Theoretical Astro–Particle Physics (SS 25)
Homework no. 8 (June 26, 2025)

1 Toy Model for Baryogenesis

In class we analyzed a toy model for baryogenesis, which featured two (real) bosons X, Y coupling to four kinds of fermions $f_{1,2,3,4}$; the Lagrangian only contained bilinears $\overline{f_1} f_2$, $\overline{f_3} f_4$ and their hermitean conjugates coupling to X, whereas Y coupled to $\overline{f_1} f_3$ and $\overline{f_2} f_4$ and their hermitean conjugates. Here we analyze a variant of this model featuring only a single real scalar boson, coupling to a larger variety of bilinears:

$$\mathcal{L}_{\Delta B} = X \left(a \overline{f_1} f_2 + b \overline{f_3} f_4 + c \overline{f_1} f_4 + d \overline{f_3} f_2 \right) + h.c. \,, \tag{1}$$

where a, b, c, d are dimensionless complex couplings.

1. Show by explicit calculation that at the tree–level, the partial widths for $X \to f_1 \overline{f_2}$ and $X \to f_2 \overline{f_1}$ are the same, even if f_1 and f_2 have non–vanishing and different masses. *Hint:* Recall that X is real, i.e. X is its own antiparticle.

2. Draw a one–loop diagram for $X \to f_1 \overline{f_2}$ involving the couplings b, c and d (or their complex conjugates).

3. Show that the interference between this diagram and the tree–level diagram has a contribution that is sensitive to some combination of complex phases of the couplings a, b, c, d, if $m_X > m_{f_3} + m_{f_4}$.

4. The phases of the couplings in the Lagrangian (1) are not well defined. Consider phase redefinitions of the form
$$f'_j = \mathrm{e}^{i\alpha_j} f_j \,,$$
where $j \in \{1, 2, 3, 4\}$ is *not* summed. Rewrite the Lagrangian (1) in terms of the f'_j, absorbing the phases into new couplings a', b', c', d'. Show that the complex phase you found in the previous point is *invariant* under this redefinition of phases. *Note:* This must be true for all "physical" (combinations of) phases, i.e. for phases directly appearing in expressions for observables like (differential) cross sections or partial widths. This "rephasing invariance" is in fact a good check on the calculation.

5. The phase transformation introduced in the previous point can also be used to make some of the couplings in the Lagrangian real. How many?

2 Scalar Field Dynamics in the Expanding Universe

Starting from the action for a minimally coupled real scalar field ϕ,

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] \,, \tag{2}$$

where $V(\phi)$ is the potential (which does not contain spacetime derivatives) and g is the determinant of the FRW metric, show that a spatially homogeneous ϕ satisfies the equation of motion

$$\frac{d^2\phi}{dt^2} + 3H\frac{d\phi}{dt} + \frac{dV(\phi)}{d\phi} = 0\,, \tag{3}$$

where $H = (dR/dt)/R$ is the usual Hubble parameter. *Hint:* Use the Euler-Lagrange equation for the action (2), and make use of the fact that $d^3\sqrt{-g} = R(t)^3 d^3\bar{x}$, where the $\bar{x}_i$ are dimensionless spatial coordinates.