Potentially Large One-loop Corrections of Dark Matter Relic Density

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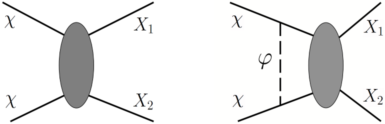
based on M. Drees, JMK, and K. Nagao
arXiv:0911.3795

Outline

Formalism: Sommerfeld enhancement [Iengo: 0902.0688]

$\chi(p_1)+\chi(p_2)\rightarrow X_1(p_1')+\bar{X}_2(p_2')$.
$(P=\frac{p_1+p_2}{2}=\frac{p_1'+p_2'}{2}, p=\frac{p_1-p_2}{2}, p'=\frac{p_1'-p_2'}{2};$
in the CM $P_0=\sqrt{p^2+m^2}, \vec{P}=0, p_0=0)$



$A(p,p';P_0)_L$: Amplitude for the annihilation from the partial wave ("L") of two χ by a boson exchange,

$A(|\vec{p}|,p')_L=A_{0,L}(|\vec{p}|,p')+\delta A(|\vec{p}|,p')_L$,

$$\delta A_L(|\vec{p}|,p')=ig^2\bar{v}(p_2)\int\frac{d^4q}{(2\pi)^4}\frac{\not{q}-\not{P}+m_\chi}{(q-P)^2-m_\chi^2+i\epsilon}(\gamma_5)^{n_L}\frac{\not{q}+\not{P}+m_\chi}{(q+P)^2-m_\chi^2+i\epsilon}$$
$$\times\frac{1}{(p-q)^2-\mu^2+i\epsilon}\tilde{A}_{0,L}(|\vec{q}|,p')\,u(p_1)$$

Let us consider the S-wave annihilation (n_L=0):

- NR limit 1:
$$\frac{1}{k^2-\mu^2} = -\frac{1}{\vec{k}^2+\mu^2} + \frac{k_0^2}{(k^2-\mu^2)(\vec{k}^2+\mu^2)}$$
$$\simeq -\frac{1}{\vec{k}^2+\mu^2}, \text{ where } k = p - p'.$$

  Using
  $\Lambda^{\pm} = \frac{\omega \pm H}{2\omega}$; $H = \beta\vec{\gamma}\cdot\vec{q} + \beta m$ $\omega = \sqrt{m^2+q^2}$,
$$\frac{(\not{P}+\not{q}+m)_1}{(P+q)^2-m^2+i\epsilon}\frac{(\not{P}-\not{q}+m)_2}{(P-q)^2-m^2+i\epsilon} =$$
$$\left(\frac{\Lambda_1^+(\vec{q})}{q_0+P_0-\omega+i\epsilon} + \frac{\Lambda_1^-(\vec{q})}{q_0+P_0+\omega-i\epsilon}\right)\gamma_1^0\left(\frac{-\Lambda_2^-(-\vec{q})}{q_0-P_0-\omega+i\epsilon} + \frac{-\Lambda_2^+(-\vec{q})}{q_0-P_0+\omega+i\epsilon}\right)\gamma_2^0.$$

- NR limit 2:
  The residue at $q_0 = \omega - P_0$:
$$\Lambda_1^+(\vec{q})\left(\frac{\Lambda_2^-(-\vec{q})}{2P_0} - \frac{\Lambda_2^+(-\vec{q})}{2(\omega-P_0)}\right)\gamma_1^0\gamma_2^0 \to -\frac{\Lambda_1^+(\vec{q})\Lambda_2^+(-\vec{q})}{2(\omega-P_0)}\gamma_1^0\gamma_2^0.$$

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$$\Rightarrow A(\vec{p},p';P_0) = A_0(\vec{p},p';P_0) + \frac{g^2}{(2\pi)^3}\int \frac{dq^3}{(\vec{p}-\vec{q})^2+\mu^2}\frac{A(\vec{q},p';P_0)}{2(\omega-P_0)}.$$

Solving it iteratively for the massless vector boson gives us Sommerfeld enhancement: $|A_s|^2 = \frac{2\pi\alpha}{v}|A_0|^2$.

- Maximal if $\mu = 0, v = 0$.
- Can enhance σ a lot.
  - γ from galactic center. [Hisano et al.(2005)]
  - PAMELA/Fermi anomaly. [Arkani-Hamed et al.(2009), Lattanzi & Silk(2009), ...]

