Physikalisches Institut
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Theoretische Physik

Exercises 6
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Exercises on Theoretical Particle Physics II

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6.1 The MSSM Higgs sector (*20 credits*)

(a) Using the R-parity preserving part of the MSSM Superpotential,

$$\mathcal{W} = \mu H\bar{H} + y_{\mathrm{E}} LH\bar{E} + y_{\mathrm{D}} QH\bar{D} + y_{\mathrm{U}} Q\bar{H}\bar{U}\,,$$

obtain the contributions to the Higgs scalar potential in terms of the Higgs scalars $H|_{\theta=0} = h = (h^0, h^-)$ and $\bar{H}|_{\theta=0} = \bar{h} = (\bar{h}^+, \bar{h}^0)$. (*2 credits*)

(b) Consider also the contributions which come from the D-terms of the electroweak gauge multiplets. The Kähler potential reads

$$\mathcal{K} \supset \bar{H}^\dagger e^V \bar{H} + H^\dagger e^V H$$

with $V = g_1 Y V_1 + g_2 T^a V_2^a$, Y being the Hypercharge, T^a the $SU(2)_L$ generators. (*4 credits*)

(c) Show that at this stage electroweak symmetry breaking is not possible. (*1 credit*)

(d) Therefore we include a soft SUSY breaking sector

$$\mathcal{L}_{\mathrm{soft}} = -m^2_{\mathrm{soft},1}|h|^2 - m^2_{\mathrm{soft},2}|\bar{h}|^2 - m^2_{\mathrm{soft},3}\left(\bar{h}h + \mathrm{h.c.}\right)\,,$$

where $|h|^2 = h^\dagger h$ and $\bar{h}h = \epsilon^{ab}\bar{h}^a h^b$. We can use an SU(2) rotation to set $\langle h^-\rangle = 0$. Show that being in a minimum then implies that $\langle \bar{h}^+\rangle = 0$ so that electromagnetism is restored. Show that using further phase rotations we can make $m^2_{\mathrm{soft},3}$, $\langle h^0\rangle$ and $\langle \bar{h}^0\rangle$ real. At the end the potential should read

$$V(h,\bar{h}) = m_1^2|h|^2 + m_2^2|\bar{h}|^2 + m_3^2\left(\bar{h}h + \mathrm{h.c.}\right) + \frac{g_1^2+g_2^2}{8}\left(|h|^2 - |\bar{h}|^2\right)\,.$$

Identify the mass parameters m_1^2, m_2^2 and m_3^2. (*3 credits*)

(e) To obtain electroweak symmetry breaking we require the potential to be bounded from below and that the point $h^0 = \bar{h}^0 = 0$ is not a minimum. Show that this leads to the requirements

$$2m_3^2 < m_1^2 + m_2^2\,, \tag{1a}$$
$$m_3^4 > m_1^2 m_2^2\,. \tag{1b}$$

(*3 credits*)

(f) Show that $|\mu|^2$, $m^2_{\text{soft},1}$, $m^2_{\text{soft},2}$ and m_3^2 can be related through m_Z^2 if we require agreement with experimental result for the Higgs vev:

$$v_{\text{SM}}^2 = \langle h^0\rangle^2 + \langle \bar{h}^0\rangle^2 = \frac{4m_Z^2}{g_1^2+g_2^2} \approx (246GeV)^2$$

Check that these relations indeed fulfill the requirements (1). Since only the sum of the squares of $\langle h^0\rangle$ and $\langle \bar{h}^0\rangle$ is fixed experimentally, the parameter β is introduced to parameterize the remaining freedom. One defines $\tan\beta = \bar{v}/v = \langle \bar{h}^0\rangle/\langle h^0\rangle$. *(4 credits)*

(g) After electroweak symmetry breaking, three of the eight real scalar degrees of freedom of the two Higgs multiplets are swallowed to give mass to the Z^0 and $W^\pm$ bosons. The remaining physical fields are usually named A^0 (a neutral CP-odd pseudoscalar), $H^\pm$ (two charged scalars that are conjugates to each other), H_0 and h_0 (a heavy and a light CP-even scalar filed).

Obtain the mass matrix for H_0 and h_0. Show that m_{h^0} has an upper bound.

Hint: H_0 and h_0 are a mixture of Re$(h^0) - \langle h^0\rangle$ and Re$(\bar{h}^0) - \langle \bar{h}^0\rangle$. You can use $m^2_{A^0} = 2m_3^2/\sin 2\beta$ to simplify the notation. *(4 credits)*

(h) Making the universal assumption $m_1^2 = m_2^2$ does not fulfill (1). However, when we impose it on a high unification scale, we have to take into account the renormalization group equations. For the Higgs mass parameters they are given by

$$\frac{\mathrm{d}\log m_i^2}{\mathrm{d}\log\Lambda} = \frac{1}{16\pi^2}\left[\sum_a |y_a|^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2\right] ,$$

where $i = 1, 2$, Λ is the renormalization scale and the sum runs over all diagonal Yukawa matrix entries y_a to which the respective Higgs couples. For simplification we neglected the off-diagonal Yukawa matrix entries. Write down the precise expressions for the two Higgses. Show that making the universal assumption at a high scale, the radiative corrections can still lead to electroweak symmetry breaking. *(3 credits)*