Abundance of Thermal WIMPs in Non-standard Cosmological Scenarios

Mitsuru Kakizaki (Bonn Univ.)

September 27, 2007

First Annual School of EU Network
"UniverseNet" – The Origin of the Universe

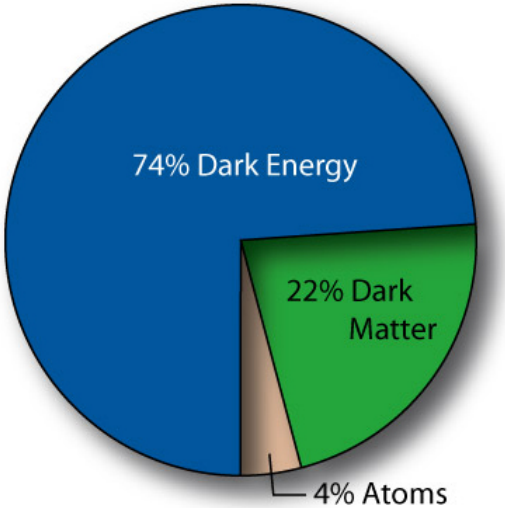
In collaboration with

- Manuel Drees (Bonn Univ.)
- Hoernisa Iminniyaz (Univ. of Xinjiang)

Refs:

- PRD73 (2006) 123502 [hep-ph/0603165]
- arXiv:0704.1590 [hep-ph], to appear in PRD

1. Motivation



[http://map.gsfc.nasa.gov]

- Observations of
  - cosmic microwave background
  - structure of the universe
  - etc.

→ **Non-baryonic dark matter:** $0.08 < \Omega_{\rm DM} h^2 < 0.12$

- Weakly interacting massive particles (WIMPs) χ are good candidates for cold dark matter (CDM)

The predicted thermal relic abundance naturally explains the observed dark matter abundance: $\Omega_{\chi,\rm standard} h^2 \sim 0.1$

- Neutralino (LSP); 1st KK mode of the B boson (LKP); etc.

Investigation of early universe using CDM abundance

- The relic abundance of thermal WIMPs is determined by the Boltzmann equation: $\dot{n}_\chi + 3Hn_\chi = -\langle\sigma_{\rm eff} v\rangle(n_\chi^2 - n_{\chi,{\rm eq}}^2)$

  (and the reheat temperature: T_R)

- The (effective) cross section $\sigma_{\rm eff}$ can be (hopefully) determined from collider and DM detection experiments

We can test the standard CDM scenario and investigate conditions of very early universe: $T_R, H, \cdots$

- Standard scenario or non-standard scenarios?

  [Scherrer; Salati; Rosati; Profumo, Ullio; Pallis; Maisero, Pietroni, Rosati; ...]

Outline

This work

- We provide an approximate analytic treatment that is applicable to low-reheat-temperature scenarios
- Based on the assumption that dark matter is made of thermal WIMPs,
  - we derive the lower bound on the reheating temperature
  - we constrain possible modifications of the Hubble parameter

c.f. Cosmic p^{+}, γ $\Rightarrow$ Bounds on pre-BBN expansion

[Schelke, Catena, Fornengo, Masiero, Pietroni PRD74 (2006); Donato, Fornengo, Schelke, JCAP0703 (2007)]

1. Motivation
2. Standard calculation of WIMP relic abundance
3. Low-temperature scenario
4. Constraints on the very early universe from WIMP dark matter
5. Summary

2. Standard calculation of the WIMP relic abundance

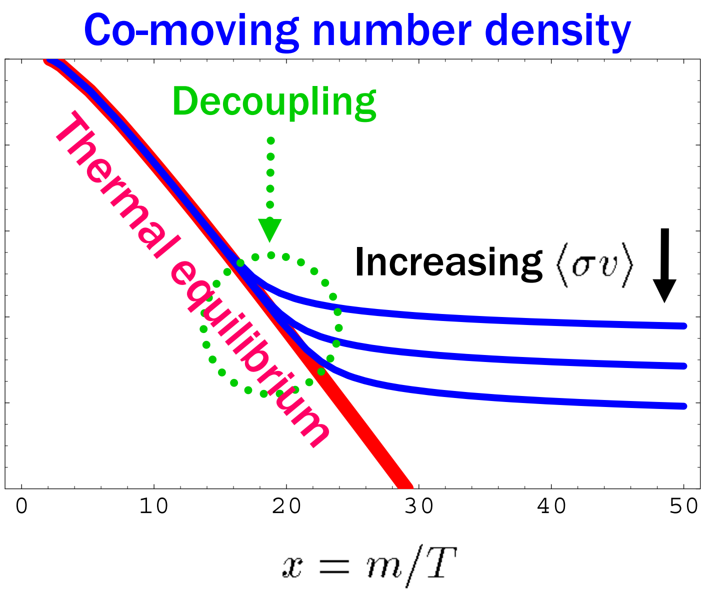
[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); …]

