Exercises on Theoretical Particle Physics II

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16. Gauge coupling unification *(14 credits)*

(a) The running of gauge couplings g_i at one loop is given by

$$8\pi^2 \frac{dg_i^2}{dt} = b_i g_i^4, \qquad t = \log \frac{\mu}{\mu_0}, \qquad i \in \{1,2,3\}$$

with

$$b_i = -\frac{11}{3}C + \frac{2}{3}\sum_i T_i(R) + \frac{1}{3}\sum_j T_j(R)$$

where C is the quadratic Casimir operator of the adjoint representation and the sum over i is over fermions and the sum over j over scalars. We further have for the standard model gauge groups

$$\begin{aligned} \mathrm{SU}(N): &\qquad C = N, \qquad T_i(R) = \frac{1}{2} \qquad \text{for each particle} \\ \mathrm{U}(1): &\qquad C = 0, \qquad T_i(R) = \frac{Y^2}{4} \qquad \text{with } Y = \text{Hypercharge.} \end{aligned}$$

Show that in the standard model

$$b_1 = \frac{41}{6}.$$

(2 credits)

(b) Repeat the analysis from part (a) for g_2 and g_3 and determine

$$b_2 = -\frac{19}{6}, \qquad b_3 = -7.$$

(2 credits)

(c) Show that in a supersymmetric theory

$$b_i = -3C + \sum_i T_i(R)$$

where the sum is over left chiral superfields.

(2 credits)

(d) Use the result from part (c) and show that in the MSSM

$$g_1 = 11, \qquad g_2 = 1, \qquad b_3 = -3.$$

(2 credits)

(e) Solve the differential equation given in part (a) and show that the result can be written as

$$g_i^2(\mu) = g_i^2(\mu_0)\left(1 - \frac{1}{8\pi^2} g_i^2(\mu_0) b_i \log\frac{\mu}{\mu_0}\right)^{-1}.$$

(2 credits)

(f) Use the result from part (e) together with the coupling constants at the weak scale and plot the evolution of the coupling constants with a computer program like `Mathematica` or `Gnuplot`. Make two plots, one for the standard model and one for the MSSM. You will observe the famous gauge coupling unification in the MSSM around $\mu = 2 \cdot 10^{16}$ GeV. Use the values

$$g_1^2(M_Z) \approx 0.1277, \qquad g_2^2(M_Z) \approx 0.424, \qquad g_3^2(M_Z) \approx 1.495$$

and rescale g_1^2 by a factor of $\frac{5}{3}$. This factor is necessary beacuse otherwise g_1 is not appropriatly normalized to be part of a higher gauge group like SU(5).

(4 credits)

17. Decoupling gravity in the scalar potential *(6 credits)*

(a) We want to obtain the resulting scalar potential in the gravity decoupling limit $M_{\text{Pl}} \to \infty$ of the locally supersymmetric scalar potential

$$V_{\text{SUGRA}} = -e^{-G}\left(3 + G^{i\bar{j}} G_i G_{\bar{j}}\right),$$

where

$$G = -K - \log|W|^2, \qquad G_i = \frac{\partial G}{\partial \phi_i}, \qquad G^{i\bar{j}} = (G^{-1})^{i\bar{j}}.$$

In the expression for G we have set $M_{\text{Pl}} = 1$ where M_{Pl} is the Planck scale. What are the mass dimensions for K and W? What is the mass dimension of G? Rewrite G and V_{SUGRA} by multiplying the terms with appropriate powers of M_{Pl}.

(2 credits)

(b) Write V_{SUGRA} in terms of K and W. Assume a canonical Kähler potential of the form $K = \Phi^\dagger \Phi$.

(2 credits)

(c) Show that in the gravity decoupling limit $M_{\text{Pl}} \to \infty$ one obtains the globally supersymmetric scalar F-term potential

$$V_{\text{SUSY}} = F^* F.$$

(2 credits)