Exercises on Theoretical Particle Physics II

Dr. S.Förste

Due 06/14/2010

On this exercise sheet we will examine the easiest supersymmetric extension of the Standard Model of Particle Physics (SM), namely the **M**inimal **S**upersymmetric **S**tandard **M**odel (MSSM). The gauge sector is a four-dimensional super Yang-Mills theory with the SM gauge group $G_{SM} = SU(3) \times SU(2) \times U(1)$ coupled to a matter sector of chiral multiplets.
The "Minimal" attribute in the MSSM means that the gauge group of the MSSM is exactly that of the SM and the particle content is extended in a minimal way. This means that for each SM particle one superpartner is introduced. Additionally, the MSSM contains **two** Higgs doublets. The particles and their representation with respect to G_{SM} read:

$$\begin{array}{llll} \text{Quarks:} & Q_i = (\mathbf{3},\mathbf{2})_{1/6} & \bar{u}_i = (\bar{\mathbf{3}},\mathbf{1})_{-2/3} & \bar{d}_i = (\bar{\mathbf{3}},\mathbf{1})_{1/3} \\ \text{Leptons:} & L_i = (\mathbf{1},\mathbf{2})_{-1/2} & \bar{e}_i = (\mathbf{1},\mathbf{1})_{1} & \\ \text{Higgs:} & H = (\mathbf{1},\mathbf{2})_{-1/2} & \bar{H} = (\mathbf{1},\mathbf{2})_{1/2} & \end{array} \tag{1}$$

where conventionally the right-handed particles are given as their conjugate states, the index i labels the three families, and the hypercharge is given as a subscript.
The ingredients for the MSSM Lagrangian are as follows:

- The superpotential W is chosen such that the correct Yukawa couplings and the scalar Higgs potential arise,
- The Kähler potential generating the kinetic terms is the canonical one, $K = \sum_i \Phi_i^\dagger e^V \Phi_i$,
- SUSY is broken by soft SUSY-breaking terms.

These ingredients will be studied subsequently on this and on the next exercise sheet.

7.1 The superpotential for the MSSM matter sector (*11 credits*)

(a) Give the gauge transformations for the component fields (like e.g. L_i) in (1). (*1 credit*)

(b) Write down the most general, renormalizable, gauge-invariant superpotential W for the superfields. Show explicitly that the terms are gauge invariant.
Hint: W must be a holomporphic function of the superfields. Denote the superpartners of the fields using the same letter but with a tilde. (*4 credits*)

(c) Give (at least) two reasons why another Higgs doublet $\bar{H}$ has to be introduced in the MSSM. (*2 credits*)

(d) Identify those terms in the superpotential which preserve baryon and lepton number and those which do not. Give the relevant Feynman diagram which leads to proton decay in case the baryon and lepton number violating terms are allowed. (*2 credits*)

(e) Define R-parity as $R_p = (-1)^{3B+L+2s}$ with s being the spin and B and L the baryon and lepton number, respectively. Calculate the R-charge of all fields and verify that R-parity exactly forbids the dangerous terms. (*2 credits*)

7.2 The MSSM Higgs potential *(7 credits)*

(a) Using the R-parity preserving part of the superpotential, calculate the scalar potential and identify the mass terms for $\text{scalar}(H) = h = (h^0, h^-)$ and $\text{scalar}(\bar{H}) = \bar{h} = (\bar{h}^+, \bar{h}^0)$. Do not forget the μ term $\mu H\bar{H}$. *(2 credits)*

(b) Now we add the D-term contribution from the gauge couplings of the scalar potential:

$$V_D = \frac{1}{2}g_1^2 \left(\sum_{\varphi} \varphi^* Y \varphi \right)^2 + \frac{1}{2}g_2^2 \sum_{a=1}^{3} \left[\sum_{i=1}^{2} (\varphi^*, \varphi^*)_i \, T^a \begin{pmatrix} \varphi \\ \varphi \end{pmatrix}_i \right]^2 . \tag{2}$$

Here, g_1 and g_2 are the $U(1)$ and $SU(2)$ gauge couplings, Y and $T^a = \frac{\sigma^a}{2}$ denote the hypercharge and $SU(2)$ generators, $\varphi \in \{h^0, h^-, \bar{h}^+, \bar{h}^0\}$, and $(\varphi, \varphi)_1 = (h^0, h^-)$, $(\varphi, \varphi)_2 = (\bar{h}^+, \bar{h}^0)$. Deduce this form of the scalar potential and expand the sums. *(4 credits)*

(c) By considering the full scalar Higgs potential in unbroken SUSY, argue whether or not it is possible to break the electroweak symmetry. *(1 credit)*