- Conventional assumptions:
  - $\chi = \bar{\chi}$, single production of χ is forbidden
  - Thermal equilibrium was maintained:

    $T_R(\text{Reheat temp}) \geq T_F(\text{Freezeout temp})$
- For adiabatic expansion the Boltzmann eq. is

$$\frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = -\frac{\langle\sigma v\rangle s}{Hx}(Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2),$$

$$Y_{\chi(,\mathrm{eq})} = \frac{n_{\chi(,\mathrm{eq})}}{s}, x = \frac{m_\chi}{T}$$



- χ decoupled when they were non-relativistic in RD epoch:

$$\langle\sigma v\rangle = a + 6b/x + \mathcal{O}(1/x^2), \quad n_{\chi,\mathrm{eq}} = g_\chi \; (m_\chi T/2\pi)^{3/2} \mathrm{e}^{-m_\chi/T}$$

$$\boxed{\Omega_{\chi,\mathrm{standard}} h^2 \simeq 0.1 \times \left(\frac{a + 3b/x_F}{10^{-9}\ \mathrm{GeV}^{-2}}\right)^{-1} \left(\frac{x_F}{22}\right) \left(\frac{g_*}{90}\right)^{-1/2} \sim \Omega_{\mathrm{DM}} h^2}$$

3. Low-temperature scenario

- **Assumptions:** $T_R \leq T_F;\ Y_\chi(x_0) = 0\,,\ x_0 = m_\chi / T_R$
- **Zeroth order approximation:** For $T_R \ll T_F$, χ annihilation is negligible:

$$\boxed{\frac{dY_0}{dx} = 0.028\, g_\chi^2 g_*^{-3/2} m_\chi M_{\rm Pl} e^{-2x} x \left(a + \frac{6b}{x}\right)}$$

The solution Y_0 is proportional to the cross section:

- **First order approximation:**
  - Add a correction term describing annihilation to Y_0 : $Y_1 = Y_0 + \delta\ (\delta < 0)$
  - As long as $|\delta| \ll Y_0$, the evolution equation for δ is

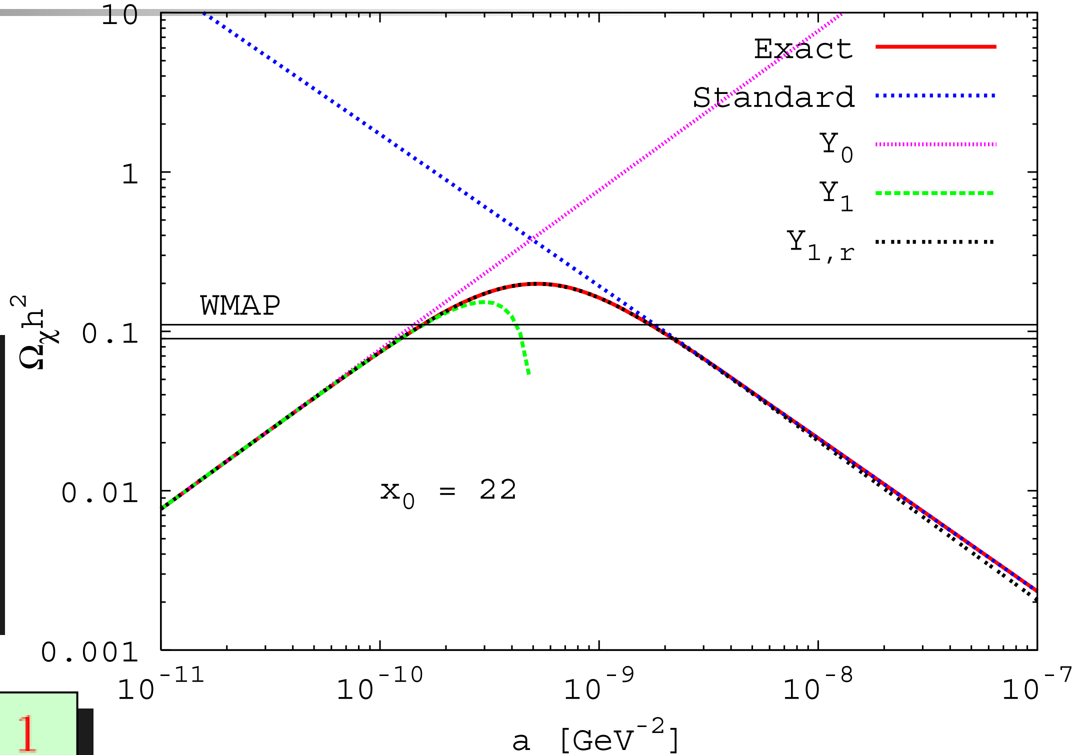
$$\boxed{\frac{d\delta}{dx} = -1.3\,\sqrt{g_*} m_\chi M_{\rm Pl} \left(a + \frac{6b}{x}\right) \frac{Y_0(x)^2}{x^2}}$$

