Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercises 1
17.10.2016
WS 2016/17

Exercises on Theoretical Particle Physics I

Prof. Dr. H.P. Nilles

Due 24.10.2016

1. Natural units (*5 credits*)

Using the system of natural units ($\hbar = c = k_B = 1$) express the following quantities in GeV or powers of GeV:

(a) $1\,\mathrm{K}$

(*1 credit*)

(b) $1\,\mathrm{g}$

(*1 credit*)

(c) $1\,\mathrm{cm}$

(*1 credit*)

(d) $1\,\mathrm{mb}$ (millibarn)

(*1 credit*)

(e) Hubble constant $H_0 = 72\,\mathrm{km\ s^{-1}\ Mpc^{-1}}$

(*1 credit*)

2. The Lorentz group Part I (*15 credits*)

The Lorentz group is defined as the set of transformations

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

that leave the bilinear form $\langle x, y\rangle = \eta_{\mu\nu} x^\mu y^\nu$ invariant ($\mu, \nu = 0, \ldots, 3$). Use the mostly negative Minkowski metric

$$(\eta_{\mu\nu}) = \mathrm{diag}\,(1, -1, -1, -1)\,.$$

(a) Show that the bilinear form is invariant under the transformation $\Lambda^\mu{}_\nu$ if

$$\eta_{\mu\nu}\Lambda^\mu{}_\rho\Lambda^\nu{}_\sigma = \eta_{\rho\sigma}.$$

(*2 credits*)

(b) Use part (a) to show that $\det \Lambda = \pm 1$ and $|\Lambda^0{}_0| \geq 1$. Argue that this splits the Lorentz group into four (disconnected) branches. Which branch contains the identity element?

(*5 credits*)

(c) Identify the Lorentz transformations associated to time and parity reversal and relate them to the respective branches from part (b).

(*2 credits*)

(d) The restricted or proper orthochronous Lorentz group generates Lorentz boosts and rotations and is denoted as

$$L^{\uparrow}{}_{+} = \mathrm{SO}^{+}(1,3;\mathbb{R}) = \left\{ \Lambda \in \mathrm{O}(1,3;\mathbb{R}) \mid \det\Lambda = 1, \Lambda^0{}_0 \geq +1 \right\}.$$

Show that this restricted Lorentz group indeed forms a group.

(*4 credits*)

(e) In the neighbourhood of the identity the Lorentz transformation $\Lambda \in L^{\uparrow}{}_{+}$ can be written as

$$\Lambda^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} + \omega^{\mu}{}_{\nu}$$

where ω is an infinitesimal parameter. What conditions must be placed on ω so that the infinitesimal expansion satisfies the criteria from part (a)? Calculate the variation of a four-vector $\delta x^{\mu} = x'^{\mu} - x^{\mu}$.

(*2 credits*)