Physikalisches Institut
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Theoretische Physik

Exercises 8
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Exercises on Theoretical Particle Physics II

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8.1 The MSSM Higgs Sector (*10 credits*)

Recall the study you performed in exercise 7.2, now we will analyse further properties on the MSSM Higgs sector.

(a) Include the following **soft SUSY breaking** terms in the scalar potential

$$\mathcal{L}_{\text{soft}} = -m^2_{\text{soft},1}|h|^2 - m^2_{\text{soft},2}|\bar{h}|^2 - m_3^2(\bar{h}h + c.c.)\,, \tag{1}$$

where $|h|^2 = h^\dagger h = |h^0|^2 + |h^-|^2$ and $\bar{h}h = \bar{h}^a h^b \varepsilon_{ab}$. The resulting potential's minimum should break the electroweak symmetry.[1]

Show that the scalar potential can be written as

$$V(h,\bar{h}) = m_1^2|h|^2 + m_2^2|\bar{h}|^2 + m_3^2(\bar{h}h + c.c.) + \frac{g_1^2 + g_2^2}{8}\left(|h|^2 - |\bar{h}|^2\right)^2 . \tag{2}$$

How are m_1^2 and m_2^2 defined? (*2 credits*)

(b) One requirement for successful electroweak symmetry breaking is a negative (mass)^2 term for at least one linear combination of the Higgs fields. Derive an inequality for m_3^2 to achieve this? The second requirement is that the potential should be bounded from below. When is this guaranteed? (*3 credits*)

(c) Show that $|\mu|^2$, $m^2_{\text{soft},1}$, $m^2_{\text{soft},2}$ and m_3^2 can be related through m_Z^2 if we require agreement with experimental result for the Higgs vev:

$$v^2_{\text{SM}} = \langle h^0\rangle^2 + \langle\bar{h}^0\rangle^2 = \frac{4m_Z^2}{g_1^2 + g_2^2} \approx (246GeV)^2 \tag{3}$$

Since only the sum of the squares of $\langle h^0\rangle$ and $\langle\bar{h}^0\rangle$ is fixed experimentally, the parameter β is introduced to parameterize the remaining freedom. One defines $\tan\beta = \bar{v}/v = \langle\bar{h}^0\rangle/\langle h^0\rangle$. (*3 credits*)

(d) Check that the relations you found satisfy the constraints in 8.1(b). (*2 credits*)

(e) After electroweak symmetry breaking, three of the eight real scalar degrees of freedom of the two Higgs multiplets are swallowed to give mass to the Z^0 and $W^\pm$ bosons. The remaining physical fields are usually named A^0 (a neutral CP-odd pseudoscalar), $H^\pm$ (two charged scalars that are conjugates to each other), H_0 and h_0 (a heavy and a light CP-even scalar filed).

Obtain the mass matrix for H_0 and h_0. Show that m_{h^0} has an upper bound. (*4 credits*)

Hint: H_0 and h_0 are a mixture of $Re(h^0) - \langle h^0\rangle$ and $Re(\bar{h}^0) - \langle\bar{h}^0\rangle$. You can use $m^2_{A^0} = 2m_3^2/\sin 2\beta$ to simplify the notation.

[1] It is possible to set $\langle\bar{h}^+\rangle = \langle h^-\rangle = 0$ through a SU(2) gauge transformation. $\langle\bar{h}^0\rangle$ and $\langle h^0\rangle$ can be made real and positive by a phase redefinition.

8.2 Evolution of the gauge couplings *(11 credits)*

The running of the coupling constants of a general non-abelian gauge theory, comming from the renormalization group equations is given by

$$\frac{8\pi^2}{g^2(q^2)} = \frac{8\pi^2}{g^2(\Lambda^2)} + b_0 \log(q^2/\Lambda^2), \tag{4}$$

where Λ is the cutoff scale of the theory and q is a typical momentum transfer in a process. The coefficient b_0 depends on the parameters: cuadratic casimir c_i in a given representation ($\mathrm{tr}(T^a T^b) = c_i \delta^{ab}$), number of fermions in the ith representation $n_f^{(i)}$ and number of bosons in the ith representation $n_b^{(i)}$. b_0 is given by

$$b_o = 11/3 c_A - 2/3 c_i n_f^{(i)} - 1/3 c_i n_b^{(i)} \tag{5}$$

For the group $SU(N)$, the quadratic casimir in the adjoint $f^{abc} f^{bcd} = c_A \delta^{ab}$ is $c_A = N$. For the fundamental representation $c_{fund.} = 1/2$. In the case of $U(1)$ field, the same formula applies with $c_A = 0$ and a state of charge q_i has $c_i \sim q_i^2$.

(a) Consider the case of a non-supersymmetric $SU(N)$ theory with N_f fermions in the fundamental, and N_f fermions in the antifundamental,which is the value of b_0? Now consider the addition of a supersymmetry to the previous theory, which is the value of b_0 in this case? *(2 credits)*

(b) Compute the value of b_0 for the three different gauge groups of the Standard Model, with a number of generations N_G . Take for the c_i of the $U(1)_Y$ the normalization $c_i = 3/5 q_i^2$, why we follow this convention? *(3 credits)*

(c) Consider the MSSM with N_G number of generations, obtain the new values of the b_0 coefficients for the three gauge factors. Specify to $N_G = 3$, compare the asymptotic behaviour of the $SU(2)$ factors of MSSM and the SM. *(4 credits)*

(d) Take into account the gauge coupling unification for $SU(5)$, working for the SM using the measured values for the couplings $SU(2)$ g_2 and $U(1)$ g_1 compute the value of the unification scale M_Z. Determine the value of the coupling g_3 at this scale. *(1 credit)*

(e) Now work for the MSSM, determine using initial values for g_2 and g_1, the scale at which the gauge couplings meet. *(1 credit)*