Physikalisches Institut
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Theoretische Physik

Mock Exam
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SS 13

Theoretical Particle Astrophysics

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–Mock Exam–

1. Overview *20 points*

(a) Give three reasons why we think that the Standard Model of particle physics is incomplete. (*3 points*)

(b) Which cosmological parameters can be found from measuring the primordial abundances for light elements? (*1 point*)

(c) What are the properties of an axion-like particle? (*1 point*)

(d) Why do we think that there is dark matter in the universe? What are the properties of any dark matter candidate? (*2 points*)

(e) State the hierarchy problem. (*1 point*)

(f) The CMB spectrum gives a picture of a particular moment in the history of the universe. What did happen at that particular moment? (*1 point*)

(g) What is a domain wall? (*1 point*)

(h) Name three dark matter candidates. (*1 point*)

(i) What is the particle content of the standard model, and what is the corresponding gauge symmetry? (*2 points*)

(j) How does the particle number density scale with the temperature (i) in the non-relativistic case ($T \ll m$) and (ii) in the ultra-relativistic limit ($T \gg m$). (*2 points*)

(k) What is the physical reason for the freeze-out of a given particle species in the early universe? (*1 point*)

(l) What are the assumptions behind the FRW metric? (*1 point*)

(m) Give a motivation for considering Grand Unified Theories (GUTs). What are the usual drawbacks of these models? (*2 points*)

(n) Argue why a baryon asymmetry is needed in the early universe? (*1 point*)

2. Toy Model for Baryogenesis

20 points

Consider a real scalar boson X and four fermions f_i with the following interaction terms:

$$\mathcal{L}_{\Delta b} = X\left(a\overline{f_1}f_2 + b\overline{f_3}f_4 + c\overline{f_1}f_4 + d\overline{f_3}f_2\right) + h.c.\,, \tag{1}$$

where we assume that the fermion bilinears have a net baryon number, and a, b, c, d are dimensionless complex couplings.

(a) The couplings a, b, c, d are not necessarily real. Consider phase redefinitions of the form

$$f_j' = \mathrm{e}^{i\alpha_j} f_j\,,$$

where $j \in \{1, 2, 3, 4\}$ is *not* summed. This redefinitions can also be used to make some of the couplings in the Lagrangian real. The physically meaningful phases are those you can not transform away. (*5 points*)

(b) Show that at tree–level, the partial widths of $X \to f_1\overline{f_2}$ and $X \to f_2\overline{f_1}$ are the same, even if f_1 and f_2 have non–vanishing masses. *Hint: You do not need to compute the kinematical factors.* (*3 points*)

(c) Assume that $m_X > m_{f_3} + m_{f_4}$. Draw a one–loop diagram for $X \to f_1\overline{f_2}$ involving the couplings b, c and d (or their complex conjugates). (*4 points*)

(d) Show that the interference between this diagram and the tree–level diagram has a contribution that is sensitive to the phase(s) you identified in (b). (*4 points*)

(e) Compare this result to what you expect for the loop corrected process $X \to f_2\overline{f_1}$ and argue why C and CP are violated. (*4 points*)

3. Scalar Field Dynamics

15 points

The action of a real scalar field ϕ with potential $V(\phi)$ is given by

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

with metric $g_{\mu\nu}$ and $g = \det(g_{\mu\nu})$.

(a) By varying the action with respect to ϕ, derive the Klein-Gordon equation,

$$\Box\phi = V'\,.$$

(*3 points*)

(b) First show that the variation of the metric determinant yields

$$\delta g = -g g_{\mu\nu}\delta g^{\mu\nu}$$

(*4 points*)

(c) By varying the action with respect to $g^{\mu\nu}$, show that the stress-energy tensor associated with the scalar field is

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial^\lambda\phi\partial_\lambda\phi - V(\phi)\right) .$$

(*3 points*)

(d) Assuming the flat FRW metric for $g_{\mu\nu}(t)$ and restricting to the case of a homogeneous field, $\phi = \bar{\phi}(t)$, show that the stress-energy tensor takes the form of a perfect fluid. Show that the equation of state of the fluid is

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\bar{\phi}} - V(\bar{\phi})}{\frac{1}{2}\dot{\bar{\phi}} + V(\bar{\phi})} .$$

(*3 points*)

(e) What is the scaling of the energy density, $\rho_\phi(a)$ in the limits $\frac{1}{2}\dot{\bar{\phi}} \ll V$ and $\frac{1}{2}\dot{\bar{\phi}} \gg V$? Give a physical interpretation of each case. (*2 points*)