An analytic expression for δ can be obtained and is proportional to σ^3

Resummed ansatz

- We can obtain an excellent analytic formula for $T_R \leq T_F$ through resummation:

$$Y(x) = Y_0 + \delta = Y_0\left(1 + \frac{\delta}{Y_0}\right) \simeq \frac{Y_0}{1-\delta/Y_0} \equiv Y_{1,r}$$



$x_0 \to x_F$ **Standard formula**

- For $|\delta| \gg Y_0$, $Y_{1,r}(x) \simeq -\frac{Y_0^2}{\delta} \propto \frac{1}{\sigma}$

At late times, $Y_{1,r}(x \to \infty) = \dfrac{x_0}{1.3\sqrt{g_*}m_\chi M_{\rm Pl}(a + 3b/x_0)}$

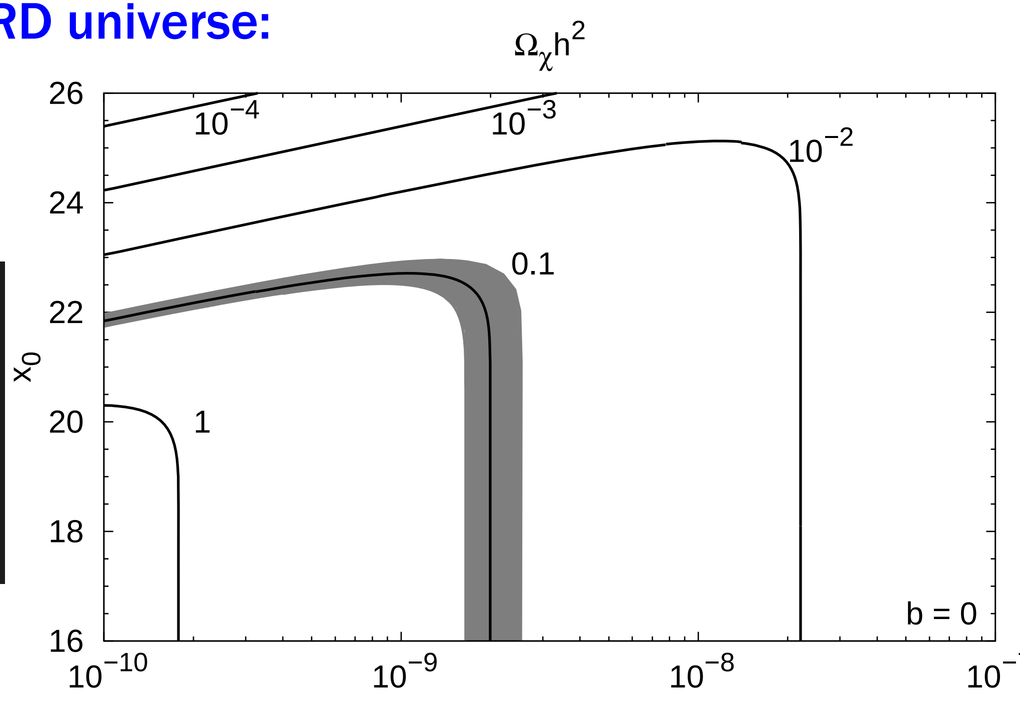
4. Constraints on the very early universe from WIMP DM

