# Exercises on Theoretical Particle Physics II

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## 7.1 Supergravity as gauged Supersymmetry (*14 credits*)

In this exercise we want to gauge supersymmetry to obtain supergravity and its coupling to matter. For this we first consider a simple massless chiral superfield in the on-shell formulation, consisting of a complex scalar $\phi$ and a Weyl spinor $\psi$. The Lagrangean reads

$$\mathcal{L}_0 = \partial_\mu \phi^* \partial^\mu \phi + \mathrm{i}\bar{\psi}\bar{\sigma}^\mu \partial_\mu \psi \,.$$

It is invariant up to a total derivative under global SUSY transformations

$$\delta\phi = \sqrt{2}\epsilon_W \psi \,, \qquad \delta\psi = -\mathrm{i}\sqrt{2}\left(\partial_\mu \phi\right)\sigma^\mu \bar{\epsilon}_W \,,$$

where $\epsilon_W$ denotes a Weyl transformation parameter. By iterating the Noether procedure we want to obtain a locally supersymmetric Lagrangean up to a certain order.

(a) We rewrite the fields as $\phi = \frac{1}{\sqrt{2}}\left(A + \mathrm{i}B\right)$ and $\Psi = \binom{\psi}{\bar{\psi}}$ where $A, B$ are real scalar fields and $\Psi$ is a Majorana spinor. Show that the action becomes

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu A \partial^\mu A + \frac{1}{2}\partial_\mu B \partial^\mu B + \frac{\mathrm{i}}{2}\bar{\Psi}\gamma^\mu \partial_\mu \Psi \,,$$

and the SUSY transformations become

$$\delta A = \bar{\epsilon}\Psi \,, \qquad \delta B = \mathrm{i}\bar{\epsilon}\gamma_5 \Psi \,, \qquad \delta\Psi = -\mathrm{i}\gamma^\mu \partial_\mu \left(A + \mathrm{i}\gamma_5 B\right)\epsilon \,,$$

where $\epsilon = \binom{\epsilon_W}{\bar{\epsilon}_W}$. (*3 credits*)

(b) Now we replace $\epsilon$ by $\epsilon(x)$. Show that this leads to a variation of $\mathcal{L}_0$ of the form

$$\delta\mathcal{L}_0 = \left(\partial_\mu \bar{\epsilon}\right) j^\mu \,, \qquad \text{with } j^\mu = \gamma^\nu \left(\partial_\nu \left(A - \mathrm{i}\gamma_5 B\right)\right)\gamma^\mu \Psi \,.$$

(*3 credits*)

(c) We want to cancel this variation by adding a Rarita–Schwinger field $\Psi_\mu$ transforming as $\delta\Psi_\mu = 2\kappa^{-1}\partial_\mu \epsilon$. How does $\mathcal{L}_1$ have to look in order to cancel $\delta\mathcal{L}_0$ with the variation of $\Psi_\mu$? Find for the total variation of the new action

$$\delta(\mathcal{L}_0 + \mathcal{L}_1) = \mathrm{i}\kappa\bar{\Psi}^\mu \gamma^\nu \epsilon T_{\mu\nu} + \frac{\mathrm{i}\kappa}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu \gamma_\mu \partial_\rho \epsilon A \overleftrightarrow{\partial_\sigma} B + \frac{\mathrm{i}\kappa}{2}\epsilon^{\mu\nu\rho\sigma}\partial_\rho \bar{\Psi}_\mu \gamma_\mu \epsilon A \overleftrightarrow{\partial_\sigma} B \,,$$

with $A\overleftrightarrow{\partial_\sigma}B := A\partial_\sigma B - B\partial_\sigma A$. Identify the energy momentum tensor $T_{\mu\nu}$. (*4 credits*)

(d) To cancel the first term one needs[1] to add a spin two field transforming as $\delta h_{\mu\nu} = -\frac{\mathrm{i}}{2}\bar{\epsilon}\left(\gamma_\mu \Psi_\nu + \gamma_\nu \Psi_\mu\right)$. Show that the required Lagrangean is precisely the linearized coupling of gravity to matter. *Hint: Replace* $\eta_{\mu\nu} \to g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $\gamma^\mu \partial_\mu \to \gamma^\nu \left(\delta^\mu_\nu - \frac{1}{2} h_{\nu\rho}\eta^{\rho\mu}\right)\partial_\mu$ *and* $d^4x \to \sqrt{-g}\,d^4x$ *in the action with* $g = \det g_{\mu\nu}$ *and expand* $\mathcal{L}_0$ *to first order in* $h_{\mu\nu}$. $\det(\mathbb{1} + H) = 1 + \operatorname{tr} H + \mathcal{O}(H^2)$ . (*4 credits*)

### 7.2 Decoupling Gravity in the Scalar Potential (*6 credits*)

We want to find the globally supersymmetric Lagrangean

$$\mathcal{L}_{\text{glob}} = \int \mathrm{d}^4\theta K(\Phi, \Phi^\dagger) + \int \mathrm{d}^2\theta W(\Phi) + \text{h.c.}\,.$$

We want to obtain the resulting scalar potenital in the gravity decoupling limit of the locally supersymmetric scalar Lagrangean

$$V_{\text{loc}} = -e^{-G}\left(3 + G^{i\bar{j}} G_i G_{\bar{j}}\right)\,, \tag{1}$$

where

$$G = -K - \log|W|^2\,, \tag{2}$$

and $G_i = G_{,i}$, $G^{i\bar{j}} = (G^{-1})^{i\bar{j}}$.

(a) In the expression for $G$ we have set $M = 1$ where $M$ is the gravity scale. What are the mass dimensions for $K, W$? What is the mass dimension of $G$? Rewrite (2) and (1) by multiplying the terms with appropriate powers of $M$. (*2 credits*)

(b) Write $V_{\text{loc}}$ in terms of $K$ and $W$. Assume a canonical Kähler potential of the form $K = \Phi^\dagger \Phi$. (*2 credits*)

(c) Show that in the gravity decoupling limit $M \to \infty$ one obtains the globally supersymmetric scalar potential $V_{\text{glob}}$. (*2 credits*)

[1] This can be shown but is rather tedious.