4. SU(5) GUTs and Proton Decay *18 points*

Consider an SU(5) Grand Unified Theory (GUT). The smallest irreducible representations are the (anti-) fundamental **5** ($\overline{\mathbf{5}}$), the two index antisymmetric **10** and the adjoint **24**. By considering the decompositions of certain products of these, we see that any of the following products

$$(\mathbf{24})^3\,,\ (\mathbf{24})^2\,,\ (\mathbf{24}\cdot\mathbf{5}\cdot\overline{\mathbf{5}})\,,\ (\mathbf{24}\cdot\mathbf{10}\cdot\overline{\mathbf{10}})\,,\ (\mathbf{10}\cdot\overline{\mathbf{5}}\cdot\overline{\mathbf{5}})\,,\ (\mathbf{10}\cdot\mathbf{10}\cdot\mathbf{5})\,, \tag{2}$$

can be made gauge invariant by a suitable contraction of the gauge indices for the representations involved in it. In terms of $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$, the adjoint decomposes as

$$\mathbf{24} \to (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{3},\mathbf{2})_{-\frac{5}{6}} \oplus (\overline{\mathbf{3}},\mathbf{2})_{\frac{5}{6}} . \tag{3}$$

(a) Give the branchings for the **10** and the $\overline{\mathbf{5}}$ in terms of G_{SM} representations and explain how one can get one family of particles out of these two SU(5) representations. (*3 points*)

(b) The Higgs field originates from a **5**-plet, so that in addition to the Higgs doublet H one also gets a color tripet T. From the Yukawas at the SU(5) level

$$\mathbf{10}_M\mathbf{10}_M\mathbf{5}_H \quad \mathbf{10}_M\overline{\mathbf{5}}_M\overline{\mathbf{5}}_H\,,$$

identify all couplings involving T. Sketch how T can mediate the decay of the proton. The subindices H and M are used to distinguish between Higgs and matter fields. (*6 points*)

(c) Assume that SU(5) is broken to G_{SM} by a VEV of a scalar field transforming in the adjoint representation. Use the decomposition given in (3) to identify the vector bosons which get massive after symmetry breaking. *(Hint: Consider the kinetic term for the adjoint scalar)* *(3 points)*

(d) Consider the kinetic terms for the matter fields (remember that such terms contain covariant derivatives). Find all couplings between matter fields and the massive vector bosons. Do these couplings respect baryon number? *(6 points)*

5. The Flatness Problem *12 points*

Consider an FRW universe dominated by a perfect fluid with pressure $p = w\rho$ with constant w. Remember the definition of the energy density parameter

$$\Omega(t) = \rho(t) \left(\frac{3H^2}{8\pi G} \right)^{-1} .$$

(a) Show that

$$\frac{d\Omega}{d \ln a} = (1 + 3w)\Omega(\Omega - 1) . \tag{4}$$

(6 points)

(b) How does $\Omega(a)$ evolve for $\Omega = 1 + \epsilon$, 1, $1 - \epsilon$ and $w = 0, -1$. Illustrate your findings. For any of these cases, explain how big was ϵ at earlier times compared to its value today. *(6 points)*

6. Stable relics out of Equilibrium *15 points*

Assume there is a stable particle χ of mass m_χ, which never was in equilibrium with the thermal bath due to its very small production cross section. The scaled abundance for χ is governed by the Botzmann equation

$$\frac{dY_\chi}{dx} = -\frac{1.32 M_{\rm Pl}\sqrt{g_*} m_\chi}{x^2} \langle \sigma v \rangle \left[Y_\chi^2 - \left(Y_\chi^{\rm eq}\right)^2 \right] , \tag{5}$$

were $x = m_\chi/T$, $Y_\chi = n_\chi/s$ with s being the entropy density, and g_* is the effective number of relativistic degrees of freedom.

1. Rewrite this equation using the explicit expressions for s and for the equilibrium density $Y_\chi^{\rm eq}$. *Hint:* Assume $T \ll m_\chi$, i.e. non–relativistic χ particles. *(3 points)*

2. Assume that at some initial temperature $T_i \ll m_\chi$ we have $n_\chi(T_i) = 0$. At least initially the annihilation term on the right–hand side of eq. (5) can then be neglected. Assume further that $\langle \sigma v \rangle = a$ is a constant. Show that the Boltzmann eq. can then be written as

$$\frac{dY_\chi}{dx} = \kappa x \mathrm{e}^{-2x} , \tag{6}$$

where κ is a (positive) constant. *(4 points)*

3. Solve eq. (6) explicitly. *(4 points)*

4. Plot both Y_χ and $Y_\chi^{\rm eq}$ and discuss the range of validity for the solution you just obtained. Which kind of particles could feature a relic abundance that fits this model? *(4 points)*