- **Out-of-equilibrium case:** $\sigma \nearrow \Rightarrow \Omega h^2 \nearrow$; $T_0 = m_\chi / x_0 \nearrow \Rightarrow \Omega h^2 \nearrow$

  Equilibrium case: $\sigma \nearrow \Rightarrow \Omega h^2 \searrow$; $\Omega_\chi h^2$ is independent of T_R

- **Thermal relic abundance in the RD universe:**

$0.8 < \Omega_{\rm DM} h^2 < 0.12$



Requirement that $\Omega_\chi h^2 \simeq 0.1$

$\Downarrow$

Lower bound on the reheat temperature: $T_R > m_\chi / 23$

Modified expansion rate

- Various cosmological models predict a non-standard early expansion

  ⟹ Predicted WIMP relic abundances are also changed

- Idea:

  Once we know σ as well as $\Omega_{\rm DM}h^2$,
  we can constrain the expansion rate at around WIMP decoupling
  within the framework of thermal WIMP production

- Parametrization: $A(z) = H_{\rm st}(z)/H(z)\,, \quad z \equiv T/m_\chi = 1/x$

- We need to know $A(z)$ only for $z_{\rm BBN} = 10^{-5} - 10^{-4} \le z \le z_F \sim 1/20 \ll \mathcal{O}(1)$

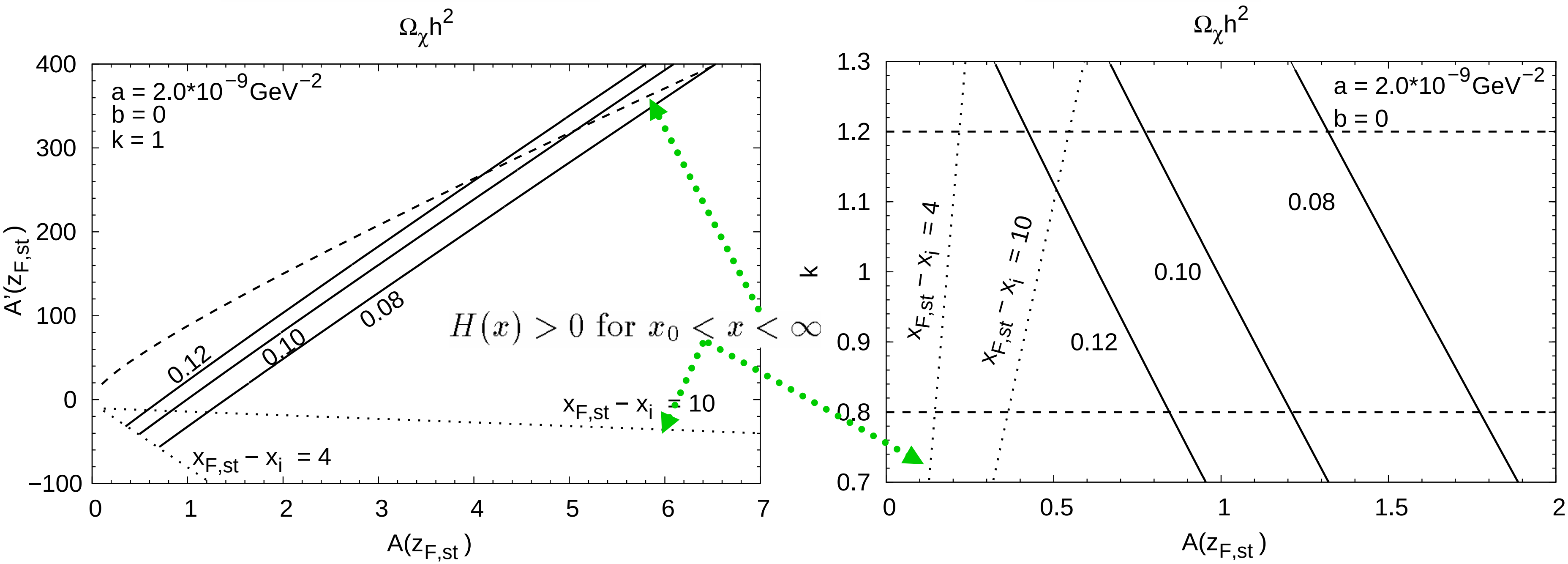
  ⟹ Taylor expansion of $A(z)$ in powers of $(z - z_{F,\rm st})$:

$$A(z) = A(z_{F,\rm st}) + (z - z_{F,\rm st})A'(z_{F,\rm st}) + \frac{1}{2}(z - z_{F,\rm st})^2 A''(z_{F,\rm st})$$

  subject to the BBN limit: $0.8 \le k \equiv A(z \to z_{\rm BBN}) \le 1.2$

Constraints on modifications of the Hubble parameter

- **Case for** $k \equiv A(z \to z_{\mathrm{BBN}}) = 1$
- **Case for** $A''(z_{F,\mathrm{st}}) = 0$



x_i : **Maximal temperature where our expansion is valid**

$\Omega_\chi h^2$ depends on all $H(T_{\mathrm{BBN}} < T < T_F)$ ⇨ Weaker constraints on $H(T_F)$

5. Summary

- Using the CDM relic density we can examine very early universe at around $T \sim m_\chi/20 \sim \mathcal{O}(10)\ \mathrm{GeV}$ (well before BBN $T_{\mathrm{BBN}} \sim \mathcal{O}(1)\ \mathrm{MeV}$)

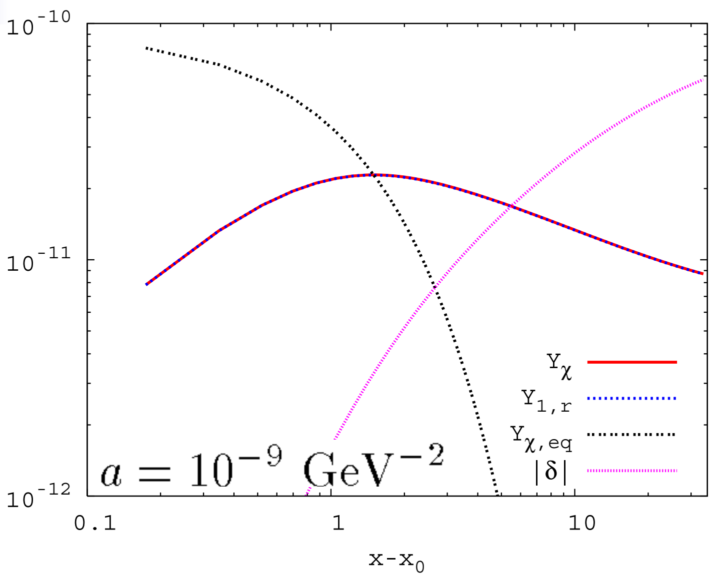
- We find an approximate analytic formula that is valid for all $T_R \leq T_F$

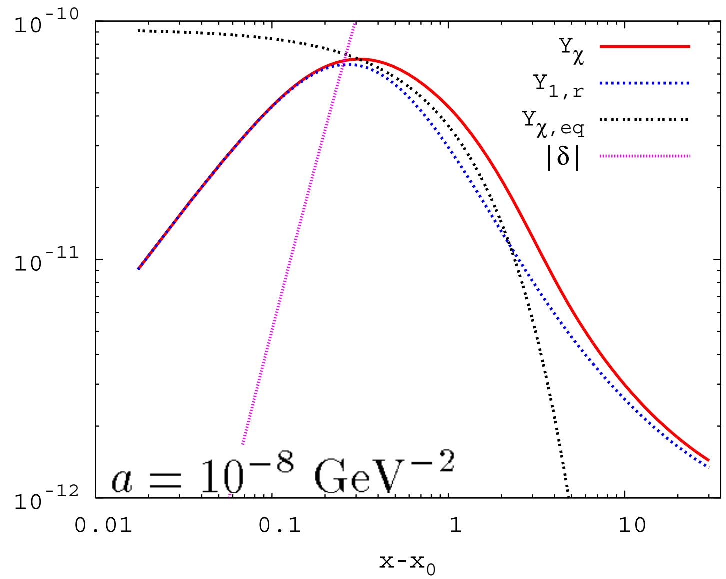
- By requiring $\Omega_{\chi,\mathrm{thermal}} h^2 = \Omega_{\mathrm{DM}} h^2$,
  we found the lower bound on the reheat temperature: $T_R > m_\chi/23$