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Formalism: Dark Matter annihilation

Consider Higgs exchange between neutralino DMs. Replace $A(\tilde{p},p';P_0)$ by $A_0(\tilde{p},p';P_0)$
$\Rightarrow A(\tilde{p},p';P_0) = A_{0,S}(1+\frac{g^2}{(2\pi)^3}\int\frac{dq^3}{(\vec{p}-\vec{q})^2+\mu^2}\frac{1}{2(\omega-P_0)})$.

Taking NR limits and using $x=|\vec{q}|/|\vec{p}|$,
$\delta A=\frac{g^2}{4\pi^2 v}I_S\cdot A_0$,
where $I_S(r)=\int\frac{xdx}{(x^2-1)}ln\frac{(1+x)^2+r}{(1-x)^2+r}$, $r=\mu^2/p^2$.
Cf. P-waves: Extra factor of $-\frac{\vec{\gamma}\cdot\vec{q}}{2m}$. (E.g. A^0 exchange)

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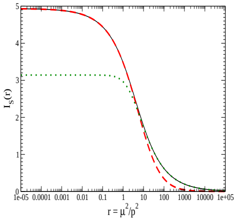
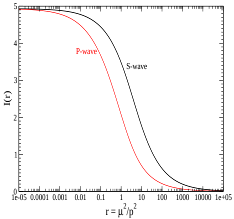
Numerically,

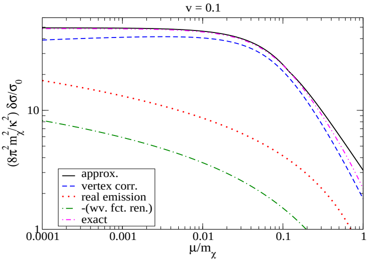
- S-wave:
  - large r (green):$I(r) = \frac{2\pi}{\sqrt{r+1}}(\frac{r+1}{r+2})$.
  - small r (red):$I(r) = \frac{\pi^2/2}{1+\sqrt{r}/\pi+r/\pi^2}$.
- P-wave:
  - large r:$I(r) = \frac{2\pi}{3\sqrt{r+1}}(1 + \frac{1.3}{r+1})$.
  - small r:$I(r) = \frac{\pi^2/2}{1+3\sqrt{r}/\pi+r/\pi^2}$.





Comparison to a full one-loop calculation

(A full one-loop calculation for a purely scalar theory)
$=$ (Vertex correction) $+$ (Wave-function renormalization)
$+$ (Real emission)



Dark Matter Relic Density

1) Standard computation

WIMP annihilations to SM particles: $\chi\chi \leftrightarrow f_1 f_2$

The Boltzman equation is written as:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\sum_{f1,f2} \int d(PS)(2\pi)^4 \delta^{(4)}(p_{\chi_1} + p_{\chi 2} - p_{f_1} - p_{f_2}) \cdot$$

$$\cdot \Big[|M(\chi\chi \to f_1 f_2)|^2 f_\chi(E_{\chi 1}) f_\chi(E_{\chi 2})(1 \pm f_1)(1 \pm f_2)$$

$$- |M(f_1 f_2 \to \chi\chi)|^2 f_1(E_{f_1}) f_2(E_{f_2})(1 \pm f_\chi)(1 \pm f_\chi) \Big],$$

with $n_\chi = \frac{g_\chi}{(2\pi)^3} \int d^3 p_\chi f_\chi$.

CP invariance and CDM $\Rightarrow \frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{ann} v \rangle (n_\chi^2 - n_{\chi,0}^2)$.

NR limit

$\Rightarrow \langle \sigma_{ann} v \rangle \simeq (\frac{m_\chi}{T})^{3/2} \frac{1}{2\sqrt{\pi}} \int dv v^2 e^{-m_\chi v^2/4T} \sigma_{ann}$, where

$\sigma_{ann} v = A + Bv^2 + \cdots$

$\Rightarrow \langle \sigma_{ann} v \rangle = A + \frac{6BT}{m_\chi}$.

