Physikalisches Institut
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Theoretische Physik

Exercises 10
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Exercises on Theoretical Particle Physics II

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10.1 Tensor scalar duality and the Stückelberg mass (*8 credits*)

We first begin with a four dimensional theory of a massless two-form tensor field B_2. The action is given by

$$S = \int H_3 \wedge *H_3 \sim \int \mathrm{d}^4x \; H_{\mu\nu\rho} H^{\mu\nu\rho} \, ,$$

where $H_3 = \mathrm{d}B_2$.

(a) What is the gauge symmetry which leaves the action invariant? How many degrees of freedom does B_2 have? (*2 credits*)

(b) We can reparametrize the theory by regarding H_3 as fundamental field. Then we have to enforce $\mathrm{d}H_3 = 0$ using a Lagrange multiplier ϕ. Show that integrating out H_3 leads to an action for the massless scalar ϕ. What is the symmetry of ϕ? (*3 credits*)

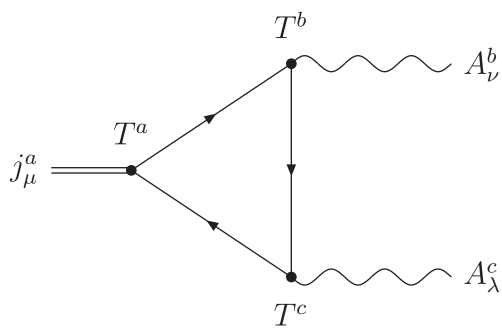
(c) We go back to the tensor theory and add a Chern–Simons coupling to a U(1) gauge theory, i.e.

$$S = \int H_3 \wedge *H_3 + cB_2 \wedge F_2 + F_2 \wedge *F_2 \qquad (1)$$

with $F_2 = \mathrm{d}A_1$. Repeat the above procedure to eliminate H_3. Show that in order to make S gauge invariant, ϕ has to transform as an axion. Show that you can gauge away ϕ to obtain a massive vector boson theory. (*3 credits*)

10.2 Anomaly computations (*8 credits*)

Consider a four dimensional theory with (Abelian or non-Abelian) gauge fields A^a_μ and a bunch of left-chiral Weyl fermions Φ_r with gauge charges q^a_r. In four dimensions the Feynman graph responsible for the anomaly is:



where $j^a_\mu = \frac{\delta S}{\delta A^{a,\mu}}$ is the current coupling to the gauge field A^a_μ and a, b, c label various gauge symmetries which can occur in the theory. The T^a stand either for the Abelian charges q^a or the non-Abelian generators in the respective representation. The particles running in the loop are all chiral fermions in the theory. This graph leads to a anomalous variation of the path integral measure which leads to an effective change of the action like

$$\delta S_{\text{anom}} \propto \int \mathrm{d}^4 x \lambda^a F^b \wedge F^c \,,$$

where λ^a is the gauge parameter.

(a) We first discuss the case where all gauge symmetries are abelian. Show that including the charges q^a_r of the fermions in the graph, the cancellation of the anomaly leads to the condition

$$\sum_r q^a_r q^b_r q^c_r = 0\,. \tag{2}$$

(*2 credits*)

(b) Take the familiar Standard Model of particle physics and $a = b = c =$ Hypercharge. Show that (2) is indeed fulfilled. (*1 credit*)

(c) Next, consider one U(1) and one non-Abelian symmetry (e.g. SU(N)) with the particles transforming only in the trivial or in the fundamental and anti-fundamental representation. Show that including group theory factors in the Feynman graphs leads to the constraint

$$\sum_r l(r) q^a_r = 0\,. \tag{3}$$

Here $l(r)$ is the quadratic Casimir in the respective representation, i.e.

$$l(r) = \mathrm{tr}_r\, T^a T^a$$

Why is there no constraint containing two Abelian charges? (*2 credits*)

(d) Check again that in the Standard Model the $U(1)_\mathrm{Y} - SU(2)_\mathrm{L} - SU(2)_\mathrm{L}$ anomaly vanishes. (*1 credit*)

(e) Finally we replace two gauge fields A^b_μ by universal graviton couplings. Show that this leads to the constraint

$$\sum_r q^a_r = 0\,. \tag{4}$$

Check that also this is fulfilled in the Standard Model. (*2 credits*)

10.3 Massless self-dual Tensor (*2 credits*)

Consider a $D = 2N$ dimensional theory with a $N-1$ form field C_{N-1} with field strength $F_N = \mathrm{d}C_{N-1}$ and action

$$\int_{M^D} F_N \wedge *F_N\,.$$

Show that an additional self-dualify constraint $F_N = *F_N$ neccessarily implies that C_{N-1} is massless by looking at the equations of motion.