- The sensitivity of $\Omega_{\chi,\mathrm{thermal}} h^2$ on $H(T_F)$ is weak
  because $\Omega_{\chi,\mathrm{thermal}} h^2$ depends on all $H(T_{\mathrm{BBN}} < T < T_F)$

Backup slides

Evolution of solutions

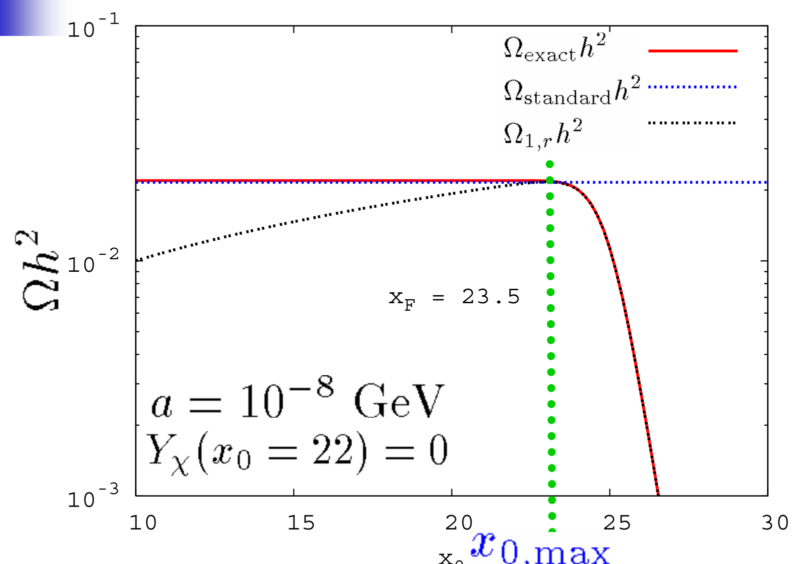


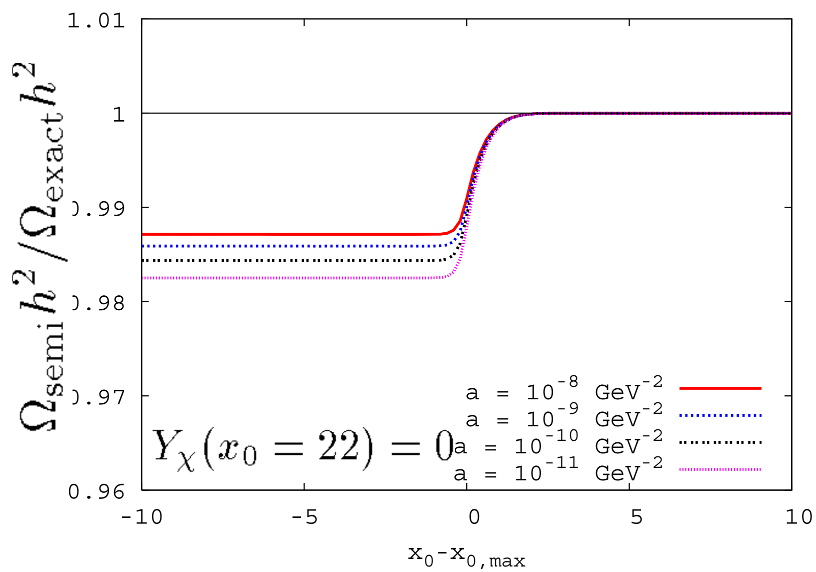


Y_χ : Exact result, $Y_{1,r}$: Re-summed ansatz, $b = 0,\ Y_\chi(x_0 = 22) = 0$

- The re-summed ansatz $Y_{1,r}$ describes the full temperature dependence of the abundance when equilibrium is not reached
- For larger cross section the deviation becomes sizable for $x - x_0 \sim 1$, but the deviation becomes smaller for $x \gg x_0$

Semi-analytic solution





- $Y_{1,r}(x_0, x \to \infty)$ $(\propto \Omega_{1,r} h^2)$ has a maximum (left)
- New semi-analytic solution can be constructed: $\Omega_{\rm semi} h^2$ (right)

For $x_0 > x_{0,\max}$, use $Y_{1,r}(x_0)$; for $x_0 < x_{0,\max}$, use $Y_{1,r}(x_{0,\max})$

The semi-analytic solution $\Omega_{\rm semi} h^2$ reproduces
the correct final relic density $\Omega_{\rm exact} h^2$ to an accuracy of a few percent