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Using $Y_\chi = n_\chi / s$,

$$Y_\chi = \frac{1}{1.32\sqrt{g_*} m_\chi M_P J(x_F)},$$

where $J(x_F) = \int_{x_F}^{\infty} dx x^{-2} \langle \sigma v \rangle$.
The present relic density of WIMPs:

$$\Omega_\chi h^2 = \frac{8.5 \times 10^{-11} x_F GeV^{-2}}{\sqrt{g_*} J(x_F)}.$$

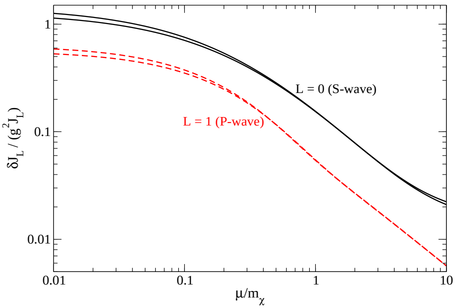
2) Including one-loop corrections

$\sigma_L = \sigma_{0,L} + \delta\sigma_L$

$(\frac{\sigma v}{\sigma_0 v})_S = \frac{2A_0 \delta A}{|A_0|^2} = \frac{g^2}{2\pi^2 v} I_S,$

$\langle \delta\sigma v \rangle_S = \langle \sigma_0 v \rangle_S \cdot \frac{x^{3/2}}{2\sqrt{\pi}} \cdot \langle \frac{I_S}{v} \rangle \cdot \frac{g^2}{2\pi^2},$

where $\langle \frac{I_S}{v} \rangle = \int v^2 \cdot \frac{I_S}{v} e^{-xv^2/4}$, $x = \frac{m_\chi}{T}$



- The corrections are less important for the P-wave annihilation.
- Analytically, $\delta J_L / J_L \propto \sqrt{x_F}$ as $\mu \to 0$, but independent of x_F for $\mu \gtrsim 0.3 m_\chi$.
- The loop corrections are significant only for $\mu \lesssim m_\chi$.

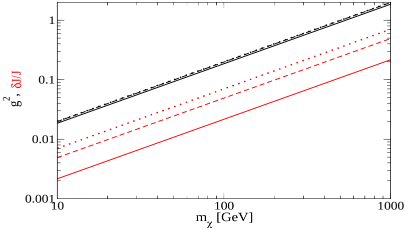
Applications

1) Scalar singlet WIMP [Burgess et al.(2001)]

- $\mathcal{L} \ni -\frac{k}{2}\chi^2|h|^2 \Rightarrow$ trilinear scalar interaction, $V\chi^2\phi$. $(V = 246\mathrm{GeV})$.
- $\Omega_\chi h^2 \simeq \Omega_\chi h^2_{WMAP} \Leftrightarrow k \simeq 0.28 m_\chi/(1TeV)$ [Davoudiasl et al. (2005)].
- $g^2 = k^2V^2/(2m_\chi^2) \simeq 0.0012$.
  $\Rightarrow$ Radiative correction always negligible!

2) Fermionic singlet WIMP [Y. G. Kim et al.(2008)]

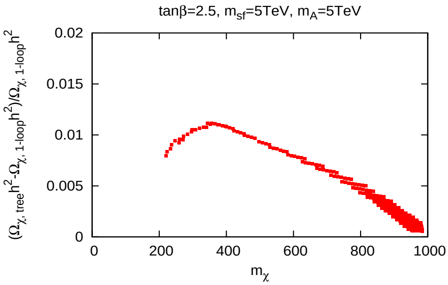
- $\mathcal{L} \ni g\bar{\chi}\chi\phi + A\phi|h|^2$.
- $\Omega_\chi h^2 \simeq \Omega_\chi h^2_{WMAP} \Leftrightarrow g^2 \simeq 0.2m_\chi/(100GeV)$.
- The correction can be as large as 10%!



3) The lightest neutralino in the MSSM

- $\Omega_\chi h^2 \simeq \Omega_\chi h^2_{WMAP}$, with light $\tilde{\chi}^0_1$
  $\Rightarrow$ Mixed state!
  $\Rightarrow$ Well-Tempered Neutralino [Arkani-Hamed et al.(2006)]
- Largest couplings to the Higgs
- One-loop correction maximal!

We fix $M_1 \& |\mu|$ at the weak scale by WMAP and take $M_2 = 2M_1$:



Summary

- A cheap way to include (important) 1-loop corrections: Purely perturbative.
- The corrections are independent on final states, but dependent on the partial wave of initial states.
- A relative size of the corrections to the annihilation integral: model-independent
- Corrections very small for a scalar singlet WIMP, can-be-large for a Dirac fermion singlet WIMP, comparable to PLANCK precision for the neutralino WIMP only if the $m_{\chi_1^0} \simeq 350\text{GeV}$.
- Co-annihilation case work in progress.

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