Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercises 8
5.12.2016
WS 2016/17

Exercises on Theoretical Particle Physics I

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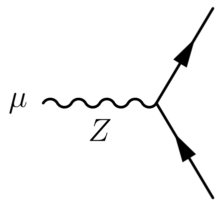
DUE 12.12.2016

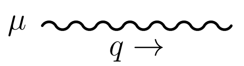
14. Electron-Positron annihilation reloaded *(20 credits)*

(a) The Feynman rules for Z bosons are



$$= -\frac{ig}{\cos\theta_W}\gamma^\mu \frac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)$$

and



$$= \frac{-i\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2}\right)}{q^2 - M_Z^2}.$$

For charged leptons there is $c_A^f = -1/2$ and $c_V^f = -1/2 + 2\sin^2\theta_W$, where θ_W is the electroweak mixing angle. One may further define $g_V = 1 - 4\sin^2\theta_W = -2c_V^f$. Consider the high energy limit and assume negligible lepton masses $m_e = m_\mu = 0$. Draw the Feynman graph in analog to part (a) of exercise 11 for the process $e^-e^+ \to \mu^-\mu^+$ for the tree level exchange of a photon or a Z boson. Use the Feynman rules to determine the matrix element for the Z boson exchange $\mathcal{M}_Z$.

(3 credits)

(b) Show

$$\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5) = -4i\epsilon^{\mu\nu\rho\sigma}.$$

(1 credit)

(c) Use part (a), part (b) and the result for $\mathcal{M}_\gamma$ from exercise 11 and exercise 12 to calculate

$$\sum_{\text{spins}} |\mathcal{M}_\gamma + \mathcal{M}_Z|^2$$
$$= \left[32\frac{e^4}{s^2} + 4\frac{e^2g^2(g_V^2+1)}{s(s-M_Z^2)\cos^2\theta_W} + \frac{g^4(g_V^4+6g_V^2+1)}{8(s-M_Z^2)^2\cos^4\theta_W}\right](p_2\cdot p_4)(p_1\cdot p_3)$$
$$+ \left[32\frac{e^4}{s^2} + 4\frac{e^2g^2(g_V^2-1)}{s(s-M_Z^2)\cos^2\theta_W} + \frac{g^4(g_V^2-1)^2}{8(s-M_Z^2)^2\cos^4\theta_W}\right](p_1\cdot p_4)(p_2\cdot p_3).$$

(10 credits)

(d) Simplify the result from part (c) with the help of the relations

$$R(s) = \frac{s}{(s - M_Z^2)\sin^2(2\theta_W)}, \qquad e = g\sin\theta_W$$

where $R(s)$ is called the resonance factor.

(1 credit)

(e) Assume that the scattering angle in the center-of-mass frame is θ. Use part (d) to derive the differential cross section.

(4 credits)

(f) Derive the total cross section σ.

(1 credit)