass from quadratic corrections. Unfortunately, no experiment has detected any superpartner of the known particles so far. This means that SUSY must necessarily be broken at the energies accessible to current experiments. The breaking should, however, be such that the good features of SUSY are maintained.
In this exercise we look at *spontaneously* broken SUSY, which means that vacuuum does not respect the symmetry of the Lagrangian. In the first exercise we will investigate how SUSY can be broken and then apply one SUSY breaking scheme in the second exercise.

5.1 SUSY Breaking (*7 credits*)

As mentioned in the introductory text, SUSY is broken spontaneously iff (meaning if and only if) $Q_\alpha \neq 0$. This, in turn, is equivalent to the existence of an $|X\rangle$ such that $\langle X|Q_\alpha|0\rangle \neq 0$, or

$$\langle 0|\{Q_\alpha, \hat{X}\}|0\rangle = \langle \delta_{(\epsilon,\bar{\epsilon})}\hat{X}\rangle \neq 0, \tag{1}$$

where $\hat{X}$ is any operator in the theory and $\langle \delta_{(\epsilon,\bar{\epsilon})}\hat{X}\rangle$ denotes the VEV of the SUSY variation of the operator $\hat{X}$. We will consider the classical limit at tree level (without quantum corrections) in which $\langle \delta_{(\epsilon,\bar{\epsilon})}\hat{X}\rangle = \delta_{(\epsilon,\bar{\epsilon})}X$ for a classical field X.

(a) While SUSY should be broken, Poincaré invariance should be maintained. Which operators $\hat{X}$ can be allowed to develop a VEV $\langle \hat{X}\rangle$ without breaking the Poincaré invariance of the vacuum? (*1 credit*)

(b) Look at the SUSY variations of all fields in the chiral multiplet, which you calculated on exercise sheet 2 in 2.1)j). What are the consequences of the vanishing of $\delta_{(\epsilon,\bar{\epsilon})}\psi$ for SUSY breaking and for the potential $V = |F|^2$? When is SUSY broken? (*2 credits*)

(c) Now look at the vector multiplet with component fields V^μ, λ, and D. Their SUSY variations are

$$\delta_{(\epsilon,\bar{\epsilon})}V^\mu = -i\bar{\lambda}\bar{\sigma}^\mu\epsilon + i\bar{\epsilon}\bar{\sigma}^\mu\lambda, \quad \delta_{(\epsilon,\bar{\epsilon})}\lambda = \bar{\sigma}^{\mu\nu}\epsilon F_{\mu\nu} + i\epsilon D, \quad \delta_{(\epsilon,\bar{\epsilon})}D = -\epsilon\sigma^\mu D_\mu\bar{\lambda} - D_\mu\lambda\sigma^\mu\bar{\epsilon} \tag{2}$$

where D_μ denotes the covariant derivative. Perform the same analysis as in (b). Note that in this case the potential is $V = \frac{1}{2}D^2$.
(*2 credits*)

(d) Alternatively, we can look at the SUSY algebra itself. Express the Hamiltonian $\mathcal{H}$ in terms of Q_α and $\bar{Q}_{\dot{\alpha}}$ and infer an inequality for the energy E on the spectrum of any SUSY theory. When is the inequality an equality? The results from b) and c) are reproduced by using the potential $V = |F|^2 + \frac{1}{2}D^2$ for a chiral and a vector-multiplet. (*2 credits*)

As we have seen in this exercise, SUSY is broken iff the auxiliary fields develop a non-zero VEV. For this reason, we distinguish *F-term* and *D-term* SUSY breaking, depending on whether $\langle F\rangle = 0$ or $\langle D\rangle = 0$. As we have seen in the last part of the exercise, having $\langle V\rangle = 0$ for unbroken SUSY is a generic feature of the SUSY algebra. In the next exercise, we will investigate F-term breaking and the next exercise sheet will deal with D-term breaking.

5.2 F-term breaking in the O'Raifeartaigh model (*10 credits*)

In the O'Raifeartaigh model, there are three (left-)chrial superfields X,Y, and Z. Let us denote the component fields of X by (x, ψ_x, F_x) (and analogously for Y and Z). As we have seen on exercise sheet 3, the kinetic terms of the fields arise from the highest component (the D-term) of the Kähler potential. We take again the easiest choice for K, such that

$$\mathcal{L}_D = K(X, Y, Z)|_{\theta^2\bar{\theta}^2} = (X^\dagger X) + (Y^\dagger Y) + (Z^\dagger Z), \tag{3}$$

where the vertical bar means restriction to the highest component. Furthermore, this term contributions quadratic terms of the auxiliary fields like $|F_\bullet|^2$. The superpotential is given by

$$W(X, Y, Z) = \lambda X(Z^2 - M^2) + gYZ, \tag{4}$$

where λ, M, and g are real parameters.

(a) Calculate the scalar potential $V(x, y, z)$ by extending the results of exercise 3.3)c), i.e.

$$V(x, y, z) = |F_x|^2 + |F_y|^2 + |F_z|^2 \quad \text{and} \quad F_\varphi^* = -\frac{\partial W(x, y, z)}{\partial \varphi} \quad \text{for} \ \ \varphi = x, y, z. \tag{5}$$

(*1 credit*)

(b) Show that the VEVs of F_x, F_y, and F_z cannot vanish simultaneously. Hence the O'Raifeartaigh model implements F-term SUSY breaking. (*1 credit*)

(c) Check that the minimum of the potential $V(x, y, z)$ is at $y = z = 0$ when $M^2 < \frac{g^2}{2\lambda^2}$. (Checking that it is indeed a minimum is rather involved.)

(*2 credits*)

(d) Calculate the masses of the scalars. To do so, expand the fields in terms of fluctuations around their background value defined by their VEVs (e.g. $x \to \langle x \rangle + x$). Insert the expansion into the potential and extract the terms quadratic in the fields. In order to diagonalize the mass matrix for z, use the ansatz $z = \frac{1}{\sqrt{2}}(a + ib)$. (*3 credits*)

(e) Calculate the masses of the fermions. To do this, combine ψ_y and ψ_z into a Dirac fermion ψ_D:

$$\psi_D = \begin{pmatrix} \psi_y \\ \psi_z \end{pmatrix}.$$

As the VEV of x is undetermined, the term $x\psi_z\psi_z$ does not constitute a mass term.

(*3 credits*)