Physikalisches Institut
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Theoretische Physik

Exercise 6
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WS 13/14

Exercises on Theoretical Particle Physics

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H 6.1 Vector- and Axial Gauge Couplings

2+2.5+2.5+2 = 9 points

We consider the fermion kinetic terms in the standard model (SM) Lagrangian, which were already given in exercise H 5.2. Here it suits better to work with Dirac spinors, hence we write

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = P_L \begin{pmatrix} \Psi_\nu \\ \Psi_e \end{pmatrix} , \qquad R = e_R = P_R \Psi_e \, ,$$

where we have used the chiral projection operators $P_{L/R} = \frac{1}{2}\left(1 \mp \gamma^5\right)$.

(a) Use the covariant derivative formulated in terms of the propagating gauge fields $W^\pm_\mu$, Z_μ and A_μ (see exercise H 5.2 (h)) to rewrite the kinetic terms

$$\mathscr{L}_{\text{Lept}} = \overline{R}(\mathrm{i}\gamma^\mu D_\mu)R + \overline{L}(\mathrm{i}\gamma^\mu D_\mu)L$$

in a more explicit way. You should arrive at

$$\begin{aligned} \mathscr{L}_{\text{Lept}} = & \, \overline{\Psi}_e \left(\mathrm{i}\gamma^\mu\partial_\mu\right)\Psi_e + \overline{\Psi}_\nu \left(\mathrm{i}\gamma^\mu\partial_\mu\right) P_L \Psi_\nu \\ & + eA_\mu \overline{\Psi}_e \gamma^\mu \Psi_e \\ & - \frac{g}{\sqrt{2}} W^+_\mu \overline{\Psi}_\nu \gamma^\mu P_L \Psi_e + \text{h.c.} \\ & + Z_\mu \Big(\overline{\Psi}_e \gamma^\mu \left(c^e_V + c^e_A \gamma^5\right) \Psi_e + \overline{\Psi}_\nu \gamma^\mu \left(c^\nu_V + c^\nu_A \gamma^5\right) \Psi_\nu \Big) \, . \end{aligned} \tag{1}$$

From this we see that the photon couples non-chirally to the electron, whereas the $W^\pm$ bosons couple the neutrino only to the left-chiral part of the electron. The Z couples to a mixture of the vector- and the axial current. Determine the respective coupling constants c^i_V and c^i_A for $i = e, \nu$.

(b) Draw the interaction vertices and give the vertex factors.

(c) Now we introduce quark fields.

	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$D = d_R$	$U = u_R$
Hypercharge Y	$1/3$	$-2/3$	$4/3$
$\mathrm{SU}(2)_L$ rep.	**2**	**1**	**1**
Lorentz rep.	$(1/2, 0)$	$(0, 1/2)$	$(0, 1/2)$

Perform the above analysis for the quarks using the quark kinetic terms

$$\mathscr{L}_{\text{Quark}} = \overline{Q}(\mathrm{i}\gamma^\mu D_\mu)Q + \overline{D}(\mathrm{i}\gamma^\mu D_\mu)D + \overline{U}(\mathrm{i}\gamma^\mu D_\mu)U\,.$$

What are the electric charges of the up and down quarks?

(d) Out of the gauge kinetic terms for the gauge bosons one also obtains some interactions. In the standard model one has interaction terms for the carriers of the $\mathrm{SU}(2)_L$ symmetry: W^a_μ $(a = 1, 2, 3)$. Why is this not the case for B_μ? Write the W^a_μ in terms of the propagating fields $W^\pm_\mu$, Z_μ and A_μ and identify the interaction vertices.

H 5.3 Fermion Mass Eigenstates *1.5+2+1.5+2.5+1.5+2 = 11 points*

Here we want to investigate the mass generation mechanism for the fermions in the standard model. Note first that since all SM fermions are chiral, we can not write gauge invariant mass terms for them. Instead, we can employ the Higgs mechanism to generate their masses. Here we assume to have N generations of quarks and leptons, so we label our fermion fields as L_i, Q_i, R_i, U_i and D_i, with the index i running from 1 to N.

(a) The electron Yukawa couplings read

$$\mathscr{L} \supset -G_e^{ij}\overline{L}_i \Phi R_j - \text{h.c.}\,,$$

where G_e^{ij} are some general $N \times N$ matrices known as *Yukawa matrices*. Using biunitary transformations: $e^i_R \to V_e^{ij} e^j_R$ and $e^i_L \to U_e^{ij} e^j_L$ one can diagonalize the Yukawa matrices. Show that after spontaneous symmetry breaking the masses for the electrons are $m^i_e = \frac{\lambda^i_e v}{\sqrt{2}}$ where λ^i_e are the eigenvalues.

(b) Perform the transformation on the interaction terms in (1). The index structure is here diagonal, i.e. the family indices are contracted with a Kronecker delta. Show that the interactions with A_μ and Z_μ stay diagonal. Show that with a proper transformation of the neutrinos also the $W^\pm_\mu$ terms stay diagonal. Why is this transformation on the neutrinos allowed? This shows us that within the standard model there is no mixing between the leptons.

(c) Now we want to consider quark masses. We can immediately write down analogous Yukawa terms for the down quarks,

$$\mathscr{L} \supset -G_d^{ij}\overline{Q}_i \Phi D_j - \text{h.c.}\,.$$

Show that they are gauge invariant.

(d) In order to write down a Yukawa coupling for the up-quark we consider the following field $\tilde{\Phi} := \mathrm{i}\sigma_2\Phi^*$. Show that $\tilde{\Phi}$ transforms as a doublet (**2**) of $\mathrm{SU}(2)_L$. What is the hypercharge of $\widetilde{\Phi}$? Use these results to show that the coupling

$$\mathscr{L} \supset -G_u^{ij}\overline{Q}_i\tilde{\Phi}U_j - \text{h.c.}\,.$$

is indeed gauge invariant. Show that inserting the Higgs VEV results in mass matrices for the up quarks. *Hint: You may first want to show that* $\sigma^2\sigma^i\sigma^2 = -\sigma^{i*}$ *and then write* $U \in \mathrm{SU}(2)_L$ *as* $U = \exp\{i\vec{a}\cdot\vec{\sigma}\}$.

(e) Again we perform biunitary transformations on the left- and right-handed up- and down quarks to diagonalize G_d and G_u. Then we insert the mass eigenstates into the gauge interactions. Show that the couplings to A_μ and Z_μ stay diagonal. This fact is known as the absence of flavour changing neutral currents (FCNC).

(f) Now we investivate the charged currents. Show that the couplings to the $W_\mu^\pm$ now are mixed by a generally non-diagonal matrix, called the *Cabibbo–Kobayashi–Maskawa* matrix V_{CKM}. Identify this matrix in terms of the unitary transformation matrices.