Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercises 11
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SS 2014

Exercises on Theoretical Particle Physics II

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Due 30.6.2014

20. Kaluza-Klein theories in 5 dimensions

(*10 credits*)

(a) For a compactification of gravity from 5 to 4 dimensions we make the general ansatz

$$G_{MN} = \phi^{\beta} \begin{pmatrix} g_{\mu\nu} + \phi A_\mu A_\nu & \phi A_\mu \\ \phi A_\nu & \phi \end{pmatrix}, \qquad G^{MN} = \phi^{-\beta} \begin{pmatrix} g^{\mu\nu} & -A^\mu \\ -A^\nu & \phi^{-1} + A_\mu A^\mu \end{pmatrix}.$$

The 5 dimensional Einstein Hilbert action is

$$S_5 = -\frac{1}{2\kappa_5^2} \int d^5x \sqrt{-G} R_5, \qquad G = \det G_{MN}.$$

Show that

$$G^{-1} = \phi^{-5\beta} \left(g^{-1}(\phi^{-1} + A_\mu A^\mu) - \sum_{\mu,\nu} A^\mu A^\nu g^{-1} g_{\mu\nu} \right), \qquad g = \det g_{\mu\nu}.$$

(*3 credits*)

(b) Show that the result from part (a) simplifies to

$$G^{-1} = \phi^{-5\beta-1} g^{-1}.$$

(*1 credit*)

(c) Use your result from part (b) to calculate $\sqrt{-G}$.

(*1 credit*)

(d) We have

$$S_4 = -\frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} R_4.$$

Use the result from part (c) to show that $\beta = -\frac{1}{3}$. What do you need to know about R_5 and R_4? Identify κ_4.

(*3 credits*)

(e) Show that 5 dimensional general coordinate transformations

$$G_{MN} \to \frac{\partial x^K}{\partial x'^M} \frac{\partial x^L}{\partial x'^N} G_{KL}$$

induce 4 dimensional gauge transformations when reparametrizing the circle coordinate as

$$y \to y + \lambda(x^\mu), \qquad x^\mu \to x^\mu.$$

(*2 credits*)

21. Fermions in higher dimensions (*10 credits*)

(a) Generalizations of gamma matrices to higher dimensions have to satisfy

$$\{\Gamma^M, \Gamma^N\} = 2\eta^{MN}\mathbb{1}$$

where we use the convention $\eta^{MN} = \mathrm{diag}(-, +, \dots, +)$, $M, N = 0, \dots, D-1$. Define in addition an analogue to γ^5 in 4 dimensions

$$\Gamma = i^\alpha \Gamma^0 \dots \Gamma^{D-1}$$

where $\alpha \in \mathbb{Z}$. Show that Γ anticommutes with Γ^M if D is even and commutes if D is odd.

(*2 credits*)

(b) Show that $\Gamma \propto \mathbb{1}$ if D is odd and determine α by the requirement $\Gamma^2 = \mathbb{1}$.

(*2 credits*)

(c) Assume D is even and show that

$$\left[\Gamma, \Sigma^{MN}\right] = 0$$

with

$$\Sigma^{MN} = \frac{i}{4}\left[\Gamma^M, \Gamma^N\right].$$

Interpret this result.

(*2 credits*)

(d) The 5 dimensional action for fermions is

$$S_5 = \int d^5x \bar{\Psi}\Gamma^M \partial_M \Psi, \qquad \bar{\Psi} = \Psi^\dagger \Gamma^0.$$

Compactification on a circle S^1 will not lead to a chiral theory in 4 dimensions. Why? How can you realize chirality in 4 dimensions?

(*4 credits*)