Physikalisches Institut
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Exercise 00
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Exercises on String Theory I

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–Class Exercises–

Exercise 0.0: Nambu-Goto– and Polyakov–like action for point particles

In this exercise we examine the action of point particles. As in string theory, the action is given by the area that is swept out by the particle. We start with the Nambu-Goto–like action for a point particle with mass m and coordinates $x^\mu(\tau)$,

$$S_{\mathrm{NG}} = -m \int d\tau \, \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} \, . \tag{1}$$

(a) Show that the action is invariant under worldline reparametrizations $\tau \mapsto \tau'(\tau)$ and spacetime reparametrizations $x^\mu \mapsto x^{\mu\prime}(x)$.

(b) Assume that the time parameter is affine, i.e. $g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu \equiv \dot{x}^2 = \text{const.}$ Derive the geodesic equation as the equation of motion.

Now we consider the Polyakov–like action. For this we introduce an auxiliary metric h on the worldline such that $ds^2 = h_{\tau\tau} d\tau^2$. The new action reads

$$S_{\mathrm{P}} = \frac{1}{2} \int d\tau \, \sqrt{h_{\tau\tau}} \left(h_{\tau\tau}^{-1} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - m^2 \right) \tag{2}$$

(c) Show that S_{P} is again invariant under reparametrizations of the worldline.

(d) Assume $m \neq 0$. Derive the Nambu–Goto action S_{NG} from S_{P}.

(e) Show that massless particles move on lightlike geodesics.

Exercise 0.1: Energy–Momentum tensor

In this exercise we examine the Energy–Momentum tensor in curved space-time. It is defined as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} \, , \tag{3}$$

where $g := \det(g_{\mu\nu})$.

(a) For calculating $T_{\mu\nu}$ we often need the functional derivative of g. To caluclate it show that $\det e^M = e^{\mathrm{tr}\, M}$ for some invertible matrix M. *Hint: Use Jordan Normal Form.*

(b) Use the formula of (a) to calculate $\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}}$.

(c) Calculate $T_{\mu\nu}$ for $S^\Lambda = \int d^4x \sqrt{-g}\,(-\Lambda)$ and for S_{NG} in (1).