Theoretical Astro–Particle Physics (SS 25)
Homework no. 10 (July 10, 2025)

1 Description of inflaton decay

In class we had described inflaton decay by adding a term $\Gamma_\phi \dot\phi$ to the left–hand side of the equation of motion for the inflaton field ϕ; the first term of this equation was $\ddot\phi$. Show that this is compatible with the usual definition of the inflaton decay width, $\dot n_\phi = -\Gamma_\phi n_\phi$ in flat space, where n_ϕ is the inflaton number density. *Hint:* Express n_ϕ in terms of ϕ and $\dot\phi$, as in the Klein–Gordon theory for scalar fields.

2 Dilution of inflaton decay products

Assume that inflaton decays produce a (massive) particle X with branching ratio $\epsilon_X \ll 1$, while most inflaton decays directly lead to light SM particles. Estimate the fractional X number density $Y_X \equiv n_x/s,\ s$ being the entropy density, at the end of reheating. Express the result in terms of the reheat temperature T_R defined in class as well as the inflaton mass m_ϕ (and ϵ_X, of course). Assuming the entropy remains constant after the end of reheating, show that this can easily give too large a relic density of stable X particles, unless they thermalize after reheating (in which case the usual freeze–out calculation will determine their relic density). *Hint:* Work in the approximation of instantaneous inflaton decay, where all X particles and the total entropy density are produced simultaneously at time $t = 1/\Gamma_\phi$.

3 Power–law inflation

This type of inflation can e.g. be described by introducing a canonically normalized scalar field ϕ (whose kinetic energy term in the Lagrangian is given by $\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$) with *exponential* potential,

$$V(\phi) = V_o \exp\left(-\phi/M_{\rm Pl}\right) , \tag{1}$$

where V_0 is a constant. Find an *exact* solution for $\phi(t)$ and the scale factor $R(t)$ to the flat–space ($k = 0$) Friedmann equation and the full scalar equation of motion, i.e. *without* taking the slow–roll approximation. Show that this also leads to an “inflationary” epoch, where the scale factor $R(t)$ grows faster than the “Hubble horizon” H^{-1}. *Hint:* Assume that ϕ dominates the energy density of the Universe and has no spatial dependence, and make the ansatz $\phi(t) = \alpha \ln(\beta t)$, where α and β are constants.